Proving Incentive Compatibility in Multi-Attribute Auctions

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by

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Abstract

Proving the incentive compatibility of an auction mechanism is always a hard but essential work in auction mechanism design. In this paper we discuss the strategy proofness of a multi-attribute auction mechanism using three different approaches: the analysis of the mechanism properties, a mathematical analysis to determine if agents can take advantage of dishonest strategies and the use of simulations to study the behaviour of bidders.

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1 Introduction

In auction design, a mechanism is said to be incentive-compatible or strategyproof if all of the participants maximize their utility when they are bidding truthfully, revealing their private information and true values on the bid [1]. This reduces the chances of a bidder trying to manipulate market prices as bidders obtain their maximum benefit when they bid honestly. Proving incentive compatibility in auctions is always laborious. However, this is a must for any researcher providing a new auction mechanism.

There are three main ways of demonstrating that a mechanism fulfills such property. First, by proving that the mechanism satisfies certain sufficient properties or conditions which make truthful bidding the dominant strategy (the strategy which maximizes its revenue if assuming that the rest of bidders are acting rationally) for bidders. Second, by formulating the rules that are used to determine the winners and compute the paymets and the utility of the participants and mathematically derving the demonstration. Third, incentive compatibility can be also derived from empirical experimentation, analyzing the revenue and utilities obtained by bidders who bid honestly and comparing them with those obtained by bidders who bid following other strategies.

These are alternative and complementary mehtods. The sufficient conditions method seemos to be the simplest one, but it is not always neither easy nor intuitive to achieve. Being successfull on the second methodology depends on the complexity of the analyzed mechanism. When these methods cannot be used, the third method is the single alternative.

In this work we explore the uabilitiy of the methods in the partiuclar case of the multi-attribute mechanism described in [2]. In that doing, we introduce as a first time the use of a SMT solver in the second method, saving a lot of time to researchers.

This paper is organized as follows. First, in Section 2 we comment on some related work to contextualize our research. Second, In Section 3 we describe the properties required for a mechanism in order to be incentive compatible. Then, Section 4 provides the details of the multi-attribute auction we are going to analyze. In Section 5 we discuss the strategy proofness of the mechanism through three different approaches. Finally, we end the paper with a summary in Section 6.

2 Related Work

In multi-attribute auctions, a key work is [3], where the author presents different scenarios regarding the payment rule and postulates that in second price multi-attribute auctions, in in order to be incentive compatible, the payment attributes should match the valuation of the second best bid but not necessarily the same attribute combiantion. The mechanism we analyze in this paper [2] follows a similar philosophy.

Some multi-attribute auction mechanisms have proved to be incentive compatible under certain conditions. E.g. [4] proposes an adaptation of the Vickrey-Clarke-Groves [5] (VCG) for multi-attribute auctions under an iterative schema. That means that bidders are allowed to modify their bids in response to the bids from other agents. The analyzed mechanism do not allows these kind of bidding as it uses sealed bids. Other multi-attribute auctions which claim to be incentive compatible only use the non-economic attributes in certain steps of the mechanis. For example, [6] presents a mechanism for auctions with temporal constraints based on VCG with a new payment method. Time constraints are used to filter the participating bids, but time is not considered when evaluating the bids, leaving aside whether time improves a bid or not. Conversely to this work, the analyzed mechanism uses non-economic attributes along the whole auction process.

3 Incentive-compatible properties

For an auction mechanism to be incentive-compatible is sufficient to satisfy the conditions of exactness, participation, monotonicity and criticality[7].

- **Exactness** postulates that a single minded bidder receives exactly the set of goods it desires or nothing. In single-item auctions this means that the bidder will get the item it bidded for or nothing.
- **Participation** requests that unsatisfied bidders pay zero and their utility is zero. In other words, a non-participating or non-winning bidder neither pays nor receives any money.
- **Monotonicity** requires that if a bidder increases its bid (decreases in reverse auctions) the bidder still wins the auction.
- **Criticality** claims that each winning bidder pays (receives in reverse auctions) the lowest value it could have declared and still be allocated the good it requested.

4 The Mechanism: Multiattribute Auction

In a multiattribute auction each bid B_i is characterized by a set of attributes, at_i, in addition to a price b_i, thus $B_i = (at_i, b_i)$. The winner determination problem consist on finding the bid which best satisfies the auctioneer regarding the price but also the rest of attributes according to the auctioneers utility function. As an optimization problem, results depend on the goal of the auctioneer or its objective optimization function, also known as scoring rule [3]. We can use $V(at_i, b_i)$ to denote that function. Thus, the winner determination problem can be defined as:

$$maximize_i(V(at_i, b_i)) \tag{1}$$

We are interested in proving the mechanism defined in [2, 8]: a reverse second price multi-attributte auction mechanism used to allocate tasks to resource providers which bases its payment in the commitment of the agreements made during the auction. In this mechanism, the

	Product	Sum	Weighted Sum
$V(b, f(at))$	$b * f(at)$	$b + f(at)$	$\mu_1 b + \mu_2 f(at)$
TB payment	$\frac{b_2 * f(at_2)}{f(at_1)}$	$b_2 + f(at_2) - f(at_1)$	$\frac{\mu_1 b_1 + \mu_2 t_2 - \mu_2 f(at_1)}{\mu_1}$
UB payment	$\frac{b_1 * f(at_1)}{f(at_1^v)}$	$b_1 + f(at_1) - f(at_1^v)$	$\frac{\mu_1 b_1 + \mu_2 f(at_1) - \mu_2 f(at_1^v)}{\mu_1}$

Table 1: Payment functions (p) in the different cases (TB: truthful bidding; UB: untruthful bidding) when using the product, the sum and the weighted sum as evaluation functions $V(b, f(a_t))$. at_1, b_1, at_2 and b_2 are the components of the winner bid and the second best bid, respectively, and the superscript^v referes to the attributes that the bidder has finally delivered to the auctioneer (e.g. being at^v the real end time of a task whilst at the end time indicated in the bid).

evaluation of the score of a bid is based on an aggregation function $f(at)$ [9] that reduces all the attributes at_i to a single value. Since the mechanism deals with reverse auctions, the goal of the auctioneer is to minimize the economic cost and the rest of attributes (e.g. to minimize the duration of a task) Thus, the winner determination problem consists on selecting the best bid, as follows:

$$mimimize_i(V(f(at_i), b_i)) \tag{2}$$

with $f(at_i) \in \Re$. Note that $f(at_i)$ should decrease when the values of at_i are better.

Three different evaluation functions are proposed: sum (Equation 4), product (Equation 3) and weighted sum (Equation 5).

$$V(f(at_i), b_i) = b_i * f(at_i) \tag{3}$$

$$V(f(at_i), b_i) = b_i + f(at_i) \tag{4}$$

$$V(f(at_i), b_i) = \mu_1 b_i + \mu_2 f(at_i) \tag{5}$$

where $\mu_1 + \mu_2 = 1$.

The payment mechanism is based in the classical Vickrey auction (where the winner pays the amount offered in the second best bid). However, it takes into account if the delivered item respects the attributes which where bidded or not, adapting the payment to this circumstance. To compute the payment, all the attributes from the winning bid and the valuation of the second best bid are taken into account, thus payment is strictly conditioned by the evaluation function. Table 1 shows the payment functions p derived from the evaluation functions proposed in Equations 3 to 5. The first row corresponds to the definition of the evaluation function; the second row corresponds to the payment when the delivered attributes are the ones which were bidded. Following a second price philosophy, the price p to pay by the winner must be equal to the amount the winner should have bid in order to obtain the same evaluation that obtained the second best bid. Finally, the third row corresponds to the case where the bidder offers a set of attributes worse than those it had tendered. In this case, payment p corresponds to the amout the bidder should have tendered in order to win the auction but with the real delivered attributes.

5 Demonstrating Incentive Compatibility

In this section we discuss the incentive compatibility of the previous mechanism following three different demonstrations: analyzing of the mechanism properties, trying to find a case which contradicts the incentive compatibility by means of a constraint solver and, at last, simulating the behaviour of the mechanism.

5.1 Meeting Sufficient conditions

Section 3 states that for an auction mechanism to be incentive compatible it sufficient to fullfill certain conditions. This subsection informally explains how the mechanism described in Section 4 commits these requirements when assuming that there are no externalities (bidders are just focused in the present auction without taking into account if there will be more auctions).

- **Exacteness:** In the presented mecahnism the auctioned item is asigned to only one winner, moreover, the only way of winning the auctioned item is to participate in the auction offering the best bid. In consequence, bidders can not obtain items for which they have not bid: they get the item for which they have bid or nothing.
- **Participation:** In the mechanism there is no fee to access the auction. Thus, if an agent does not win, it does not pay anything, obtaining 0 utility.
- **Monotoniciy:** This propertie is strictly related to the evaluation function. When using a monotonic function as evaluation function (e.g. the sum, the product or the weighted sum) the act of improving any of the attributes causes the bidder to gain a higher evaluation (see equations below). In consequence, improving a winning bid cannot cause a bidder to lose the auction.

$$\begin{array}{ccc} V(f(at_i),b_i) = f(at_i)+b_i & V(f(at_i),b_i) = f(at_i)*b_i & V(f(at_i),b_i) = \mu_1 f(at_i)+\mu_2 b_i \\ f(at_i)+b_i < f(at_i)+(b_i+1) & f(at_i)*b_i < f(at_i)*(b_i+1) & \mu_1 f(at_i)+\mu_2 b_i < \mu_1 f(at_i)+\mu_2(b_i+1) \\ f(at_i)+b_i < (f(at_i)+1)+b_i & f(at_i)*b_i < (f(at_i)+1)*b_i & \mu_1 f(at_i)+\mu_2 b_i < (\mu_1 f(at_i)+1)+\mu_2 b_i \end{array}$$

  In reverse auctions same thing happens, improving any of the attributes it will decrease the valuation of the bid. Since the winner of the auction is the one with the lowest valuation, improving an atributte will not cause a bidder to lose the auction.

- **Criticality:** The payment mechanism computes the payment by matching the evaluation of the second best bid with the evaluation of the payment and the attributes of the winning bid ($V(f(at_2),b_2) = V(f(at_1),p)$). This ensures that the winning bidder will receive the amount it should have tendered to obtain the same evaluation as the second best bid, ensuring that the payment is the minimum amount need to bid to win the auction with the bided attributes, thereby respecting criticality.

Since the mechanism has the four mentioned above properties, we can deduce that the mechanism is incentive compatible.

5.2 Seeking for a Counterexample with a Constraint Solver

To prove that truthful bidding is the dominant strategy we have to prove that, for any feasible bid, the utility of a bidder is higher or equal when bidding truthfully than when providing false attributes ($f(at_i) \neq f(at_i^v)$) or an economic bid different from the true value ($b \neq b_i$):

$$\forall i \in \mathbb{N}, \forall (b_i, b_i^v, at_i, at_i^v) \in \mathbb{R} > 0 : \left\{ (u_i(B_i, p) \leq u_i(B^v, p'))/B_i = (b_i, at_i) \wedge B_i' = (b_i^v, at_i^v) \right\} \quad (6)$$

where $u_i(B, p)$ corresponds to the utility function of the bidder i, b_i^v to its true value and at_i^v to its real or delivered attributes, B_i its bid, B_i' a bid compound by its real values p the payment the bidder will receive and p' the payment it would have received bidding trutfully.

Thereby, if we model the auction mechanism as a constraint satisfaction problem we can try to find a counterexample which contradicts Equation 6, a case where the utility of a bidder would have been higher when lying than when telling the truth. If this case exists, then the mecanism is not incentive compatible. Thus, our goal is to find out whether these cases exist or not.

5.2.1 Problem Definition

The auction mechanism and the counterexample can be modeled as an inequation system. If a SMT constraint solver is able to find a solution for the inequation system it will show that exists, at least, one case in which the utility of the bidder is higher when lying than when bidding honestly, refuting the hypothesis that the mechanism is incentive compatible. It is important to take into account the kind of functions which defines the auctioneer utilities, the winner determination problem and the payment rule as this will condition if the solver must support linear or non-linear arithmetics.

Assuming that the bidder utility function is given by the following expression:

$$u_i(B_i, p) = \begin{cases} p - b_i^v & \text{if } p > 0 \\ 0 & \text{otherwhise} \end{cases} \quad (7)$$

we can obtain the following inequation systems when the product (Equation 8), the sum (Equation 9) and the weighted sum (Equation 10) are used as evaluation function:

- Inequation system when using product as evaluation function

$$\left.\begin{array}{ll} \text{(a)} & f(at_1) \neq f(at_1^v) \vee b_1 \neq b_1^v \\ \text{(b)} & b_1 * f(at_1) > b_2 * f(at_2) \\ \text{(c)} & win = \begin{cases} 1 & \text{if } (b_2 * f(at_2) < b_1^v * f(at_1^v)) \\ 0 & \text{otherwise} \end{cases} \\ \text{(d)} & win * (\frac{b_2 * f(at_2)}{f(at_1^v)} - b_1^v) < (\frac{b_1 * f(at_1)}{f(at_1^v)} - b_1^v) \end{array}\right\} \quad (8)$$

where Eq.8a defines that bidder 1 is lying about the attributes it bids or its economic price, Eq.8b indicates that bidder 1 is the winner of the auction, *win* in Eq.8c defines if the bidder would have won the auction if it had told the truth and Eq.8d compares the obtained utility and the utility it would have obtained by bidding truthfully.

- Inequation system when using sum as evaluation function:

$$\left.\begin{array}{ll}
\text{(a)} & f(at_1) \neq f(at_1^v) \vee b_1 \neq b_1^v \\
\text{(b)} & b_1 + f(at_1) < b_2 + f(at_2) \\
\text{(c)} & win = \begin{cases} 1 & \text{if } (b_2 + f(at_2) > b_1^v + f(at_1^v)) \\ 0 & \text{otherwise} \end{cases} \\
\text{(d)} & win * (b_2 + f(at_2) - f(at_1^v) - b_1^v) < (b_1 + f(at_1) - f(at_1^v) - b_1^v)
\end{array}\right\} \tag{9}$$

where Eq.9a to d follow the same structure as Equation 8.

- Inequation system when using weighted sum as evaluation function:

$$\left.\begin{array}{ll}
\text{(a)} & f(at_1) \neq f(at_1^v) \vee b_1 \neq b_1^v \\
\text{(b)} & \mu_1 b_1 + \mu 2 f(at_1) < \mu 1 b_2 + \mu 2 f(at_2) \\
\text{(c)} & win = \begin{cases} 1 & \text{if } (\mu_1 b_2 + \mu_2 f(at_2) > \mu_1 b_1^v + \mu_2 f(at_1^v)) \\ 0 & \text{otherwise} \end{cases} \\
\text{(d)} & win * (\frac{\mu_1 b_2 + \mu_2 f(at_2) - \mu_2 f(at_1^v)}{\mu_1} - b_1^v) < (\frac{\mu_1 b_1 + \mu_2 f(at_1) - \mu_2 f(at_1^v)}{\mu_1} - b_1^v) \\
\text{(e)} & \mu_1 + \mu_2 = 1 \\
\text{(f)} & 0 < \mu_1 < 1 \\
\text{(g)} & 0 < \mu_2 < 1
\end{array}\right\} \tag{10}$$

where Eq.10a to d follow the same structure as Equation 8 and Eq.10e to f delimit the values of μ between 0 and 1.

5.2.2 Satisfactibility Test

To test the satisfactibility of the inequation systems 8 to 10 we used Microsoft Z3 solver [10]. When trying to figure out if an inequation system is satisfiable, three results can be obtained: satisfiable, meaning that the inequation system can be solved and that the mechanism is not incentive compatible; unsatisfiable, meaning that the system cannot be solved thus the best strategy is truthfulb bidding; and unknown, meaning that the solver cannot obtain a result in a reasonable time and we cannot determine whether or not the mechanism is incentive compatible. Moreover, the variable domains and the type of logic used can be bounded in order to facilitate the problem solution. In the satisfactibility tests performed real arithmetic logic has been used.

Z3 found that all the defined inequation systems are unsatisfiable, pointing that, as stated in

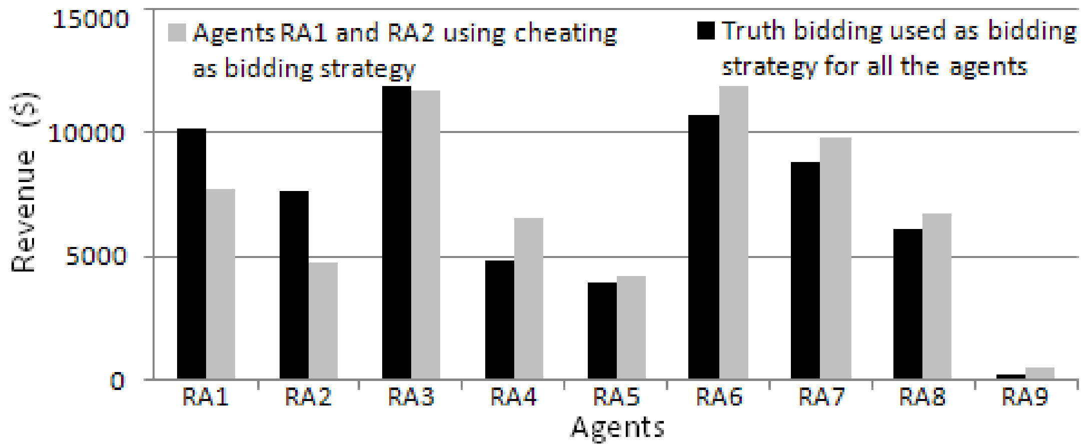

Figure 1: Mean revenue of the bidders in Scenario1.

the previous subsection, with these three evaluation functions the auction mechanism remains incentive compatible.

5.3 Experimental Validation

In addition to the previous tests, we have performed an empiric test in a simulated environment presented in [11], where bidders are resource agents that compete for tasks (reverse auction) and where two attributes are involved in the auction: economic cost and delivery time. The experiments are evaluated in terms of economic utility (agent benefits and costs). The task allocation follows these procedure:

Every time an agent needs to externalize a task:

1. Request for proposals. The auctioneer needs a resource to deploy a task. The auctioneer expects the task to be done the sooner the better, however it also expects to pay a low price for it. The auctioneer summons an auction to demand the resource.

2. Bidding. Bidder agents who have the capacity of performing the task can participate in the auction. They offer a bid with the economic amount they expect to earn with the task and its delivery time.

3. Winner determination. The auctioneer evaluates the bids using the product as an evaluation function (product). It is important to notice that, as we are dealing with a reverse auction, the aim of the auctioneer is to minimize the result of the evaluation function. Thus, the agent with the lowest score will win the auction.

4. Payment. When the winning agent completes the task, it receives a payment p. The payment will depend on whenever the task has been delivered according to the bidded time or not.

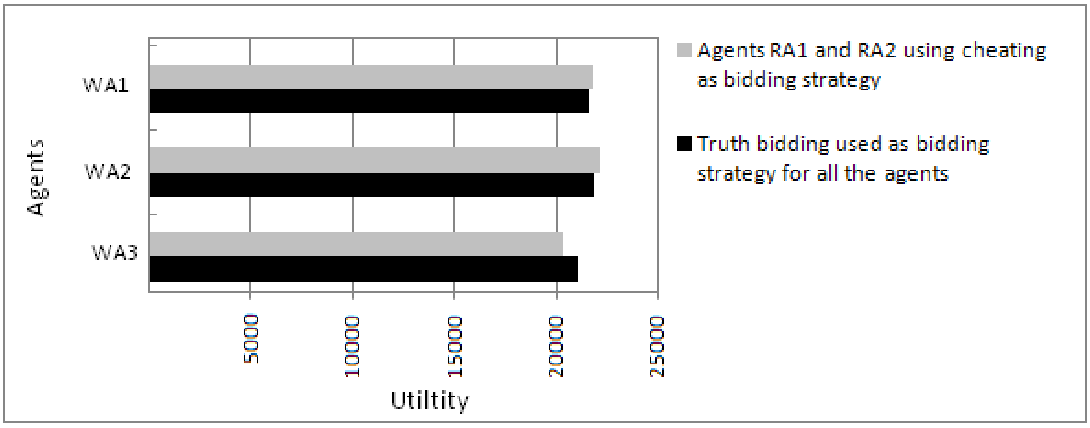

Figure 2: Mean utility of the auctioeners in Scenario1.

5.3.1 Experimental Set Up

To study the behavior of the mechanism against cheating agents we used two different kinds of agents: ones which try to maximize their utility adapting their bids [12] and ones which cheat and try to win the maximum number of auctions. In the experiments we compare the benefits that the cheating agents obtain with the ones the would have obtained if they had bid truthfully. Moreover, in a second experiment, a smart agent uses probabilistic reinforcement learning [13] to discover which is its best bidding strategy. In both cases the simulation has a length of 200 time units.

The configuration of the experiment is the following:

- **Scenario 1:** This scenario has three different auctioneers while nine bidders. In this experiment, product is used as evaluation function. Our hypothesis is that truthful agents have a higher profit than cheater agents. The main goal of this scenario is to evaluate if for a bidder agent is better to lie or to bid truthfully. To test the hypothesis, the experiment is first conducted with two cheating agents (B1 and B2) and seven adaptive agents (learn to adapt their bids from one auction to another; then the experiment is repeated but changing the bidding strategy of B1 and B2 from cheating to truthful bidding. The experiment has been repeated 100 times in order to obtain significant results.

- **Scenario 2:** In the second experiment the same number of agents take part in the simulation (three auctioneer and nine bidders). Eight of the bidders use honest bidding while the nineth tries to learn which is the best strategy to follow using a reinforcement learning algorithm. The smart agent can change its bids within a range of values defined by its true values (it can bid from 0.8 times its true value to 1.2 times). The bidder updates the probability of choosing one or another strategy according to the utility it obtained in previous actions.

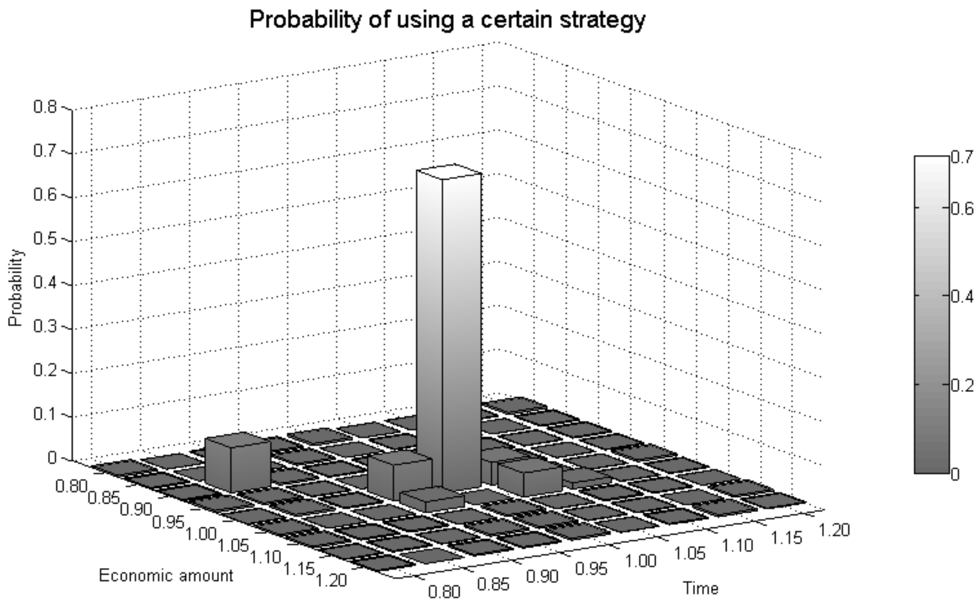

Figure 3: Results of the strategy learning of the smart agent. Z-axis shows the probability of the agent of choosing a certain bidding strategy. X-axis (delivery time) and Y-axis (economic cost) show the available strategies: being x and y the desviation of the agent true values (e.g. x=1 and y=1 correspond to the honest bidding strategy).

5.3.2 Results

Figures 1 and 2 show the results of the first experiment, the first shows the mean revenue of each bidder after the simulation when each agent bids truthfully (black bars) and when bidders 1 and 2 use cheating as bidding strategy (grey bars). It can be clearly seen how B1 and B2 earn more benefits when they are bidding according to their true values. This reafirms the hypotesis that the mechanism is incentive compatible and that bidders obtain more benefits when they reveal their true values. Moreover, we can see how the rest of the agents earn more money when B1 and B2 do not follow the truthful bidding strategy. Figure 2 shows that strategy choosen by B1 and B2 does not significantly affect the budgets of A1, A2 and A3 as they spend a similar amount of money.

Figure 3 corresponds to the second scenario. It illustrates the results of the smart agent learning process, each bar corresponds to a possible strategy. Z axis corresponds to the probability of choosing a certain strategy, X axis corresponds to the delivery time attribute whilst Y axis corresponds to the economic amount bidded. A value below 1 corresponds to an overbidding strategy (saying that it will finish earlier than it can or asking less money than his true value), a value higher than 1 corresponds to underbidding (offering values worse than his true values) whilst when both values are set to one the strategy corresponds to honest bidding. The barchart clearly points that the strategy which has more chances to be choosen is honest bidding (Probability = 0.701), meaning that this is the strategy which has provided a highest utility to the bidder and that the bidder has learnt that its dominant strategy is to reveal its real values.

Given that in this scenario the rest of bidders are using truthfull bidding as strategy, the experiment can only guarantee that the mechanism is Bayes-Nash equilibrated (the best strategy for a participant is to reveal its true values given that others are doing so [14]). However, if we take into account the results of the previous experiment and prior demonstrations (which pointed that honest bidding is the dominant strategy), we can say that a reasonable learning agent which plays in an auction game like the one analyzed in this work will be able to learn its dominant strategy, being this one honest bidding.

6 Conclusions and future work

In this work we have proven that the multi-attribute auction mechanism presented in [2] is incentive-compatible, at least, when using the sum, the weighted sum and the product as evaluation functions. During the paper we have provided three different demonstrations to verify this hypothesis. The first one shows that the mechanism has the exactness, participation, monotonicity and criticality properties required for a mechanism to be incentive compatible. The second one consisted in using a constraint solver to prove that there are no cases where a bidder can obtain higher utility by lying than by revealing its true values. For that purpose the mechanism, the utilities of agents and the bidders strategies have been modeled under an inequation system. Z3 solver determined the unsatisfactibility of the inequation systems, confirming the presented hypotesis. Finally, two resource allocation simulations have been performed to complete the demonstration. In the first we compare the benefits which obtain agents when they bid truthfuly with the benefits obtained when lying. In the second one, a smart agent follows a reinforcement learning algorithm in order to find out which is its optimal bidding strategy. In both cases the results concord with the first two demonstrations.

The three demonstrations presented clearly show that the analyzed multi-attribute mechanism is incentive compatible when assuming that there are no externatilities. The paper has shown three different approaches which can be used to test the incentive compatibility of a mechanism. Particularly, the authors consider that the second test methodology (Section 5.2) is the simplest and easiest way to ensure the strategy proofness of an auction mechanism. However, some mechanisms may be too complex for solvers to determine the satisfactibility of the inequation systems which define them, reducing the utility of this demonstration to a restricted number of mechanisms. When this occurs, demonstrations from Section 5.1 and 5.3 should be used.

As future work we will also try to demonstrate that incentive-compatibility of the mechanism for both repetitive and multi-objective auctions.

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