Mathematics Education Research Journal
https://doi.org/10.1007/s13394-025-00519-2

ORIGINAL ARTICLE

Responsive mathematics teaching and mathematics teacher noticing: a systematic review in early childhood and primary education

Ángel Alsina[1] · Nataly Pincheira[1] · Berta Barquero[2] · Isabel Sellas[3] · Marta Marimon-Martí[3]

Received: 16 April 2024 / Revised: 6 February 2025 / Accepted: 9 February 2025

Abstract

At a time when more student-centred teaching methodologies are being introduced, it is essential to investigate how educational research in teacher professional development has progressed. In this study, we focus especially on mathematics teaching to promote responsive teaching and noticing since both practices place students in the foreground in the teaching and learning processes. To this end, we carried out a systematic review of research articles published between 2010 and 2023 in the Web of Science and Scopus databases on responsive teaching and noticing in mathematics teacher education programmes of early childhood and primary school levels. Using the Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA) statement, 40 articles were identified. The general results show that the most productive countries were the United States, Spain and Australia; that the most addressed mathematical contents were those related to the teaching of numbers, operations and their properties; that there are more studies focused on primary school than on preschool; and that most studies used qualitative methodologies. The specific results show that the research topics focus on three aspects: teachers' decision-making in relation to culturally responsive mathematics teaching; the identification of general characteristics of the noticing competence in mathematics teaching; and the specific development of professional noticing skills.

Keywords Teaching mathematics · Responsive teaching · Noticing · Systematic review · Primary education · Early childhood education

Extended author information available on the last page of the article

Published online: 27 February 2025

Introduction

The professional development of mathematics teachers has had a highly productive research agenda in mathematics education in recent years. This article investigates the studies that, to our understanding, have suggested that the teaching of mathematics is only possible if it is done from the students' actions and ideas of the students, granting these students the category of epistemic agents (e.g. Marimon-Martí et al., 2023; Robertson et al., 2016; Sherin et al., 2011). From this perspective, we refer to two concepts that are closely connected but are indeed different teacher practices: *responsive teaching* and *noticing*. On the one hand, the term culturally responsive teaching, which connects students' cultures, languages and life experiences with what they learn in school, was introduced by Gay (2000) to refer to a skill that requires teachers to systematically make agile decisions as they observe and analyse student behaviour. It is this observation and these analysis tools that inform their subsequent teaching and pedagogical decisions. On the other hand, the term *noticing* can be conceptualised generically as "the act of perceiving and realising something" (López, 2021, p. 79). This term emerged around 2002 with the work of Mason (2002) and van Es and Sherin (2002). Later, Jacobs et al. (2010), Robertson et al. (2016) and Sherin and Jacobs (2011) pointed to three traits associated with this ability: paying attention to the students' mathematical thinking, interpreting it and making decisions to act in accordance with this thinking. Based on the characterisation of these three traits, *noticing* has been placed as an important focus of research in mathematics education.

To date, several systematic reviews have been conducted separately on both constructs. Some systematic reviews have been carried out on *responsive teaching* (e.g., Miller et al., 2023; Young & Young, 2023) which, although they consider mathematics education, they do not focus on it. And Mason (2002) explains that while professionals can perform noticing, not all of them have developed this skill, which is why training and shared knowledge seem necessary. In this regard, some systematic reviews have been carried out that try to synthesise the main advances in the literature (e.g., Amador et al., 2021; López, 2021). Some previous reviews have identified several key aspects of responsive teaching and noticing but showed some limitations: first, due to the lack of specificity concerning the teaching of mathematics in culturally diverse contexts; and second, the lack of analysis on teacher observation skills. The review we present in this paper aims to address these limitations by using specific analysis standards and focusing on mathematics education at the early childhood and primary school levels.

With this framework, we are interested in conducting a systematic review of both constructs because, despite referring to different teacher practices, they share a way to approach teaching in which students' knowledge, ideas and reasoning are placed in the foreground during the teaching and learning processes (Marimon-Martí et al., 2023; Robertson et al., 2016; Sherin et al., 2011). Our article focuses on some aspects of this professional competence, such as the relationship between teacher perception and decision-making and the identification of the elements of teacher education that facilitate these competences.

In this context, the aim of this article is to carry out a systematic review of studies published in Web of Science and Scopus that have linked these constructs exclusively to mathematics education in the stages of early childhood and primary education, with the purpose of complementing the previous reviews mentioned before. We assume that this study will help us to better understand both practices, assuming that teacher noticing is more about teachers' vision and view of either other teachers or themselves (externally), whereas culturally responsive practices are in situ classroom practices. To this end, we initially carry out a general analysis of each article, followed by an analysis of more specific aspects, such as the content standards that the studies meet.

Theoretical framework

Responsive teaching has been conceptualised and enacted in distinct ways. Nevertheless, there are three common features in the different conceptualisations: foregrounding students' ideas, recognising the disciplinary connections within students' ideas and pursuing the substance of student thinking (Robertson et al., 2016).

Responsive teaching is a construct that encompasses a series of methodological approaches, such as culturally relevant, culturally sensitive, congruent and contextualised pedagogies (Gay, 2000) in which culture takes on a leading role, which is why the term *culturally* responsive teaching is often used in the literature. This approach places students at the centre of the learning process and uses "cultural knowledge, prior experiences, frames of reference and performance styles of ethnically diverse students to make learning encounters more relevant to and effective for them" (Gay, 2000, p. 29). From this point of view, culturally responsive teaching is distinguished by its emphasis on validating, facilitating, liberating and empowering students by "cultivating their cultural integrity, individual abilities, and academic success" (Gay, 2000, p. 44) and is based on four pillars: (1) the attitude and expectations of the teacher; (2) cultural communication in the classroom; (3) the culturally diverse context in the curriculum; and (4) culturally congruent instructional strategies. In later works, Gay (2002) adds a fifth element and defines them as follows: (1) development of a knowledge base on cultural diversity; (2) a culturally relevant curriculum design; (3) demonstration of cultural care and creation of a learning community; (4) intercultural communication; and (5) cultural congruence in classroom teaching.

Robertson et al. (2016) provide depictions of responsive teaching in the literature: Cognitively Guided Instruction (CGI), mathematics teacher noticing, discursive studies and case studies of responsive teaching. The Cognitively Guided Instruction (CGI) programme aims to help teachers adapt their mathematics instruction with regard to students' mathematical thinking, focusing on how teachers make sense of research-based frameworks regarding student thinking. Research on mathematics teacher noticing focused "on how the teachers themselves distil key features of students' mathematical thinking from a chaotic, complex instructional environment" (Robertson el al., 2016, p. 38). The basis of discursive studies of responsive teaching is the discursive approach to defining and evaluating such teaching in classroom interactions. Finally, Robertson et al. (2016) present case studies divided into two subsets: first-hand practitioner accounts of responsive teaching and researchers' analyses of examples of responsive teaching. Based on these initial approaches, the conceptualisation of responsive teaching has evolved towards the idea of the vision of a professional with a particular and more sophisticated look at the objects and processes involved in their profession.

When talking about responsive teaching, awareness should be considered as evidence that a teacher is attending and responding to a situation. van Es and Sherin (2002), for example, point out that noticing is about developing the ability to detect what is important about a situation in the classroom, to establish connections between specific situations and general pedagogical principles and to use what is known to reason about situations observed in the classroom. In a more recent publication, van Es and Sherin (2021) introduce a third dimension, "shaping", which expands on the first two of attending and interpreting. The expanded framework they propose considers attending (identifying and disregarding features of classroom interactions), interpreting (using knowledge and experience and adopting a stance of inquiry) and shaping, which "involves constructing interactions and contexts to gain access to additional information" (op. cit., p. 19).

Several studies (e.g. Atkins & Frank, 2015; Hammer et al., 2012; Jaber, et al., 2018; Maskiewicz & Winters, 2012) show that this teaching competence based on responsive teaching increases student commitment, promotes complex and deep learning (Kang & Anderson, 2015) and facilitates equitable participation among linguistically, culturally and socially diverse students (Acquah & Szelei, 2020; Al Aleeli, 2021; Umultu & Kim, 2020). This approach requires the acquisition, by teachers, of pedagogical knowledge and specific skills to (re)construct their knowledge from and about the children's knowledge (Kang & Anderson, 2015). More specifically, and as was noted in the introduction, Jacobs et al. (2010), and later Sherin et al. (2011) and Robertson et al. (2016), point to various features associated with mathematics teacher noticing: (a) paying attention to students' mathematical thinking; (b) interpreting their mathematical thinking; and (c) making decisions to act based on this mathematical thinking. van Es and Sherin (2021) argue that shaping differs from decision-making since it is a part of noticing. Deciding how to respond involves teachers reasoning about a potential response regarding student thinking, "in contrast, 'shaping' involves teachers and students engaging in an interaction with each other in the moment" (p. 24).

According to Zapatera and Callejo (2018), the first skill involves identifying the significant mathematical aspects in the strategies that students use because their details provide an entryway to students' mathematical thinking. To do this, the teacher must have deep mathematical knowledge for teaching that allows him or her to observe in detail the responses and actions of the students and the way in which they face problems (Sánchez-Matamoros et al., 2021; Zapatera & Callejo, 2018).

To interpret students' mathematical thinking, the teacher must connect the mathematical elements identified in the strategies with the understanding of the mathematical concepts, for which he or she needs sufficient knowledge in the field of mathematics (Jacobs et al., 2010). Mason (2002) focuses this skill on explaining and theorising observations in order to relate them to mathematical content and evaluating them to make appropriate decisions. A teacher's goal, on identifying the strategies and interpreting the understanding of the students, is to use this information to *make decisions* about what and how to guide students' mathematical work. van Es and Sherin (2002) emphasise this practical aspect of the professional view and indicate that the objective of professionally observing students' mathematical thinking is to make instructional responses effective. Sherin et al. (2011) consider that the ability to integrate these three skills is a necessary but not sufficient condition to make better decisions for the teaching and learning of students.

From this conceptual framework, some studies have focused on analysing the development of these skills. van Es (2011), for example, identifies three important aspects regarding teachers' actions. The first looks at what teachers highlight when they observe an instructional event; the second deals with teachers' strategies to analyse the observed events; and the third focuses on the level of detail that teachers provide when they explain, interpret and justify classroom observations. Later, the research of Simpson and Haltiwanger (2017) indicates that novice teachers are less effective than experienced teachers. Other studies have focused on analysing the effect of teacher training programmes, concluding that if they are well planned, they help improve teachers' skills (Hawkins & Rogers, 2016; Mitchell & Marin, 2015; Star & Strickland, 2008). Finally, other authors have focused on analysing how the ability is transformed and, more specifically, on the differences in the focus of attention and interpretation (Talanquer et al., 2013, 2015). With respect to the focus of attention, evidence has been provided that less attention is paid to general aspects of the teacher or the class, while a great deal of attention is paid to the ideas and reasoning of the students and how they solve problems. Regarding interpretation, the various studies conclude that little importance is given to evaluating ideas as correct or incorrect, whereas the nature and origin of ideas or reasoning are emphasised.

These studies show that the construction of knowledge in the classroom is a complex process, the success of which depends fundamentally on two factors: (a) the ability, knowledge and tools that the teachers have to plan interventions in the classroom in which teachers and students can develop new roles; and (b) the ability, knowledge and strategies of teachers to act in the classroom facilitating a process of knowledge construction in which the students act as active epistemic agents; that is, act in accordance with the principles of responsive teaching (Robertson et al., 2016).

Method

Our study provides a systematic literature review following the criteria and procedures of the Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA) proposed by Moher et al. (2015). The review was organised into four phases: *Phase 1*. Establish search elements and Boolean logic; *Phase 2*. Select information sources; *Phase 3*. Establish eligibility criteria; *Phase 4*. Data extraction and management to establish the sample.

In *Phase 1*, the search elements were formulated from the key terms that guide the study, considering the area of knowledge and the search context: ("mathematics education") AND ("responsive teaching" OR "culturally responsive teaching"

OR "noticing") AND ("primary education" OR "elementary education" OR "early childhood education").

To select the sources of information, corresponding to *Phase 2*, the criterion was established to consult databases that include the most relevant scientific production, internationally, in the field of educational research. We therefore considered Clarivate Analytics' Web of Science (WOS) and Elsevier's Scopus, given the impact index they constitute (JCR and SJR, respectively). From this selection, we considered the indexing of scientific articles in journals that are housed under these parameters.

In *Phase 3*, the eligibility criteria of the studies were established, that is, of inclusion and exclusion, which are set out in Table 1. Academic articles that have undergone a rigorous peer review process were considered, and thus, book chapters, conference proceedings and other types of publications were excluded. The publication period considered was between 2010 and December 2023, to complement previous reviews (e.g. Amador et al., 2021; López, 2021; Miller et al., 2023; Young & Young, 2023).

Publications written in English were included as it is the main language used in the field of educational research. Publications in Spanish were also considered to take advantage of the linguistic competence of the authors in this language and to maximise the geographical coverage of the study.

Furthermore, articles focused on the stages of early childhood education and primary education levels were included since we focus on teachers who work at these school levels. Finally, all those publications whose full-text was available for review through the university's databases were included, excluding only articles whose availability was limited. This includes peer-reviewed articles in Web of Science and Scopus, regardless of open access.

Phase 4 focused on data extraction and management and was carried out in December 2023. Boolean logic was applied to the titles, abstracts and keywords of the documents. Some eligibility criteria were applied, such as document types, publication period and language, which were filtered by the search engines of each database. The data were then exported to an MS Excel® spreadsheet. The titles, abstracts and full texts were then reviewed. Based on this, we excluded articles that did not meet the eligibility criteria, such as level or open access; that is, articles that were not focused on early childhood or primary education or available for review.

Table 1 Eligibility criteria

Criterion	Inclusion	Exclusion
Document type	Peer-reviewed article	Other formats
Publication period	2010 to 2023	Prior to 2010
Language	English and Spanish	Other languages
Level	Early childhood or primary education	Other levels
Access	Full texts	Texts not available

Finally, all duplicate documents were reviewed and excluded, based on the comparison of article titles and digital object identifier (DOI) numbers.

Sample

Figure 1 summarises the selection process of the sample of articles, consisting of 40 academic articles, which constitute the units of analysis of our research.

Analysis categories

To analyse the articles in greater depth, we established a series of analysis categories, which were the following: (a) author(s), year and country where the study was carried out; (b) mathematical content addressed by the study; (c) participants: preservice teacher (PST) or in-service teacher (IST); (d) stage: early childhood education (E) or primary education (P); and (e) method and instrument of data analysis.

Data analysis

Once the sample had been established, the data analysis was carried out based on an exhaustive reading of each of the articles, applying the established analysis

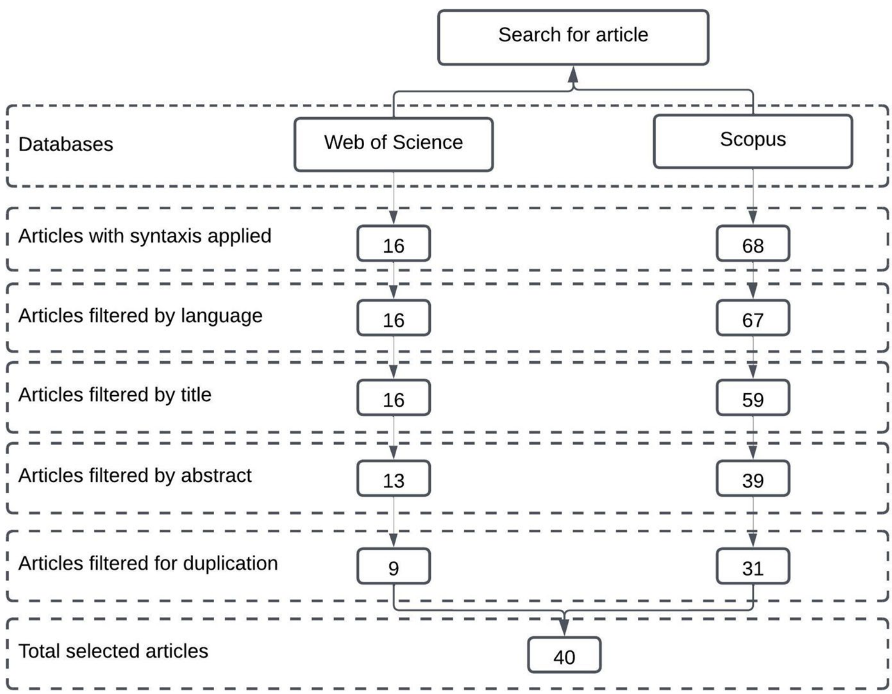

Fig. 1 Flowchart of the article selection process

categories through the content analysis technique (Krippendorff, 2019). To obtain a general categorisation of each study, a vertical or within-case analysis (Miles et al., 2013) of each of the 40 articles was carried out. Next, multiple comparisons were made between the academic articles (Miles et al., 2013) to find similarities and differences, based on a cross-sectional analysis. Based on the vertical and cross-sectional analysis of the articles, the data was described by creating analysis tables of qualitative information extracted from the review of the articles. The cross-sectional analysis made it possible to prepare a synthesis of the main results, allowing the identification of interrelated aspects between the different studies.

Results

General characterisations of the studies

Table 2 summarises an initial block of information that takes into account the author(s) and year of publication; the country where the study is carried out; mathematical content the study focuses on; the nature of the participants, i.e. preservice teachers (PST) or in-service teachers (IST); the educational stage they are teaching in, be it early childhood education (E) or primary education (P); and, finally, the research method and data collection instruments.

Year of publication and geographical distribution

In Fig. 2, you can see the temporal distribution of publications. At a general level, a growth trend can be seen in the studies focused on responsive teaching, reaching a notable increase in 2021. Thus, the temporal distribution shows that during the last ten years there has been great interest in the research community in addressing responsive teaching in teacher education at early childhood and primary school levels.

Figure 3 shows the geographical distribution of studies that were carried out in nine countries. A large majority of them were conducted in the USA (57.5%), followed by Spain (12.5%) and Australia (10%). To a lesser extent, other countries also carried out studies on this topic, such as Canada, Ireland and Turkey, with a presence of 5% each. A lower presence of studies conducted in Norway and Germany can also be seen (2.5% each).

Mathematical content addressed

The studies address various mathematical contents, evidencing a wide range of possibilities in the mathematics classroom to approach the characteristics of responsive teaching or noticing by teachers. Standing out among them are the contents related to the teaching and learning of numbers, operations and properties of these (18.1%), problem solving (18.1%), fractions (13.7%) and patterns and sequences (11.4%). Early mathematics (6.8%), mathematical reasoning, more specifically justification and generalisation (4.5%), equations, symmetry, probability, magnitude and measurement (2.2%

Table 2 General characteristics of the selected articles

Author/s (year)	Country	Math content	Participants (PST/IST)	Stage		Method	Instruments
				E	P		
Baker and Galanti (2017)	United States	Problem solving	4 IST		x	Qualitative	Video recording and survey
Brenneman et al. (2019)	United States	Early mathematics	25 IST	x		Qualitative	Video recording, survey and class observation
Callejo and Zapatera (2017)	Spain	Generalisation of patterns	38 PST		x	Qualitative	Questionnaire
Campbell and Griffin (2017)	United States	-	21 IST	x	x	Mixed	Class observation
Carter and Amador (2015)	United States	-	7 PST		x	Qualitative	Video recording
Coskun et al. (2023)	Turkey	Fractions	32 PST		x	Qualitative	Video recording
Crespo et al. (2021)	United States	-	18 PST		x	Qualitative	Class observation
Dunning (2023)	United States	Fractions	33 IST		x	Qualitative	Class observation and interviews
Ferdig and Kosko (2020)	United States	Commutative property of multiplication	34 PST	x		Mixed	Video recording and survey
Francis and Jacobsen (2013)	Canada	Problem solving	10 IST		x	Qualitative	Video recording, field notes and interviews
Graue et al. (2015)	United States	Early mathematics	55 IST	x		Qualitative	Video recording
Hernandez and Shroyer (2017)	United States	-	12 PST		x	Qualitative	Video recording, class observation and interviews
Ivars et al. (2018)	Spain	Part-whole meaning of fractions	29 PST		x	Qualitative	Video recording
Jacobs et al. (2010)	United States	Problem solving with whole numbers	131 PST/IST		x	Mixed	Written production
Jazby et al. (2023)	Australia	Numerical patterns	1 IST		x	Qualitative	Video recording and interviews
Jensen et al. (2023)	United States	-	60 PST		x	Qualitative	Video recording
Jessup (2023)	United States	Fractions	72 IST		x	Qualitative	Audio recordings and interviews
Kalinec-Craig et al. (2019)	United States	Symmetry	7 IST	x	x	Qualitative	Video and audio recordings

Table 2 (continued)

Author/s (year)	Country	Math content	Participants (PST/IST)	Stage		Method	Instruments
				E	P		
Kalinec-Craig et al. (2021)	United States	Elementary mathematics	18 PST		x	Mixed	Video recording
Kosko et al. (2021)	United States	Commutative property of multiplication	34 PST	x		Mixed	Video recording
Land et al. (2019)	United States	Problem statement	20 IST	x	x	Qualitative	Interview
Leavy and Hourigan (2016)	Ireland	Numbers (counting, cardinality, subitisation)	25 PST		x	Qualitative	Video recording
Leavy and Hourigan (2022)	Ireland	Problem solving	28 PST		x	Design-based research	Survey and field notes
Lee and Lee (2023)	United States	Generalisation of patterns	154 PST		x	Mixed	Questionnaire
Luna and Selmer (2021)	United States	-	1 IST		x	Qualitative	Interview
Melhuish et al. (2020)	United States	Mathematical reasoning: justification and generalisation	79 IST		x	Mixed	Video recording, survey and class observation
Namakshi et al. (2022)	United States	Numbers and operations	99 PST		x	Mixed	Interview and questionnaire
O'Keeffe et al. (2019)	Australia	-	28 IST		x	Qualitative	Survey
Osmanoglu et al. (2015)	Turkey	-	15 PST		x	Qualitative	Interview
Owens (2015)	Australia	-	44 IST		x	Qualitative	Interview
Prediger et al. (2022)	Germany	Probability	1 IST		x	Qualitative	Video recording and interview
Sánchez-Matamoros et al. (2021)	Spain	Magnitude and measurement	1 PST	x		Qualitative	Written production
Schack et al. (2013)	Spain	Early arithmetic	94 PST		x	Qualitative	Video recording, interview and questionnaire
Throop Robinson et al. (2021)	Canada	Problem solving and mathematical reasoning	244 IST		x	Mixed	Survey and interview
Tyminski et al. (2021)	United States	Operations with whole numbers	74 PST		x	Qualitative	Video recording and survey

Table 2 (continued)

Author/s (year)	Country	Math content	Participants (PST/IST)	Stage		Method	Instruments
				E	P		
Wager (2014)	United States	Problem solving, number sense and equation	13 IST		x	Mixed	Video recording
Wager and Parks (2016)	United States	Numbers (cardinality, correspondence, subitisation)	9 IST	x		Qualitative	Field notes
Xenofontos and Alkan (2022)	Norway	Fractions, patterns and sequences	64 PST		x	Qualitative	Written production
Xu et al. (2019)	Australia	Addition strategies and addition of fractions	2 IST		x	Qualitative	Video recording
Zapatera and Callejo (2018)	Spain	Generalisation of patterns	40 PST		x	Qualitative	Questionnaire

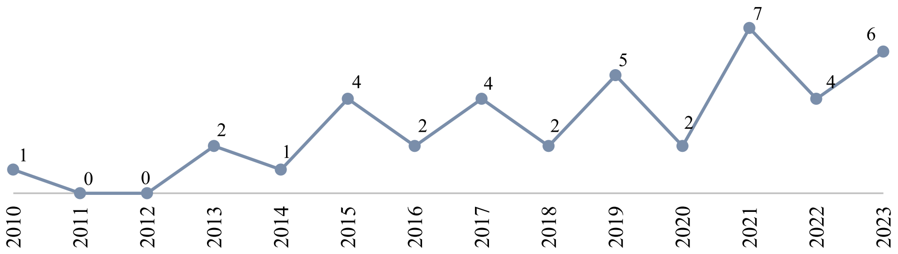

Fig. 2 Temporal distributions of investigations

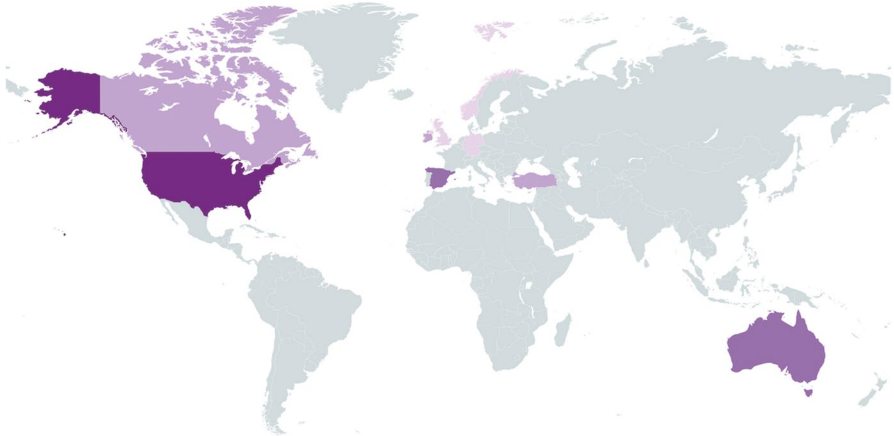

Fig. 3 Geographical distribution of the investigations (image created with Datawrapper)

each) are addressed less frequently. However, 18.1% of the studies do not carry out their investigation considering specific mathematical content, and they focus on general ideas of students' mathematical reasoning that are considered by teachers (Fig. 4).

Participants and their training

With respect to the participants and their training, it is evident that most of the studies (77.5%) were focused on addressing the responsive teaching or noticing of primary education teachers, with a small number of studies (15%) focusing on early childhood education. Only 7.5% of the studies considered both types of participants, both early childhood education and primary school teacher education. It should be noted that 50% of the studies were carried out with teachers in training, 47.5% with in-service teachers and 2.5% combined both preservice and in-service teachers.

Research method and data collection instruments

At a general level, a predominance of the qualitative approach is evident with 28 articles (70%), followed by the mixed approach with ten articles (25%). There is a

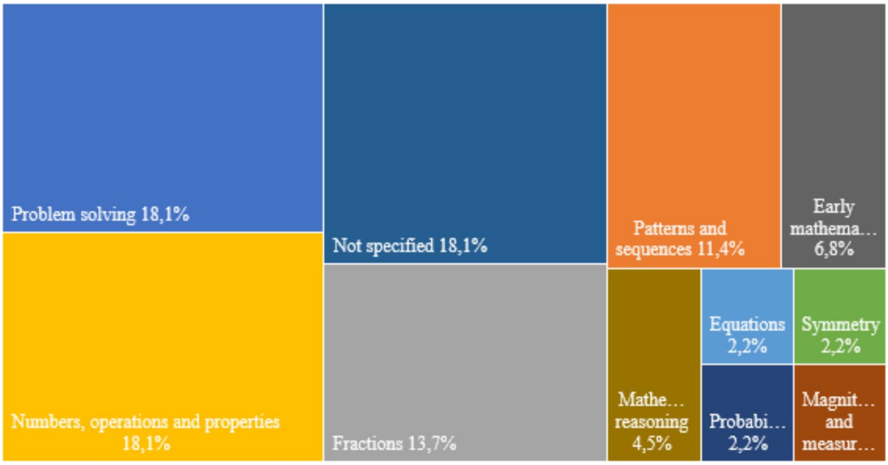

Fig. 4 Distribution of articles according to mathematical content

much lower presence of the quantitative approach and design-based research, with 2.5% each. Regarding the techniques used in the various investigations, 34.5% lean towards video recording, followed by interviews (21.3%) and, to a lesser extent, surveys (13.1%), class observation (9.8%) questionnaires (8.2%), written productions and field notes (4.9% each) and, the least used, audio recording (3.3%).

It is important to note that a study may exhibit more than one data collection instrument, depending on the research needs of the authors.

Specific characteristics of the investigations

To delve deeper into the specific characteristics of the studies, an analysis of the objectives, research questions and main results was carried out. This allowed us to determine that the research topics of the articles revolve around three main aspects:

(1) Teacher decision-making
(2) Teacher noticing competence
(3) Professional development of noticing skills and responsive practices

Below, we detail the meeting points, complements or specificities of the studies we have analysed around these three main aspects.

Teacher decision-making

Decision-making in mathematics education from a culturally responsive perspective implies that teachers adapt their practices according to the cultural knowledge and experiences of their students. This characteristic is particularly important as it allows considering the integration of cultural context in the teaching and learning of mathematics, providing more accessible and meaningful learning for students from diverse backgrounds. Studies such as those by Graue et al. (2015) and Wager and Parks (2016) highlight how this decision-making competency enhances teachers' ability to respond successfully to the possible emerging needs in the classroom, creating an environment where active participation from students is valued.

Graue et al. (2015), for example, explore the pedagogical decisions of active early childhood education for culturally responsive early mathematics through a professional development programme. Specifically, they analyse how teachers adopted the idea of involving children in mathematical experiences by linking content knowledge and practices at home from the perspective of improvisation conceptualised by the authors as "a receptive and collaborative activity through which the teachers and children generate meanings and knowledge together" (Ibid., p.13). The results reveal that teachers' improvisation in literary and mathematics practices, based on previous experiences at home and at school, provide learning opportunities, allowing children's interests, experience and abilities to be addressed. Therefore, teachers who improvise in their classrooms take on new meanings of cultural tools as they actively respond to the diverse intellectual, social and emotional experiences and needs of children, putting into practice diverse knowledge through the interactions that are generated in classrooms between students and teachers.

Wager and Parks (2016) analyse what the in-service early childhood teacher education notice about children's mathematical thinking in culturally and developmentally receptive early teaching of numbers during play. To do so, teachers might recognise the mathematics that children engage with and make decisions in the moment to support learning. The results reveal that teachers, when describing and discussing children's mathematical knowledge during play, focus on particular skills: counting with one-to-one correspondence up to a particular number, mastering cardinality, rote counting up to a particular number, grouping skills, comparing sets and identifying the successor. To support children's thinking, teachers mostly ask scaffolding questions, and they propose and implement new evaluation strategies and design more sophisticated ways to interact in class.

Leavy and Hourigan (2016) explore how lesson study can act as a vehicle to promote meaningful learning and the development of knowledge among preservice primary school teachers when teaching early number concepts. Through lesson planning, teaching and reflection, the future teachers realised that they should be able to defend or explain their decision-making processes around planned pedagogical decisions. They focused their attention on children's mathematical understanding of numeration, highlighting the importance of analysing what students say, especially when they give a mathematically incorrect answer in relation, for example, to cardinality and one-to-one correspondence. At the same time, they realised the impact that the first numerical concepts have and their importance in the didactic decision-making of teachers, such as, for example, outlining a coherent instruction sequence when teaching numbers in the first stages of schooling or selecting more appropriate math problems to pose to elementary school students to make sense of the construction of numbers.

Hernandez and Shroyer (2017) analyse the use of culturally responsive teaching strategies in future Latino teachers in the mathematics instruction process in the primary school classroom. The results show culturally responsive teaching practices in relation to the following: (a) the integration of content, since they integrate content in the teaching of mathematics by including content from the students' cultures, fostering positive relationships between teachers and students and maintaining high expectations for all students; (b) knowledge construction: they facilitate the construction of knowledge in mathematics teaching based on what the students know in order to create links between their experiences at home, in the community and what they learn in the classroom; (c) reduction of prejudice: manifested during the teaching of mathematics through the use of the mother tongue to support students to learn and understand the mathematics contents, as well as to establish relationships with Spanish-speaking parents, the promotion of positive interactions between students and the creation of a safe learning environment in which students feel free to participate in class discussions and/or the activities of the mathematics lessons. O'Keeffe et al. (2019) explore Australian preservice teachers' confidence and knowledge of culturally responsive pedagogy in teaching mathematics in the context of professional development workshops. The findings show an increase in the confidence perceived by preservice teachers since they consider the incorporation of culturally responsive strategies for teaching mathematics with Aboriginal students. Preservice teachers design lessons focusing on all the diversity in class and design learning environments to support this diversity.

Kalinec-Craig et al. (2019) examine an experience developed by active American primary school teachers using Talavera tiles as an example of a culturally responsive context for teaching geometry, more specifically symmetry. Teachers ensure that students learn more mathematics when they relate to what they hear, see and do outside the school; thus, experiences in a culturally responsive context can support students' mathematical thinking. Similarly, Owens (2015) documents the development of a project in Australian schools adopting an indigenous approach to teaching mathematics, revealing the cultural understanding of teachers when considering learning experiences in a culturally responsive context, for example for teaching arithmetic. Practising teachers stopped focusing their attention on the textbook and began to incorporate more relevant experiences for students, focusing on their participation. The results of both studies highlight the importance of teachers' decision-making to adapt their teaching to a culturally responsive context through controlled changes in mathematics lesson addressing students' cultural needs.

Throop Robinson et al. (2021) examine the impact of a training programme in relation to teachers' ability to teach mathematics in primary education through a survey of recent graduates. The survey results revealed notable findings across three themes: professional development and knowledge of contents, leadership and culturally responsive pedagogy. Improvements observed include teachers' comfort in selecting and designing rich maths tasks and the instructional option of including work groups with students to foster problem solving. With respect to culturally responsive teaching practices, practising teachers were expected to delve deeper into issues related to equity for underserved groups of students. However, only 61% of teachers reported having a deeper understanding of the systemic factors that result in the performance gap among such students.

Teacher noticing competence

Teacher noticing competence involves teachers not only identifying their students' mathematical strategies, but also interpreting students' understanding of mathematics. This competence is central for responsive teaching, as it provides a solid foundation for tailoring instruction to the students' individual needs. Studies such as Xu et al. (2019) and Osmanoglu et al. (2015) show how preservice teachers gain greater sensitivity to students' mathematical ideas, allowing for more informed and targeted pedagogical intervention to improve mathematical understanding.

Xu et al. (2019) analyse the professional perception of teachers through video observation in a fraction level. The results reveal changes in teachers' perception with regards to teaching practice, valuing highly the importance of allowing students that have a misconception to discover why their way of thinking or the answers they provided were incorrect. Likewise, based on video observation, Osmanoglu et al. (2015) investigate what future primary school teachers pay attention to in instructional classes about teaching mathematics. The results show that future teachers focused on problems related to teaching actions that reflect specific domains of teaching knowledge, such as knowledge of pedagogical content, general pedagogical knowledge and knowledge of the curriculum, and that this perception began to increase over time. More specifically, future teachers began to pay more attention to how to understand effective teaching and a reformist curriculum, facilitate and ensure student understanding, connect mathematics to real life, motivate students to think and reason, impart student-centred lessons, use multiple instructional methods, prevent misconceptions, be able to understand students' questions and ideas and ask students to explain and defend their answers.

From another perspective, Xenofontos and Alkan (2022) explore what future primary school teachers notice when they are provided learning opportunities in non-formal contexts. In the context of a mathematics fair, future teachers reflect on and discuss four central elements: (a) the mathematical task and the importance of the proposed objectives, contents and potential thereof; (b) issues related to students during the interaction with the mathematical task linked to the commitment and motivation to resolve the task, cognitive development and preference towards specific representations of the task and prior knowledge to obtain the solution; (c) aspects related to teaching and pedagogy, establishing links between their observations and theories, highlighting, for example, the importance of collaborative learning and evaluation for learning; and (d) the benefits and limitations of learning mathematics in a non-formal context, observing that the reflections and attention of the future teachers are not focused on the mathematical content but rather on the teaching experience.

Lee and Lee (2023) use a questionnaire to analyse the perception of future primary school teachers about the pattern generalisation strategies that students develop. The findings revealed that teachers pay attention to students' ways of thinking; however, teachers' actions to support student learning lacked control and appropriate tools. It is notable the low percentage of future teachers using specific tools to analyse specific aspects of the students' mathematical thinking. The authors also mention generic aspects, such as efficiency, understanding, student explanations, the presence of correct answers and the use of representation.

Carter and Amador (2015) examine the conversational components of preservice primary school teachers on professionally focusing on students' mathematical thinking, showing that they provide detailed descriptions of the students' words and actions and, occasionally, try to interpret such observations. However, upon receiving a specific prompt, they were more likely to verbalise the professional observation and engage in a conversation about one or more students' understanding.

Kalinec-Craig et al. (2021) characterise the type of language used by future teachers to realise the students' mathematical strengths through a *Lesson Sketch* experience, an approach to research and development in teacher education that relies on the creation and use of multimedia representations of practice. It can be seen that future mathematics teachers are more likely to use strengths-based language and identify the mathematical evidence in their statements, whereas unengaged language (statements that do not fit a strengths- or deficit-based coding scheme) suggests a fruitful, albeit complex, space for learning to name and notice students' mathematical strengths.

Finally, Jazby et al. (2023) examine the way an elementary school teacher directs his attention mid-lesson and the environmental structures he perceives as meaningful. The results reveal that the teacher pays attention to the students' worksheets, which become the environmental structure that allows him to be aware of their errors. He also pays attention to the students' facial expressions, verbal expressions and manipulation of materials to gather information mid-lesson.

Professional development of noticing skills and responsive practices

The development of noticing skills in mathematics teachers includes the ability to attend to, interpret and decide how to respond to students' mathematical ideas and activity. These abilities, as highlighted by Leavy and Hourigan (2022) and Coskun et al. (2023), are closely related and mutually reinforcing. Teacher education that emphasises these professional abilities allows for a substantial improvement in teaching practice, tending to facilitate the creation of a learning environment in which students are active agents of their own mathematical development.

Jacobs et al. (2010) evaluate noticing skills of 131 preservice and in-service teachers. Their study shows that knowledge in attending to children's strategies increases with teaching experience. In the attention paid by teachers to children's strategies in solving numerical problems, the way teachers describe and analyse students' mathematical activity is crucial, for instance, analysing how children counted, how they used different strategies to represent quantities or how they decomposed numbers to facilitate their manipulation. In relation to interpretating students' activities, teachers made sense of the details of each strategy and observed how these details reflected on children understanding, as well as recognising what strategies and understandings the children did not demonstrate. Finally, decision-making easily focused on posing new numerical problems, taking close control of the selection of the numbers proposed in those problems.

Leavy and Hourigan (2022) describe an assessment framework developed to focus preservice teachers' attention on identifying, analysing and modifying the characteristics of mathematical problems for use in primary school (aged 5 to 12) classrooms. The model consists of eight indicators: (a) use of a motivating and attractive context; (b) clarity in language and cultural context; (c) curricular coherence; (d) attention to cognitive demand; (e) an adequate number of solution steps to support reasoning; (f) a variety of solution strategies; (g) facilitation of multiple solutions; and (h) opportunity and success. Their study found that the preservice primary school teachers that explored the model presented a limited repertoire of contexts for posing mathematical problems, and language and terminology posed challenges to understanding mathematical problems, as did the adoption of problems from other countries both for their cultural context and for the mathematical cultural systems of the students. It is evident that the lack of coherence with the curriculum sometimes led preservice teachers to pose problems that were too cognitively demanding, which meant the students were unable to solve them. Likewise, teachers reported that the mathematical problems that present a variety of strategies for their solution provided various access routes to bring students closer to their solution, thus satisfying their different needs. Regarding multiple solutions, they seemed more difficult to pose by future teachers. Finally, with regard to opportunities for success, the preservice teachers focused on providing students, through mathematics problems, with positive mathematics experiences, thus promoting self-efficacy and self-confidence as mathematics teachers.

Callejo and Zapatera (2017) characterise profiles of the teaching skill "perceiving students' mathematical thinking". To do this, preservice primary school teachers described and interpreted the responses of three primary school students to three linear pattern generalisation problems via a questionnaire. The results show that the preservice teachers detect various mathematical elements to describe the student's responses, for example, spatial and numerical structures and functional relationships. However, they did not use the identified mathematical elements to interpret the primary school students' understanding of pattern generalisation. Later, Zapatera and Callejo (2018) did another study on this skill and pattern generalisation. The data showed that preservice teachers with a low level of mathematical knowledge, and some with a sufficient level, were unable to interpret the students' understanding. However, this paper highlights that some teachers express generic and ambiguous comments, without a clear reference to the mathematical elements that characterise the generalisation process.

Coskun et al. (2023) investigated preservice primary school teachers' perceived experience of students' mathematical thinking within the context of fractions. The findings indicate that most preservice teachers showed a limited level in all professional observation skills: paying attention, interpreting and deciding how to respond. In relation to fractions, Ivars et al. (2018) examined whether using a hypothetical learning trajectory as a guide to focusing on students' mathematical thinking could improve the professional discourse and noticing of preservice teachers. Based on the analysis of three tasks on the part-whole meaning of the fraction, preservice teachers had difficulties in identifying and using the mathematical elements to interpret the students' mathematical thinking in at least one of the tasks. It should be noted that the use of the learning trajectory benefited the development of a more detailed discourse when interpreting the students' mathematical thinking, improving their observation capacity. More specifically, the study shows that a hypothetical learning trajectory can help teachers identify learning objectives, interpret students' mathematical thinking and respond with appropriate instruction linked to educational decision-making.

In a similar context to those of the learning trajectories, Sánchez-Matamoros et al. (2021) identified characteristics of the instrumental genesis process in a preservice early childhood education when looking at a classroom situation using a learning trajectory about measure and measurements. The results reveal characteristics of the noticing competence developed by the preservice teacher: (a) giving sense or meaning to the mathematical elements of the trajectory, such as recognition of the measure and measurement of length, conservation (recognising equivalence between measurement of different lengths using different materials, construction of units of measurement, among others); and (b) considering the sequentiality of the levels of understanding through the choice and design of mathematical tasks allows the construction of use schemes to make decisions. Moreover, similarly to the results shown by Ivars et al. (2018), the authors argue that learning trajectories can be used to support professional noticing skills.

Immersive video, which records a spherical view of an environment, is another resource that has been used to deepen the perception and noticing capacity of teachers. The results show that the use of videos allows teachers to pay greater attention to mathematical and didactic strategies, as well as to pay attention to more general pedagogical aspects concerning the general organisation of the teaching and learning process (Ferdig & Kosko, 2020). They can also enable closer observation of the set of students' actions by making more sophisticated descriptions of students' thinking (Kosko et al., 2021).

Schack et al. (2013) examined the professional observation skills of preservice primary school teachers in the stages of early arithmetic learning for a year. The results indicate the potential for future teachers to develop professional observation skills. Findings suggest that preservice teachers were better able to remember before and after the professional development intervention the details of the students' strategies (attention) than to interpret the students' mathematical thinking, and they also demonstrated significant growth in the three components of noticing. Tyminski et al. (2021) analysed the perception of preservice primary school teachers regarding students' mathematical thinking. The results show the success of the future teachers in the three facets of observation (paying attention, interpreting and deciding), with a particular effect on interpretation. The ability to encourage reflection of the strategies used by the students when solving a mathematical task and to encourage students to explore additional strategies and to establish connections during the development of the task stood out. Decision-making was the most complex skill to develop for preservice teachers.

Luna and Selmer (2021) investigated how an experienced primary school teacher responds to her students' thinking as part of her teacher observation practice. To do this, the teacher investigated the students' thinking based on evidence from their lessons and discussed this thinking. Supported by the analysis of this evidence, the teacher focused on the students' need to share ideas, clarify them, develop conceptual understanding, debate and support these ideas, build connections between them, participate in problem solving and experience the activity as more manageable/understandable.

Finally, Wager (2014) analysed how in-service primary school teachers pay attention to student engagement in mathematics classrooms. Teachers focused on analysing (a) how students change their participation and behaviour when, for instance, teachers inform more of children actions, ideas or proposals; (b) what teachers had done to support students' participation; and (c) how their pedagogical practice may or may not have supported students' participation during the class. Regarding interpretation, they structured the activities to support participation and interpret how the students participated in the class. Finally, the authors noted general changes in the way teachers planned the teaching of mathematics and reported explicit changes in teachers' practice.

Discussion and conclusions

In this study, we have carried out a systematic review that reports on 40 research articles published between 2010 and 2023 on responsive teaching and/or noticing of early childhood education and primary school mathematics teachers, allowing us to investigate the research and analysis foci on which such studies have focused.

One contribution of the systematic review, which complements previous reviews (e.g. Amador et al., 2021; König et al., 2022; López, 2021; Miller et al., 2023; Young and Young, 2023), is the inclusion of various descriptive data from the studies that have been carried out so far. The most representative findings show that the studies focused on responsive teaching and/or noticing of mathematics teachers have been published since 2013, with the most productive countries being the United States (56.8%), followed by Spain (10.8%) and Australia (8.1%). The most discussed mathematical content was numbers, operations and their properties (19.5%), which is in line with the curricular guidelines of organisations with a wide international impact such as the national Council of Teachers of Mathematics (NCTM, 2000), which gives a leading role to this standard in the early childhood and primary education levels. Another representative finding is that more studies have been conducted at primary school level (75.7%) than in the early childhood stage (16.2%), and very few studies have addressed and compare both educational levels together. In relation to methodological questions, most of the studies used qualitative methodologies (70.3%), most being based on video recordings (35.1%).

To examine further the specific characteristics of the studies, an analysis of the objectives, research questions and main results was carried out. As noted above, this made possible the identification of key themes, which constitute a second contribution of this systematic review. In other words, it provides a renewed understanding of responsive teaching and observation in mathematics education. Three key features are highlighted: the culturally informed decision-making, the identification of general features of noticing competence and the development of specific professional observational skills. Responsive teaching requires teachers to be able to make decisions based on the interpretation of students' ideas, integrating students' cultural and personal knowledge on these decisions and actions. This ability not only promotes more equitable and meaningful teaching but also enables students to actively engage as epistemic agents in their learning. The findings of this systematic review highlight the importance of including specific training strategies that strengthen these skills in the preservice and in-service teacher education proposals.

Although in-depth knowledge of mathematical content is necessary for the development of advanced observational competence, the findings suggest that this knowledge alone is not sufficient. The ability to make effective didactic and pedagogical decisions also depends on factors such as practical experience, specific training in observation, the transference of tools supporting the observation and analysis and specific tools helping teachers to interpret and respond to students' suggestions in the context of their cultural and social learning. From the detailed analysis of these topics regarding responsive teaching and noticing of mathematics teaching, the following conclusions can be drawn:

– The teachers' attention must be active, not passive, and requires multiple decisions and actions about what to pay attention to and what to ignore, and it requires an interpretation of events that, in turn, informs and influences subsequent responses.
– When teachers adopt a culturally responsive stance in mathematics teaching, they integrate the knowledge and experiences of students as they occur in their cultures, homes and communities (Kalinec-Craig et al., 2019).
– Professional noticing skills (which include attending to students' strategies, interpreting students' mathematical understanding, and deciding how to respond based on students' understanding, among other gestures or actions) are interrelated and interdependent. We can therefore infer that supporting the development of teachers' specific strategies in the observation and analysis of students' actions and responses may help to raise their overall levels of attention and interpretation.
– All three observation skills can be developed. However, decision-making is the most complex skill to develop and involves several elements such as, for example, the mathematical knowledge that teachers have.
– Improvement of the noticing competence is linked to the future teachers' knowledge of the mathematical content (Ivars et al., 2018; Zapatera & Callejo, 2018).
– To advance towards the acquisition of observation skills, one training strategy that seems to be important involves the incorporation into the initial training of teachers—more specifically, in the teaching subjects of mathematics—of written or oral excerpts of the children's work in the classroom; that is, their response and ways of thinking when faced with a specific mathematical task.

With regard to the study's limitations, a main one is the exclusive consideration of scientific articles published in the Web of Science and Scopus databases, which means that other publications were not accessed (e.g. conference proceedings and book chapters of international prestige, subjected to peer review). These might have provided additional data and further strengthened the findings of the review. Another limitation is that scientific studies published in languages other than English and Spanish were not accessed. While currently most of the scientific literature is published in English, there are also publications of international impact in other languages.

With regard to future lines of research, it will be necessary to explore more data published in other kinds of publication and in other languages to complement or reinforce the obtained findings. Furthermore, from the perspective of the professional development of mathematics teachers, it will be necessary to design and implement training proposals that focus on further improving the features of responsive teaching and the training of mathematics teachers. In particular, these studies should highlight specific training techniques to promote the knowledge and skills needed in preservice and in-service teacher education.

Funding Open Access funding provided thanks to the CRUE-CSIC agreement with Springer Nature. Funding work carried out as part of 2020 ARMIF 00007, funded by the Agència de Gestió d'Ajuts Universitaris i de Recerca (AGAUR) of Catalonia (Spain). Open Access funding provided thanks to the CRUE-CSIC agreement with Springer Nature. Open Access funding provided thanks to the CRUE-CSIC agreement with Springer Nature.

Data Availability Data generated or analysed during this study are available from the authors on request.

Declarations

Ethical approval Authors declare that the study does not require ethical approval as it is based on existing literature.

Conflict of interest The authors declare no competing interests.

References

Acquah, E. O., & Szelei, N. (2020). The potential of modelling culturally responsive teaching: Pre-service teachers' learning experiences. *Teaching in Higher Education, 25*(2), 157–173. https://doi.org/10.1080/13562517.2018.1547275

Al Aleeli, S. (2021). Empowering female pre-service teachers through culturally responsive teaching: Shared and divergent views of Arab and expatriate teacher educators in the United Arab Emirates. *Education 3-13, 50*(5), 592–611. https://doi.org/10.1080/03004279.2021.1876133

Amador, J. M., Bragelmann, J., & Castro, A. (2021). Notación de futuros maestros: Una revisión de la literatura de enfoques metodológicos para apoyar y analizar la observación. *Enseñanza y Formación Del Profesorado, 99*, 103256. https://doi.org/10.1016/j.tate.2020.103256

Atkins, L. J., & Frank, B. W. (2015). Examining the products of responsive inquiry. In A. D. Robertson, R. Scherr, & D. Hammer (Eds.), *Responsive teaching in science and mathematics* (pp. 56–84). Routledge.

Baker, C. K., & Galanti, T. M. (2017). Integrating STEM in elementary classrooms using model-eliciting activities: Responsive professional development for mathematics coaches and teachers. *International Journal of STEM Education, 4*(10), 1–15. https://doi.org/10.1186/s40594-017-0066-3

Brenneman, K., Lange, A., & Nayfeld, I. (2019). Integrating STEM into preschool education; designing a professional development model in diverse settings. *Early Childhood Education Journal, 47*, 15–28. https://doi.org/10.1007/s10643-018-0912-z

Callejo, M. L., & Zapatera, A. (2017). Prospective primary teachers' noticing of students' understanding of pattern generalization. *Journal of Mathematics Teachers Education, 20*, 309–333. https://doi.org/10.1007/s10857-016-9343-1

Campbell, P., & Griffin, M. (2017). Reflections on the promise and complexity of mathematics coaching. *The Journal of Mathematical Behavior, 46*, 163–176. https://doi.org/10.1016/j.jmathb.2016.12.007

Carter, I. S. W., & Amador, J. (2015). Lexical and indexical conversational components that mediate professional noticing during lesson study. *Eurasia Journal of Mathematics, Science and Technology Education, 11*(6), 1339–1361. https://doi.org/10.12973/eurasia.2015.1392a

Coskun, S. D., Sitrava, R. T., & Bostan, M. I. (2023). Pre-service elementary teachers' noticing expertise of students' mathematical thinking: The case of fractions. *International Journal of Mathematical Education in Science and Technology, 54*(6), 982–999. https://doi.org/10.1080/0020739X.2021.1979260

Crespo, S., Bowen, D., Buli, T., Bannister, N., & Kalinec-Craig, C. (2021). Supporting prospective teachers to notice and name student language resources as mathematical strengths. *ZDM, 53*, 461–473. https://doi.org/10.1007/s11858-020-01205-2

Dunning, A. (2023). A framework for selecting strategies for whole-class discussions. *Journal of Mathematics Teacher Education, 26*, 433–454. https://doi.org/10.1007/s10857-022-09536-5

Ferdig, R. E., & Kosko, K. W. (2020). Implementing 360 video to increase immersion, perceptual capacity, and teacher noticing. *TechTrends, 64*, 849–859. https://doi.org/10.1007/s11528-020-00522-3

Francis, K., & Jacobsen, M. (2013). Synchronous online collaborative professional development for elementary mathematics teachers. *The International Review of Research in Open and Distributed Learning, 14*(3), 319–343. https://doi.org/10.19173/irrodl.v14i3.1460

Gay, G. (2000). *Culturally responsive teaching: Theory, research, and practice*. Teachers College Press.

Gay, G. (2002). Preparing for culturally responsive teaching. *Journal of Teacher Education, 53*(2), 106–116. https://doi.org/10.1177/0022487102053002003

Graue, M. E., Whyte, K. L., & Karabon, A. E. (2015). The power of improvisational teaching. *Teaching and Teacher Education, 48*, 13–21. https://doi.org/10.1016/j.tate.2015.01.014

Hammer, D., Goldberg, F., & Fargason, S. (2012). Responsive teaching and the beginnings of energy in a third grade classroom. *Review of Science, Mathematics and ICT Education, 6*(1), 51–72.

Hawkins, S., & Rogers, M. (2016). Tools for reflection: Video-based reflection within a preservice community of practice. *Journal of Science Teacher Education, 27*(4), 415–437. https://doi.org/10.1007/S10972-016-9468-1

Hernandez, C., & Shroyer, M. G. (2017). The use of culturally responsive teaching strategies among Latina/o student teaching interns during science and mathematics instruction of CLD students. *Journal of Science Teacher Education, 28*(4), 367–387. https://doi.org/10.1080/1046560X.2017.1343605

Ivars, P., Fernández, C., Llinares, S., & Choy, B. H. (2018). Enhancing noticing: Using a hypothetical learning trajectory to improve pre-service primary teachers' professional discourse. *Eurasia Journal of Mathematics, Science and Technology Education, 14*(11), em1599. https://doi.org/10.29333/ejmste/93421

Jaber, L. Z., Southerland, S., & Dake, F. (2018). Cultivating epistemic empathy in preservice teacher education. *Teaching and Teacher Education, 72*, 13–23. https://doi.org/10.1016/j.tate.2018.02.009

Jacobs, V. R., Lamb, L. L. C., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education, 4*(2), 169–202. https://doi.org/10.5951/jresematheduc.41.2.0169

Jazby, D., Widjaja, W., Xu, L., & van Driel, J. H. (2023). Noticing student thinking under pressure in primary mathematics and science lessons. *International Journal of Science and Mathematics Education, 21*, 645–666. https://doi.org/10.1007/s10763-022-10263-9

Jensen, J. L., Gallagher, M. A., Scrivner, S., & Mitchell, M. L. (2023). Preservice teachers equitably attending to and interpreting multilingual students' mathematical thinking. *School Science and Mathematics, 123*(8), 447–460. https://doi.org/10.1111/ssm.12608

Jessup, N. A. (2023). Exemplifying the situated nature of teacher noticing: Elementary teachers' use of insider knowledge. *School Science and Mathematics, 123*(8), 502–514. https://doi.org/10.1111/ssm.12598

Kalinec-Craig, C. A., Prasad, P. V., & Luna, C. (2019). Geometric transformations and Talavera tiles: A culturally responsive approach to teacher professional development and mathematics teaching. *Journal of Mathematics and the Arts, 13*(1–2), 72–90. https://doi.org/10.1080/17513472.2018.1504491

Kalinec-Craig, C. A., Bannister, N., Bowen, D., Jacques, L. A., & Crespo, S. (2021). "It was smart when: " Supporting prospective teachers' noticing of students' mathematical strengths. *Journal of Mathematics Teacher Education, 24*, 375–398. https://doi.org/10.1007/s10857-020-09464-2

Kang, H., & Anderson, C. W. (2015). Supporting preservice science teachers' ability to attend and respond to student thinking by design. *Science Teacher Education, 99*(5), 863–895. https://doi.org/10.1002/sce.21182

König, J., Santagata, R., Scheiner, T., Adleff, A.-K., Yang, X., & Kaiser, G. (2022). Teacher noticing and teacher professional vision: A systematic literature review on conceptualizations and research designs. *Educational Research Review, 36*, 100453. https://doi.org/10.1016/j.edurev.2022.100453

Kosko, K. W., Ferdig, R. E., & Zolfaghari, M. (2021). Preservice teachers' professional noticing when viewing standard and 360 video. *Journal of Teacher Education, 72*(3), 284–297. https://doi.org/10.1177/0022487120939544

Krippendorff, K. (2019). *Content analysis: An introduction to its methodology* (4th ed.). SAGE Publications, Inc. https://doi.org/10.4135/9781071878781

Land, T. J., Tyminski, A. M., & Drake, C. (2019). Examining aspects of teachers' posing of problems in response to children's mathematical thinking. *Journal of Mathematics Teacher Education, 22*, 331–353. https://doi.org/10.1007/s10857-018-9418-2

Leavy, A. M., & Hourigan, M. (2016). Using lesson study to support knowledge development in initial teacher education: Insights from early number classrooms. *Teaching and Teacher Education, 57*, 161–175. https://doi.org/10.1016/j.tate.2016.04.002

Leavy, A., & Hourigan, M. (2022). The framework for posing elementary mathematics problems (F-PosE): Supporting teachers to evaluate and select problems for use in elementary mathematics. *Educational Studies in Mathematics, 111*, 147–176. https://doi.org/10.1007/s10649-022-10155-3

Lee, M. Y., & Lee, J. E. (2023). An analysis of elementary prospective teachers' noticing of student pattern generalization strategies in mathematics. *Journal of Mathematics Teacher Education, 26*, 155–177. https://doi.org/10.1007/s10857-021-09520-5

López, L. M. (2021). Noticing: Una revisión bibliográfica sobre los orígenes y las perspectivas actuales. *Revista Chilena De Educación Matemática, 13*(3), 79–92.

Luna, M., & Selmer, S. (2021). Examining the responding component of teacher noticing: A case of one teacher's pedagogical responses to students' thinking in classroom artifacts. *Journal of Teacher Education, 72*(5), 579–593. https://doi.org/10.1177/00224871211015980

Marimon-Martí, M., & Martí, J. (Eds.) (2023). *Orientacions per introduir l'ensenyament responsiu a la formació inicial de mestres (Guidelines for introducing responsive teaching in initial teacher education).* University of Vic - Central University of Catalonia Publications Services.

Maskiewicz, A., & Winters, V. (2012). Understanding the co-construction of inquiry practices: A case study of a responsive teaching environment. *Journal of Research in Science Teaching, 49*(4), 429–464.

Mason, J. (2002). *Researching your own practice: The discipline of noticing*. Routledge.

Melhuish, K., Thanheiser, E., & Guyot, L. (2020). Elementary school teachers' noticing of essential mathematical reasoning forms: Justification and generalization. *Journal of Mathematics Teacher Education, 23*, 35–67. https://doi.org/10.1007/s10857-018-9408-4

Miles, M., Huberman, M., & Saldaña, J. (2013). *Qualitative data analysis: A methods sourcebook* (3rd ed.). SAGE Publications, Inc.

Miller, R. D., Gerzel-Short, L., Kelly, J. H., Uphold, N., Hsiao, Y.-J., Hovey, K. A., & Wei, Y. (2023). Preparing culturally responsive teachers: A systematic review. *Multicultural Learning and Teaching*. https://doi.org/10.1515/mlt-2023-0004

Mitchell, R. N., & Marin, K. A. (2015). Examining the use of a structured analysis framework to support prospective teacher noticing. *Journal of Mathematics Teacher Education, 18*, 551–575. https://doi.org/10.1007/s10857-014-9294-3

Moher, D., Shamseer, L., Clarke, M., Ghersi, D., Liberati, A., Petticrew, M., Shekelle, P., & Stewart, L.A. (2015). Preferred reporting items for systematic review and meta-analysis protocols (PRISMA-P) 2015 statement. *Systematic Reviews, 4*(1). https://doi.org/10.1186/2046-4053-4-1

Namakshi, N., Warshauer, H. K., Strickland, S., & McMahon, L. (2022). Investigating preservice teachers' assessment skills: Relating aspects of teacher noticing and content knowledge for assessing student thinking in written work. *School Science and Mathematics, 122*(3), 142–154. https://doi.org/10.1111/ssm.12522

O'Keeffe, L., Paige, K., & Osborne, S. (2019). Getting started: Exploring pre-service teachers' confidence and knowledge of culturally responsive pedagogy in teaching mathematics and science. *Asia-Pacific Journal of Teacher Education, 47*(2), 152–175. https://doi.org/10.1080/1359866X.2018.1531386

Osmanoglu, A., Isiksal, M., & Koc, Y. (2015). Getting ready for the profession: Prospective teachers' noticing related to teacher actions. *Australian Journal of Teacher Education, 40*(2), 29–51. https://doi.org/10.14221/ajte.2015v40n2.3

Owens, K. (2015). Changing the teaching of mathematics for improved Indigenous education in a rural Australian city. *Journal Mathematics Teachers Education, 18*, 53–78. https://doi.org/10.1007/s10857-014-9271-x

Prediger, S., Roesken-Winter, B., Stahnke, R., & Pöhler, B. (2022). Conceptualizing content-related PD facilitator expertise. *Journal of Mathematics Teacher Education, 25*, 403–428. https://doi.org/10.1007/s10857-021-09497-1

Robertson, A. D., Scherr, R. E., & Hammer, D. (Eds.). (2016). *Responsive teaching in Science and Mathematics*. Routledge.

Sánchez-Matamoros, G., Moreno, M., & Valls, J. (2021). Instrumental genesis of a learning trajectory: The case of Pedro's professional noticing. *Acta Scientiae (Canoas), 23*(7), 91–119. https://doi.org/10.17648/acta.scientiae.6296

Schack, E. O., Fisher, M. H., Thomas, J. N., Eisenhardt, S., Tassell, J., & Yoder, M. (2013). Prospective elementary school teachers' professional noticing of children's early numeracy. *Journal of Mathematics Teacher Education, 16*, 379–397. https://doi.org/10.1007/s10857-013-9240-9

Sherin, M. G., Jacobs, V. R., & Philipp, R. A. (2011). *Mathematics teacher noticing: Seeing through teachers' eyes*. Routledge.

Simpson, A., & Haltiwanger, L. (2017). "This is the first time I've done this": Exploring secondary prospective mathematics teachers' noticing of students' mathematical thinking. *Journal of Mathematics Teacher Education, 20*, 335–355. https://doi.org/10.1007/s10857-016-9352-0

Star, J. R., & Strickland, S. K. (2008). Learning to observe: Using video to improve preservice mathematics teachers' ability to notice. *Journal of Mathematics Teacher Education, 11*, 107–125. https://doi.org/10.1007/s10857-007-9063-7

Talanquer, V., Tomanek, D., & Novodvorsky, I. (2013). Assessing students' understanding of inquiry: What do prospective science teachers notice. *Journal of Research in Science Teaching, 50*(2), 189–208.

Talanquer, V., Bolger, M., & Tomanek, D. (2015). Exploring prospective teachers' assessment practices: Noticing and interpreting student understanding in the assessment of written work. *Journal of Research in Science Teaching, 52*(5), 585–609. https://doi.org/10.1002/tea.21209

Throop Robinson, E., Lunney Borden, L., & Carter, E. (2021). Building teacher capacity and leadership in elementary mathematics classrooms in Nova Scotia: Review of the certificate in elementary mathematics pedagogy. *Canadian Journal of Science, Mathematics and Technology Education, 21*, 856–874. https://doi.org/10.1007/s42330-021-00183-1

Tyminski, A. M., Simpson, A. J., Land, T. J., Drake, C., & Dede, E. (2021). Prospective elementary mathematics teachers' noticing of childrens' mathematics: A focus on extending moves. *Journal of Mathematics Teacher Education, 24*, 533–561. https://doi.org/10.1007/s10857-020-09472-2

Umultu, D., & Kim, C. (2020). Design guidelines for scaffolding pre-service teachers' reflection-in-action toward culturally responsive teaching. *Reflective Practice, 21*(5), 587–603. https://doi.org/10.1080/14623943.2020.1779049

van Es, E. A. (2011). A framework for learning to notice student thinking. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 117–133). Routledge.

van Es, E. A., & Sherin, M. G. (2002). Learning to notice: Scaffolding new teachers' interpretations of classroom interactions. *Journal of Technology and Teacher Education, 10*(4), 571–596.

van Es, E. A., & Sherin, M. G. (2021). Expanding on prior conceptualizations of teacher noticing. *ZDM, 53*, 17–27. https://doi.org/10.1007/s11858-020-01211-4

Wager, A. A. (2014). Noticing children's participation: Insights into teacher positionality toward equitable mathematics pedagogy. *Journal for Research in Mathematics Education, 45*(3), 312–350. https://doi.org/10.5951/jresematheduc.45.3.0312

Wager, A. A., & Parks, A. N. (2016). Assessing early number learning in play. *ZDM, 48*, 991–1002. https://doi.org/10.1007/s11858-016-0806-8

Xenofontos, C., & Alkan, S. H. (2022). Prospective primary teachers' professional noticing in non-formal learning environments: The case of a mathematics fair. *Education Sciense, 12*(1), 55. https://doi.org/10.3390/educsci12010055

Xu, L., Widjaja, W., & Ferguson, J. (2019). Seeing through the eyes of the teacher? Investigating Primary School Teachers' Professional Noticing through a Video-Based Research Methodology. *International Journal of Research & Method in Education, 42*(5), 470–484. https://doi.org/10.1080/1743727X.2018.1499016

Young, J., & Young, J. (2023). A systematic review of culturally responsive teaching self-efficacy using confidence intervals. *Multicultural Learning and Teaching, 18*(2), 251–280. https://doi.org/10.1515/mlt-2021-0011

Zapatera, A., & Callejo, M. L. (2018). El conocimiento matemático y la mirada profesional de estudiantes para maestro en el contexto de la generalización de patrones. Caracterización de perfiles. *Revista Complutense de Educación, 29*(4), 1217–1235. https://doi.org/10.5209/RCED.55070

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Authors and Affiliations

Ángel Alsina[1] · Nataly Pincheira[1] · Berta Barquero[2] · Isabel Sellas[3] · Marta Marimon-Martí[3]

✉ Ángel Alsina
angel.alsina@udg.edu

Nataly Pincheira
nataly.pincheira@udg.edu

Berta Barquero
bbarquero@ub.edu

Isabel Sellas
isabel.sellas@uvic.cat

Marta Marimon-Martí
marta.marimon@uvic.cat

[1] University of Girona, Girona, Spain

[2] University of Barcelona, Barcelona, Spain

[3] University of Vic-Central University of Catalonia, Barcelona, Spain