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Mathematical knowledge of pre-service teachers when performing measurement estimation tasks

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Abstract

Measurement estimation skills are relevant for our everyday life and should be addressed in the mathematics classroom. This study explores the measurement estimation strategies used by primary school pre-service teachers in length, area, volume, and capacity. The study participants consisted of 120 pre-service teachers who had not yet received formal university training on these concepts. While more than 80% demonstrated the ability to indicate a strategy and provide acceptable estimates for length measurements, significant difficulties were observed in estimating area, volume, and capacity, with many participants relying heavily on formulas. A series of activities were designed and implemented to improve pre-service teachers' capacity for estimation. Results indicate a notable improvement in estimation skills following completion of the designed tasks. This study highlights the importance of addressing challenges in measurement estimation among pre-service teachers and offers insights for improving their proficiency in this area.

Keywords: estimation, measurement of magnitudes, primary education, pre-service teachers, mathematical knowledge

INTRODUCTION

Measurement estimation skills are relevant for our everyday life, by both children and adults, surpassing other measurement processes and should be addressed in the mathematics classroom (Hoth et al., 2023). This prevalence stems from various factors: the unavailability or inadequacy of measurement tools, and the absence of a necessity for high accuracy in certain tasks (Çilingir Altıner, 2024; Joram et al., 2005; Kılıç & Olkum, 2013). For example, while driving a car, we keep a certain distance behind the vehicle in front of us to make sure there is enough space between us to avoid having to stop suddenly. Or when cooking, we know when we have enough rice for everyone. Therefore, in recent years, the estimation of magnitudes has taken on a relevant role within the content block of measurement and have garnered significant attention from researchers across different educational stages and perspectives (Baturo & Nason, 1996; Goya, 2011; Papadopoulos, 2010; Pizarro et al., 2014).

The mathematics educational concept of measurement skills encompasses the knowledge of various measurable attributes, the methods of measurement, and the units required to express the results. It stands as a crucial topic in mathematical education, aligning with the standards outlined in *principles and standards for school mathematics* (NCTM, 2000). Among these standards, measurement aims to equip students with the ability to identify, quantify, compare, and order the measurements of objects. Understanding the units of measurement and the correct use of measuring instruments is considered to be basic. This highlights the importance of working on estimation at primary and secondary levels and not only in the number and operation content standards, but also in the measurement (Attivo, 1979; Bright, 1976; Castillo et al., 2011; Hildreth, 1983; Pizarro et al., 2018; Segovia et al., 1989). This implies going beyond the use of formulas in the measurement processes (Barrera et al., 2019; Luelmo, 2001; Runnalls & Hong, 2020; López-Serentill, 2022), working on different strategies for estimation in contextualized, significant situations (Moreno et al., 2015).

In the same line, international reports and curriculum documents (Department of Education [England], 2014;

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Contribution to the literature

- The current study describes the methodologies used by primary school pre-service teachers when solving tasks of estimating measurements of length, area, volume and capacity.
- Pre-service teachers presented significant deficits in the knowledge and mastery of the units of measurement and their magnitudes, especially in the case of area, volume, and capacity.
- Building on insights from previous research (Bright, 1976; Castillo et al., 2011), our study offers pedagogical strategies and activities to enhance pre-service teachers' skills in estimating measurement.

MEFP, 2022; NCTM, 2000) have highlighted the importance and the need to work on the measurement estimation, something which constitutes the basis for learning measurement. However, in some countries, estimation has not been fully incorporated into teaching mathematics, neither in schools (Hogan & Brezinski, 2003; Ruwisch et al., 2015), nor in teacher education. In many cases, the teaching of measurement continues to focus on teaching the units of the decimal metric system and conversions between them. Or teaching the use of formulas for the calculation of magnitudes, thereby losing contact with key elements in the construction of the sense of magnitude. Some of these elements include discovering the main measurable magnitudes through experimentation, acquiring the notion of unit of measurement, practicing with realistic contents, discovering the meaning of approximate measurements or using estimation and measurement tools (Alsina, 2003).

Both children and adults tend to make poor measurement estimations (Er et al., 2021; Joram et al., 2005). This is unsurprising, as estimation is frequently neglected in teaching (Andrews et al., 2022; Ruwisch et al., 2015), likely due to teachers' uncertainty about how to effectively teach this skill (Joram et al., 2005; Pizarro et al., 2015). Another cause can be found by analyzing textbooks where sometimes estimation tasks are almost non-existent (Andrews et al., 2022; Castillo et al., 2011).

According to some authors (Jones et al., 2009; Joram et al., 2005) measurement estimations skills are learnable and can be improved by teaching interventions. Therefore, different teacher education may have a significant influence on students' estimation skills (Hoth et al., 2023). In order to develop an effective mathematics lesson, pre-service mathematics teachers need to have adequate knowledge for teaching mathematics (López-Serentill, 2022). The professional knowledge of teachers has been a topic of interest in the field of mathematics education in recent decades (Hill et al., 2008; Hoover, 2014; Pincheira & Alsina, 2021; Runnalls & Hong, 2020). This is due to the need for better prepared teachers to teach mathematics, especially at the early levels, as "teachers are the key to mathematics learning opportunities" (Even & Ball, 2009, p. 1-2); that is, the quality of teaching is closely related to teachers, their knowledge and their preparation for teaching, which directly impacts on the learning and development of mathematical competences of their students (Darling-Hammond & Bransford, 2005; Hattie, 2012). Therefore, if students' mathematical education needs to be improved, special attention needs to be paid to the teacher's knowledge. Given such evidence, developing mathematical knowledge for teaching (MKT) is an essential goal for teacher educators at all levels (Runnalls & Hong, 2020).

Although research on measurement estimation has increased in recent years, particularly at the primary school level (Andrews et al., 2022; Çilingir Altıner, 2024; Hoth et al. 2023) and at the secondary school level (Er et al., 2021; Huang, 2020; Satan et al., 2022), there are still too few studies focused on in-service and pre-service teachers when performing measurement estimation tasks (Castillo-Mateo et al., 2017; Pizarro et al., 2018). This study aims to contribute to this topic in two main ways. Firstly, it aims to provide information on the common content knowledge (CCK) in measurement estimation among pre-service teachers by analyzing the strategies used when performing length, area, volume, and capacity estimation tasks. Secondly, based on the literature review, the study implemented an intervention process in the classroom designed to enhance pre-service teachers' estimation capacity. The goal of this intervention is to improve pre-service teachers' measurement estimations and to equip them with practical techniques that they can utilize effectively in their classrooms. Consequently, the study formulates the following research questions: what is CCK of measurement estimation that pre-service teachers have? Furthermore, to what extent did the intervention with the task sequence improve the knowledge of the pre-service teachers?

THEORETICAL FRAMEWORK

Mathematical Knowledge for Teaching

The knowledge required for effective teaching is complex and made up of several distinct yet interactive pieces (Runnalls et al., 2020). In the domain of mathematics, several frameworks have been developed since the late 1980's to describe the knowledge required for teaching this subject. Schulman (1987) was one of the first to popularize the distinction between *content knowledge* and *pedagogical content knowledge* (PCK). Ball et al. (2008) based on the knowledge framework proposed

Table 1. Types of estimates

Unit of reference to be used is physically	Object to be estimated is absent	Object to be estimated is present
Absent	Type 1: Object absent/unit of reference absent	Type 2: Object present/unit of reference absent
Present	Type 3: Object absent/unit of reference present	Type 4: Object present/unit of reference present

by Shulman (1987), developed the model of MKT described as "the mathematical knowledge needed to carry out the work of teaching mathematics" (Ball et al., 2008, p. 395). They refine these two categories of knowledge into six domains of knowledge. The three domains under subject matter knowledge consist of CCK, specialized content knowledge (SCK), and knowledge at the mathematical horizon or horizon content knowledge (HCK). PCK is subdivided into knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of curriculum (KCC). Each of these facets of mathematical knowledge for teaching may be further explored within specific mathematical topics.

CCK refers to "mathematical knowledge and skill used in settings other than teaching" (Ball et al. 2008, p. 399) and an example would be the ability to make accurate estimating measurements. SCK is mathematical knowledge and skill that is "not typically needed for purposes other than teaching" (Ball et al. 2008, p. 400), such as knowing different measurement estimation methods and skills or understanding measurement units. HCK is defined as "awareness of how mathematical topics are related over the span of mathematics included in the curriculum" (Ball et al. 2008, p. 403) for example, experimental knowledge of magnitudes to be able to move towards the notion of unit of measurement. KCS "combines knowing about students and knowing about mathematics" (Ball et al. 2008, p. 401) and would involve understanding the specific errors or misconceptions that would lead a student to do nonacceptable measurement estimations. KCT refers to knowledge of mathematics combined with knowledge of teaching and would include knowing the different types of measurement estimation activities to improve students' estimations. Finally, KCC, "represented by the full range of programs designed for the teaching of particular subjects and topics at a given level, and the variety of instructional materials available in relation to those programs" (Ball et al., 2008, p. 391).

Measurement Estimation

Although different definitions of estimation corresponding to different tasks related to distinct areas of mathematical knowledge can be found in the literature (Pizarro et al., 2018), we understand estimation to be, according to the definition given by Segovia et al. (1989), the value judgment of the result of a numerical operation or the measurement of an amount based on the individual circumstances that it emits. As can be deduced from the above definition, two types of estimation can be differentiated: Estimation in calculation or estimative calculation, and estimation in measurement. In the first case we refer to the judgments that can be established on the results of arithmetic operations, also known as computational estimation. In the second case, which is the one that interests us, we refer to judgments that are established regarding a certain amount or the assessment that serves us as the result of a measure. Other authors, such as Hogan and Brezinski (2003) also include *numerosity* as another type of estimation, which consists of "visually estimating a number of objects arranged on a plane for a limited time" (Pizarro et al., 2015, p. 3228). When this number is small and can be evaluated quickly and accurately, the task is called *subitizing*.

If we focus on estimation in measurement, Pizarro et al. (2014, p. 528) propose as a definition: "Perceptively assign a value or a range of values and the unit corresponding to a discrete or continuous magnitude, by means of prior knowledge or by non-direct comparison to some auxiliary object". This definition is based on three essential elements: assigning a numerical value, performing the task perceptively and relating perception with previous knowledge or with the mental image of the auxiliary object. In a direct measurement process, the magnitude to be measured is compared with a pattern defined *a priori*, which acts as a unit derived from the use of a measuring instrument. On the other hand, in an estimation process there is no direct comparison with a measuring instrument, whilst a mental reference suitable for the magnitude to be estimated is used which acts as a unit to which the magnitude is being compared (Pizarro, et al. 2018).

According to Bright (1976), there are 4 types of measurement estimation activities depending on whether the unit of measure to be used in the estimate is provided and whether the object to be estimated is present or not. **Table 1** shows these four types.

When measuring estimates are made, there are different senses and skills that are worked with and therefore that must be mastered. On the one hand, in relation to the numerical sense, concepts of enumeration, number, quantity, count are worked on (Bright, 1976; Joram et al., 2005). On the other hand, in the sensory sphere, measurement estimation requires perceptual skills, since measuring instruments are not used to estimate a measurement, instead it is the senses which

Table 2. Examples of content on measurement in the Spanish curriculum

	Basic knowledge
Lower school (6-8 years)	Estimation of measures (distances, sizes, masses, capacities ...) by direct comparison with other measures.
Middle school (8-10 years)	Estimation of length, mass, & capacity measurements by comparison. Evaluation of measurement results & estimations or calculations of measurements, reasoning whether or not they are possible.
Upper middle school (10-12 years)	Measurement estimation of angles and areas by comparison. Evaluation of measurement results and estimations or calculations of measurements, reasoning whether or not they are possible.

are used, and this entails recognizing and assimilating units of measurement (Hogan & Brezinski, 2003). Finally, problem-solving skills are also needed (Pizarro et al., 2018).

Castillo et al. (2011) indicate that measurement estimation involves nine skills: understanding the quality to be estimated, perceiving what will be estimated, understanding the concept of unit of measurement, possessing a mental image of the unit of measurement to be used, possessing a mental image of referents to be used in the task, adapting the unit of measurement to be used with what is to be estimated, knowing and using appropriate terms of the estimation in measurement, selecting and using appropriate strategies to make estimations and verify the adequacy of the estimation.

From a purely quantitative perspective, when an estimation is made, we are issuing an opinion, a judgment, an assessment. So, the value we obtain is unlikely to coincide with the exact one, it's going to be an approximate value. This fact, according to Chamorro (1996) causes students to reject measurement estimation in their tasks, even in situations where it does not make sense to raise a problem of precision. Chamorro (1996) considers that in the educational scenario an idealization of objects is created, where measurements are always carried out properly and their results are mostly whole numbers. For this reason, measurement estimation lacks adequate development in school.

Estimation in the Spanish Primary Education Curriculum

Different reports and curricular documents from institutions such as the British committee "committee of inquiry into the teaching of mathematics in schools" and the American association NCTM, among others, have had a considerable influence on Spanish curricula for mathematical education (Castillo-Mateo, 2013).

The Cockcroft report (Cockcroft, 1982) points out that every adult needs to master, in a very clear and effective way, everything related to the world of magnitudes: weighing, measuring lengths, areas and volumes, knowing how to understand and understand tables and graphs, knowing how to make calculations from tables and, likewise, have the capacity to know how to make estimates and approximations (Castillo-Mateo et al., 2012).

On the other hand, one of the specific objectives of the NCTM standards of the year 2000 is to apply appropriate techniques and tools to perform measurements, where it is stated that strategies must be developed to estimate perimeters, areas and volumes of irregular shapes. The Standards propose, from early childhood education to high school, the creation of common benchmarks for both estimating and comparing. In this way, measurement estimation contributes to the development of spatial awareness and numerical concepts.

The curricular regulations in Spain (Ley Orgánica, 2006) have for some time emphasized the need to teach estimation, highlighting that the estimation of continuous quantities is an important part of the primary curriculum. In the new Spanish education law, measurement estimation appears as being basic knowledge throughout the primary education stage, being one of the objectives of the stage "to start solving problems that require the realization of elementary calculation operations, geometric knowledge and estimation, as well as being able to apply them to day-to-day situations" (BOE, 2022, p. 2439).

In **Table 2** there are some examples of how this content appears in the new Spanish curriculum.

It should be noted that in the new Spanish curriculum, in no age group, has it been possible to find any basic knowledge that explicitly refers to the understanding of the errors that are made when making measurements, when using measuring instruments or on the accuracy of these measurements according to the units of measurement used, in contrast with the NCTM that does mention these factors.

METHOD

The study is an exploratory research, since it offers a first approach to determining which methods pre-service teachers use when making estimates of measurement of magnitudes. The methodological approach is mixed (Ghauri & Gronhaug, 2010). Throw qualitative analysis, we identify the measurement estimation strategies that pre-service used to make their estimates. Additionally, quantitative analysis was conducted to compare estimates made in both the initial and final tests, providing insights into the effectiveness of interventions aimed at improving estimation skills.

Participants and Context

In order to detect and categorize the difficulties related to the measurement encountered by the students who enter primary teacher training studies, in particular the estimation of magnitude tasks, an analysis was undertaken of the initial activity carried out with four classes of 30 students (120 Spanish students in total) of the degree in primary education who had not yet received training relating to the content blocks of measurement and space and form during their degree course. In line with ethical guidelines, necessary approvals were obtained before conducting the research. The participants were explicitly informed about the study's purpose and their participation was voluntary. They were assured that their responses would be treated as confidential and solely used for research purposes. Furthermore, the participants were informed that they had the right to withdraw from the study at any point if they wished to do so.

Instruments

The activity consisted of a questionnaire with open-ended questions where all the students were asked to detail the procedure performed and justify the answers given in writing. Students had to make different estimations of the magnitudes of the square in front of the faculty: length of the street (40 m), height of the stairs (7.5 m), area of the square (185.4 m^2), volume of a bollard (75.4 m^3) and capacity of a rubbish bin (60l). The measurement unit was not given to students so they could estimate in comparison to any chosen unit. All estimation activities corresponded to task type 2 according to the classification given by Bright (1976), i.e., estimation tasks where the object to be estimated is present and the reference unit is absent.

After performing the initial test, six sessions were held to work on estimation. For the design of the sequence of measurement estimation tasks, we considered the nine factors involved in the estimation process by Castillo et al. (2011) and the indications given by Bright (1976) on how estimation should be addressed in school:

- It is advantageous for students to have the opportunity to develop an idea of the size of the unit before making estimates.
- To develop the ability to estimate, probably the best procedure is for students to first make an estimate and then perform the measurement as a test method. Bright calls this method the guess/check procedure.
- It is important that the situations posed to students are familiar to them.
- Students should be encouraged to make good estimates, but should not be penalized, however imprecise their answers may be.

Figure 1. Construction of the cubic meter (Source: Field study)

- Activities should be sequenced so that all students can improve their estimates.
- Self-assessment of estimates helps students develop self-correction skills.
- Self-checking of estimates also helps to distinguish the act of measurement from the abstract concept of measurement and provides an experimental environment for explaining measurement errors.

Two types of tasks were designed: tasks focused on re-inventing and becoming familiar with the units of measurement, and tasks focused on working on estimation. Within the latter, there were also two types of tasks. Firstly, those that were provided with the object to be estimated and the unit of measure to be used. And secondly, those where they had to choose the appropriate conventional unit of measure to estimate the indicated object. The following are some examples of each of these tasks:

(a) Units of measurement tasks: Search for parts of the body approximately one centimeter, ten centimeters, half a meter and one meter. Construction of the square meter and the cubic meter. Activity with manipulatable material to be able to determine how many cubic centimeters fit in a cubic decimeter and how many cubic decimeters fit in a cubic meter. Search for containers of different shapes that can contain one liter of water (**Figure 1**).

(b) Tasks on estimation giving the unit of measurement (estimation type 4 according to the Bright, 1976 classification): during the entire training course, at the beginning of each session an estimation task was carried out, based on the examples provided by https://estimation180.com, where they had to

Figure 2. Example of estimation task from the web estimation180.com (https://estimation180.com)

estimate, for example, how many small glasses were needed to fill the entire jug (**Figure 2**).

(c) Tasks on estimation without providing the unit of measure (type 2 estimation according to Bright classification): These tasks were undertaken after the tasks on units of measure. They were asked to make estimations of accessible and familiar objects such as the volume of the classroom, the area of the worktable, the capacity of a bath, the area of their room, etc. After making the estimates, they performed the measurement to be able to quantify and analyze the errors made.

At the end of the described sequence of activities, a final test was carried out with questions similar to the initial ones: estimating the length of a street, estimating the height of a building, estimating the area of the terrace of the bar, estimating the capacity of a container and estimating the volume of a food vending machine.

Data

To define the categories used to analyze student estimation strategies (qualitative analysis), we relied on the work on estimation strategies as performed by Castillo-Mateo (2017) and Segovia et al. (1989). The categories finally used were, as follows:

To analyze the estimates given in the initial and final test (quantitative analysis), since this is an approximate value, a criterion must be established to determine when we can consider the given value to be acceptable. In the different studies carried out in the field of estimation, researchers do not agree when establishing a criterion to determine when an estimate is acceptable, valid, correct or reasonable. For example, Attivo (1979) and Paull (1971) took 15% as the limit; Hildreth (1983) at 1/3 of the

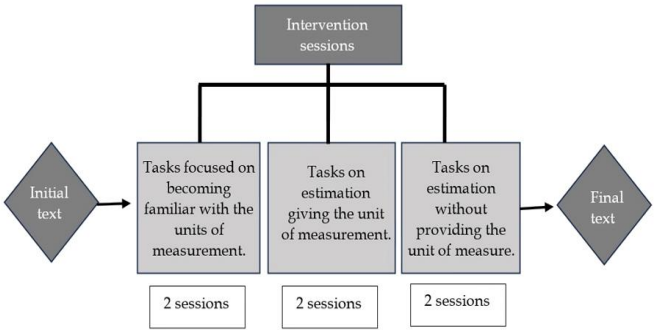

Figure 3. Description of the sequence of the intervention (Source: Authors' own elaboration)

exact value; Siegel et al. (1982) takes ±50% of the exact value as the limit or Levine (1982) establishes a graduated scale, according to which responses with an error greater than 30% are not valued. In accordance with the last author mentioned, and along with Castillo-Mateo et al. (2012) and Segovia (1997) we have decided to consider that a measure or estimate is acceptable if the relative error committed is in absolute terms less than or equal to 30% of the actual measurement.

RESULTS

The results obtained from the analysis of the pre-service teachers' responses have been grouped into two blocks. The first block refers to the initial tasks on estimation and the second block presents the comparative results between the initial and final activities so as to evaluate the impact of the execution of the sequence of estimation activities.

Measurement Estimation

This first section of results refers to the initial test that consisted of making measurement estimation tasks of length of one side of the square, area of the square, total height of some stairs, volume of a bollard and capacity of a bin. **Figure 3** shows the graph that summarizes the grouped results considering the four categories described in **Table 3** regarding the methods that pre-service teachers used to make their estimates.

As can be seen in **Figure 4**, it is in the measurements of the length of the side of the square and the height of the stairs (which in fact is also a length, but vertical) where more students used the comparison with the unit of measurement (56.7% and 33.3%, respectively) and where we find less students who made the estimates randomly (7.5% and 10%, respectively) because they have more knowledge and are more used to the use of formal length units (in this case the meter). In addition, in these two cases, when making comparisons from known references (informal units), more variety is observed, for example, for the length of the square, 43% of the students made comparisons using references such as the length of their own pace, the measurements of a single bed, the length of a pool or a football pitch and, in

Table 3. Categories used to analyze pre-service estimation processes

Category	Description	Examples
Estimation by comparison with the unit of measure	The subject compares the amount to be mentally estimated directly with a unit of measure of the decimal metric system.	To estimate the capacity of the rubbish bin they think mentally how many liters of water can fit in it.
Estimation by comparison from already known referents	The subject compares the amount to be estimated with another known amount or a multiple of this.	To estimate the length of the square, it was compared to an Olympic pool or a football pitch. To estimate the height of the stairs they thought about how many people it would take to reach the top.
Estimation using formulas.	The subject uses measurement formulas in the estimation process.	To estimate the area of the square, first estimate the width, and then the length of the square and then multiply them.
Estimation without comparison, randomly or intuitively	The subject makes an estimate without explaining how he has done it.	He indicated that he has done it "instinctively".

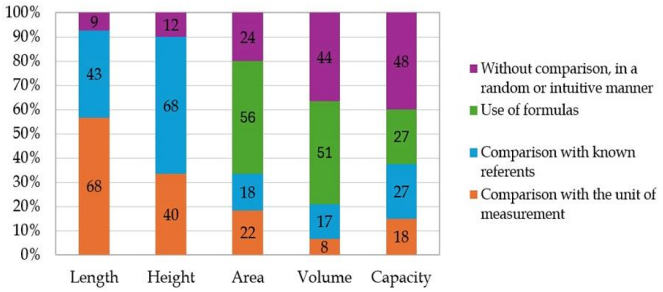

Figure 4. Relative frequencies of methods used in estimation (Source: Authors' own elaboration)

the case of the height of the staircase, 56.7% used known references, especially comparing with their height or comparing the palm of their hand with the height of a step and then multiplying by the number of steps.

In the estimates of area, volume and capacity, the percentage of students using the conventional unit of measurement is much lower (18.3% for area, 6.7% for volume, and 15% for capacity) and furthermore, as will be seen in the following section, these estimates were mostly erroneous. In the case of the area, only 15% used known referents (area of their houses, flats or rooms). In the case of volume and capacity, 14.2% and 22.5%, respectively used known referents. Most of these students, in both cases, used the capacity of a water jug or the capacity of the rubbish bags used at home (50 liters) as a reference. No specific volume reference was found. In both cases, almost 40% made estimates without random comparison and more than 50% confused volume units with capacity units. As for the use of formulas, more than 40% of students used them to make estimates of area and volume and 22.5% used them in the case of capacity. In almost half of these cases, they used them incorrectly.

To conclude this section, it should be noted that, in the reflection session that took place after the initial test, the students indicated that they had many difficulties in estimating the measurements of area, capacity and volume. This was because they were not so used to working with and using them, neither as students (most only remembered calculations from the formulas and conversion of units) nor in their day to day lives, that is, they lacked referents.

Comparative Analysis Between the Initial and Final Test Results

In order to be able to analyze how the sequence of activities contributes to the improvements identified, the results of the estimates made in the initial test and in the final test are shown below. The results have been classified into acceptable and unacceptable estimates according to whether the percentage of relative error committed is less than or greater than 30% as an absolute value (**Table 4**).

In the initial test, except in the estimates of magnitudes corresponding to a length, more than half of the students failed to make acceptable estimates, that is, they made more than 30% relative error and in the case of volume, only one in five students made an acceptable estimate. These findings align with the earlier results, where more than 30% of students performed volume and capacity estimates randomly without using any reference.

When comparing the initial and final test estimates, it is observed that, in all estimation tasks, the number of acceptable results increases, indicating enhanced estimation capacity among students. Furthermore, the increase in acceptable estimates is more pronounced for certain tasks (such as area, volume, and capacity) compared to others (like length and height). This discrepancy arises because students had familiar references for length estimates, aiding in better comparisons and estimations. However, for area, capacity, and particularly volume estimates, students lacked references during the initial evaluation. Notably, in the final test, 39.2% of students still provided unacceptable estimates for capacity, showing less improvement compared to volume estimates. This fact could be attributed to the limited number of capacity estimation tasks conducted during the working sessions, coupled with the absence of opportunities for pre-

Table 4. Students who made acceptable or unacceptable estimates in the initial and final tests

	Initial test		Final test		E. Final-initial
	E. Acceptable	E. Not acceptable	E. Acceptable	E. Not acceptable	acceptable
Length	81.7%	18.3%	90.0%	10.0%	8.33%
Height	75.0%	25.0%	85.0%	15.0%	10.00%
Area	45.0%	55.0%	76.7%	23.3%	31.67%
Volume	21.7%	78.3%	71.7%	28.3%	50.00%
Capacity	31.7%	68.3%	60.8%	39.2%	29.17%

service teachers to validate their estimates through practical experimentation across all capacity tasks.

FINAL CONSIDERATIONS

To answer the first research question: what is CCK of measurement estimation that pre-service teachers have? We analyzed the answers provided by pre-service teachers in various measurement estimation tasks and examined the methodologies they used to arrive at their estimates. The results of the initial test showed a clear deficit of the pre-service teachers in the skills to do acceptable measurement estimations, especially in the case of area, volume and capacity. This finding is consistent with the findings of the previous studies (Satan, 2022).

In the case of length magnitudes, pre-service teachers showed better estimating capacity than for area magnitudes and especially for volume or capacity. In addition, it was found that many pre-service teachers confused the magnitudes of volume and capacity. When a qualitative analysis of the strategies used was carried out, a significant difference was observed between the strategies used for estimating tasks of length and height compared to those used for estimating areas, volumes, and capacities. For length and height estimations, most pre-service teachers relied on comparisons with the unit of measurement or familiar references. In contrast, estimations of area, volume, and capacity were predominantly approached using formulas or made randomly without employing any specific estimation strategy. This is mainly due to the mastery or not of already known referents. These results are consistent with recent studies (Runnalls & Hong, 2020; Seah & Horne, 2020) who have found that pre-service teachers alike often struggle with area and volume concepts, exhibiting procedural or formula-based understanding of the topic.

The results of the initial test highlighted the need to focus more thoroughly on units of measurement, emphasizing a manipulative, experimental approach using real-life examples. This approach aims to enhance pre-service teachers' understanding of measurement concepts and improve their estimation skills contributing to the improvement of the mathematics teacher's CCK, a domain of knowledge essential to the effectiveness of their future teaching (Ball et al., 2008).

A sequence of measurement estimation tasks was designed and implemented, guided by the nine factors involved in the estimation process identified by Castillo et al. (2011) and the recommendations of Bright (1976). To evaluate the intervention, we posed the following research question: has the intervention process, designed and implemented in the classroom, resulted in improvements in pre-service teachers' estimation skills? We conducted a final test comprising different estimation tasks. The results of this final test were compared to those of the initial test, with all estimates having an error of less than 30% considered valid for the purpose of this comparison. Comparing the initial and final test estimates reveals an overall increase in acceptable results, indicating improved estimation capacity among students. Therefore, these results demonstrate that the designed learning sequence has improved the measurement estimations of the pre-service teachers. Even so, pre-service teachers continued to struggle with making accurate estimates in capacity. When discussing the results obtained and the difficulties they had with pre-service teachers, they indicated that it had been very difficult for them because the referents that had been worked on in the sessions were smaller and that they lacked larger referents to be able to use them to estimate the container. This result agrees with Hoth et al. (2023) who indicates that the size of the object to be estimated is a key factor in the estimation. In fact, some estimated it using the cubic meter as a reference and then did the conversion from units to liters.

From the results of this research, it is clear that the difficulties that pre-service teachers manifested when they faced estimation tasks are mainly due to inadequate preparation, lack of practice and poor knowledge and management of certain concepts related to units of measurement. This result is consistent with other studies conducted with primary school students that demonstrate that classroom interactions improve measurement estimates (Hoth et al., 2019, 2023; Jones et al., 2009). On the other hand, these findings highlight the importance of pre-service teachers' prior experiences and knowledge in their estimation strategies. Pre-service teachers tend to draw on their existing understanding and familiarity with length, area, volume and capacity concepts when making estimations. According to (Çilingir Altıner, 2024), this suggests that incorporating real-life examples and practical applications in mathematics education can support pre-service teachers

in developing more accurate and meaningful estimation strategies. It is also evident that there is a need to reinforce the concepts of area, coinciding with Barrera et al. (2019) and volume and capacity so that they can understand these concepts in an experimental way, and avoid focusing the teaching-learning process about measurement on purely mechanical, decontextualized and exclusively formulaic issues (Riera & Ruiz-Aguilera, 2015).

Considering the study's findings, teacher educators can enhance pre-service teachers' estimation skills by integrating a variety of measurement estimation tasks (and not only computational estimation related to arithmetic operations) across the mathematics curriculum, providing opportunities for practical experimentation and offering explicit training on estimation strategies in different situations. With the training of teachers, we must contribute to future teachers managing, in their professional capacity, activities that promote in their future students the understanding, not only of the concept of length, but also of area, volume and capacity, beyond the use of formulas. If we do not work on improving the estimation skills of pre-service teachers, it will have a negative impact on the mathematics education of their future students.

Limitations and Suggestions

While this study provides valuable insights into the estimation skills of pre-service teachers, it is essential to acknowledge certain limitations that may affect the generalizability of the findings. The results presented are the result of an experiment carried out with a sample of convenience and, therefore, with a limited representativeness. Consequently, the resulting conclusions may not be applicable to other populations with different characteristics. Therefore, caution should be exercised when generalizing the findings to other contexts. Additionally, the intervention implemented in this study was limited to a specific set of activities, and the long-term effectiveness of these interventions remains to be evaluated. Future research could explore the long-term effectiveness of interventions aimed at improving pre-service teachers' estimation skills and investigate the transferability of these skills to classroom instruction.

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REFERENCES

Alsina, A. (2003). *Desarrollo de competencias matemáticas con recursos lúdico-manipulativos para niños y niñas de 6 a 12 años* [Development of mathematical skills with playful-manipulative resources for boys and girls from 6 to 12 years old]. Narcea S.A. de Ediciones.

Andrews, P., Xenofontos, P., & Judy Sayers, J. (2022). Estimation in the primary mathematics curricula of the United Kingdom: Ambivalent expectations of an essential competence. *International Journal of Mathematical Education in Science and Technology, 53*(8), 2199-2225. https://doi.org/10.1080/0020739X.2020.1868591

Attivo, B. J. A. (1979). *The effects of three instructional strategies on prospective teacher's ability to estimate length and area in the metric system* [PhD thesis, The Pennsylvania State University].

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389-407. https://doi.org/10.1177/0022487108324554

Barrera, S. C., de Gamboa, G., & Jiménez, E. R. B. (2019). Aproximación a las conexiones matemáticas que establecen futuros maestros de primaria en tareas de medida y comparación de áreas [Approach to the mathematical connections that future primary teachers establish in tasks of measurement and comparison of areas]. In *Investigación en educación matemática XXIII* (pp. 233-242). Sociedad Española de Investigación en Educación Matemática.

Baturo, A., & Nason, R. (1996). Student teachers' subject matter knowledge within the domain of area measurement. *Educational Studies in Mathematics, 31*(3), 235-268. https://doi.org/10.1007/BF00376322

BOE. (2022). Real decreto 157/2022, de 1 de marzo, por el que se establecen la ordenación y las enseñanzas mínimas de la educación primaria [Royal decree 157/2022, of March 1, which establishes the organization and minimum teachings of primary education]. *Ministerio de Educación y Formación Profesional*. https://bit.ly/3MWojuA

Bright, G. W. (1976). Estimation as part of learning to measure. In D. Nelson, & R. E. Reys (Eds.), *Measurement in school mathematics, 1976 yearbook*. (pp. 87-104). National Council of Teachers of Mathematics.

Castillo-Mateo, J. J. (2013). *Estimación de cantidades continuas: Longitud y superficie* [Estimation of continuous quantities: Length and surface] [PhD thesis, Universidad de Granada].

Castillo-Mateo, J. J., Segovia, I., & Molina, M. (2017). Estudio comparativo de la estimación de cantidades continuas que hacen los estudiantes de

secundaria y futuros maestros [Comparative study of the estimation of continuous quantities made by high school students and future teachers]. *PNA, 12*(1), 45-62. https://doi.org/10.30827/pna.v12i1.6538

Castillo-Mateo, J., Segovia, I., Castro, E., & Molina, M. (2012). Categorización de errores en la estimación de cantidades de longitud y superficie [Categorization of errors in the estimation of length and surface quantities]. In D. Arnau, J. Lupiáñez, & A. Maz (Eds.), *Investigaciones en pensamiento numérico y algebraico e historia de la matemática y educación matemática-2012* (pp. 63-74). Departamento de Didáctica de la Matemática de la Universitat de València y SEIEM.

Chamorro, M. (1996) El currículum de medida en educación primaria y ESO y las capacidades de los escolares. *Revista de Didáctica de Las Matemáticas, 10*, 43-62.

Çilingir Altiner, E. (2024). Exploring measurement estimation strategies through concept cartoons designed with realistic mathematics education. *Humanities and Social Sciences Communications, 11*, Article 567. https://doi.org/10.1057/s41599-024-03067-5

Cockcroft, W. H. (1982). *Mathematics counts*. HM Stationery Office.

Darling-Hammond, L., & Bransford, J. (2005). *Preparing teachers for a changing world: What teachers should learn and be able to do*. Jossey-Bass.

Department of Education [England]. (2014). *The national curriculum in England framework document*. https://assets.publishing.service.gov.uk/media/5a7db9e9e5274a5eaea65f58/Master_final_national_curriculum_28_Nov.pdf

Er, Z., & Artut, P. D. (2021). Determination of measurement estimation abilities of secondary school students. *International Journal of Research in Education and Science, 7*(4), 1090-1103. https://doi.org/10.46328/ijres.2337

Even, R., & Ball, D. L. (2009). *The professional education and development of teachers of mathematics*. Springer. https://doi.org/10.1007/978-0-387-09601-8

Ghauri, P., & Gronhaug, K. (2010). *Research methods in business studies*. Prentice Hall.

Gooya, Z., Leyla G. Khosroshahi, L. G., & Teppo, A. R. (2011). Iranian students' measurement estimation performance involving linear and area attributes of real-world objects. *ZDM Mathematics Education, 43*, 709-722. https://doi.org/10.1007/s11858-011-0338-1

Hattie, J. (2012). *Visible learning for teachers*. Routledge. https://doi.org/10.4324/9780203181522

Hildreth, D. J. (1983). The use of strategies in estimating measurements. *The Arithmetic Teacher, 30*(5), 50-54. https://doi.org/10.5951/AT.30.5.0050

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education, 39*(4), 372-400. https://doi.org/10.5951/jresematheduc.39.4.0372

Hogan, T. P., & Brezinski, K. L. (2003) Quantitative estimation: One, two, or three abilities? *Mathematical Thinking and Learning, 5*(4), 259-280. https://doi.org/10.1207/S15327833MTL0504_02

Hoover, M. (2014). Commentary on section 1: Mounting progress on understanding mathematics teacher content knowledge. In L. Jane-Jane, K. Leatham, & L. R. Van Zoest (Eds.), *Research trends in mathematics teacher education* (pp. 83-90). Springer. https://doi.org/10.1007/978-3-319-02562-9_5

Hoth, J., Heinze, A., Huang, H. M. E., Weiher, D. F., Niedermeyer, I., & Ruwisch, S. (2023) Elementary school students' length estimation skills–Analyzing a multidimensional construct in a cross-country study. *International Journal of Science and Mathematics Education, 21*, 1841-1865. https://doi.org/10.1007/s10763-022-10323-0

Hoth, J., Heinze, A., Weiher, D., Ruwisch, S., & Huang, H. (2019). Primary school students' length estimation competence–A cross-country comparison between Taiwan and Germany. In J. Novotná, & H. Moraová (Eds.), *Opportunities in learning and teaching elementary mathematics* (pp. 201-211). Charles University.

Huang, H.-M. E. (2020). Effects of grade level and object size on students' measurement estimation performance. *EURASIA Journal of Mathematics, Science and Technology Education, 16*(12), Article em1923. https://doi.org/10.29333/ejmste/9342

Jones, G., Taylor, A., & Broadwell, B. (2009). Estimating linear size and scale: Body rulers. *International Journal of Science Education, 31*(11), 1495-1509. https://doi.org/10.1080/09500690802101976

Joram, E., Gabriele, A., Bertheau, M., Gelman, R., & Subrahmanyam, K. (2005) Children's use of the reference point strategy for measurement estimation. *Journal for Research in Mathematics Education, 36*(1), 4-23.

Kılıç, Ç., & Olkun, S. (2013). Primary school students' measurement estimation performance and strategies they used in real life situations. *Elementary Education Online, 12*, 295-307.

Levine, D. (1982). Strategy use and estimation ability of college students. *Journal for Research in Mathematics Education, 13*(5), 350-359. https://doi.org/10.5951/jresematheduc.13.5.0350

Ley Orgánica. (2006). *Ley Orgánica 2/2006, de 3 de Mayo, de Educación* [Organic Law 2/2006, of May 3, on Education]. https://bit.ly/3wvxhZg

López-Serentill, P. (2022). Categorización de los errores de los estudiantes para maestro de primaria en tareas de medida de magnitudes [Categorization of student errors for primary teacher in magnitude measurement tasks]. In T. F. Blanco, C. Núñez-García, M. C. Cañadas, & J. A. González-Calero (Eds.), *Investigación en Educación Matemática XXV* (pp. 363-371). SEIEM.

Luelmo, M. J. (2001). Medir en secundaria: Algo más que formulas [Medir en secundaria: Algo más que fórmulas]. *X Jornada para el Aprendizaje y Enseñanza de las Matemáticas: Actas de X JAEM, 52*, 727-737.

Moreno, M. F., Gil, F., & Montoro, A. B. (2015). Sentido de la medida [Sense of measurement]. In P. Flores, & L. Rico. (Eds.), *Enseñanza y aprendizaje de las matemáticas en educación primaria* (pp. 147-168). Pirámide.

NCTM. (2000). *Principles and standards for school mathematics*. National Council of Teachers of Mathematics.

Papadopoulos, I. (2010). "Reinventing" techniques for the estimation of the area of irregular plane figures: From the eighteenth century to the modern classroom. *International Journal of Science and Mathematics Education, 8*(5), 869-890. https://doi.org/10.1007/s10763-009-9190-y

Paull, D. R. (1971). *The ability to estimate in mathematics* [PhD thesis, Universidad de Columbia].

Pincheira, N., & Alsina, A. (2021). Teachers' mathematics knowledge for teaching early algebra: A systematic review from the MKT perspective. *Mathematics, 9*(20), Article 2590. https://doi.org/10.3390/math9202590

Pizarro, N., Albarracín, L., & Gorgorió, N. (2018). Actividades de estimación de medida: La interpretación de los docentes de educación primaria [Measurement estimation activities: The interpretation of primary education teachers]. *Bolema: Boletim de Educação Matemática, 32*(62), 1177-1197. https://doi.org/10.1590/1980-4415v32n62a21

Pizarro, N., Gorgorió, N., & Albarracín, L. (2014). Aproximación al conocimiento para la enseñanza de la estimación de medida de los maestros de primaria [Approach to knowledge for teaching measurement estimation for primary teachers]. In M. T. González, M. Codes, D. Arnau, & T. Ortega (Eds.), *Investigación en educación matemática XVIII* (pp. 523-532). SEIEM.

Pizarro, N., Gorgorió, N., & Albarracín, L. (2015). Primary teacher' approach to measurement estimation activities. In *Proceedings of the 9th Congress of the European Society for Research in Mathematics Education* (pp. 3227-3233).

Riera, J. V., & Ruiz-Aguilera, D. (2015). La medida en la formación inicial de maestros [The measure in initial teacher training]. In P. Á. Sánchez (Ed.), *17 jornadas para el aprendizaje y la enseñanza de las matemáticas* (pp. 1-12). Sociedad de Educación Matemática de la Región de Murcia, SEMRM.

Runnalls, C., & Hong, D. S. (2020). "Well, they understand the concept of area": Pre-service teachers' responses to student area misconceptions. *Mathematics Education Research Journal, 32*, 629-651. https://doi.org/10.1007/s13394-019-00274-1

Ruwisch, S., Heid, M., & Weiher, D. F. (2015). Measurement estimation in primary school: Which answer is adequate? In K. Beswick, T. Muir, & J. Fielding-Wells (Eds.), *Proceedings of 39th Conference of the International Group for the Psychology of Mathematics Education* (pp. 113-120). PME.

Satan, N., & Yetkin, İ. (2022). An analysis of the middle school students' performance in the measurement estimation. *Dokuz Eylül Üniversitesi Buca Eğitim Fakültesi Dergisi,* (53), 700-724. https://doi.org/10.53444/deubefd.1038366

Seah, R. T. K., & Horne, M. (2020). The influence of spatial reasoning on analysing about measurement situations. *Mathematics Education Research Journal, 32*, 365-386. https://doi.org/10.1007/s13394-020-00327-w

Segovia, I. (1997). *Estimación de cantidades discretas. Estudio de variables y procesos* [Estimation of discrete quantities. Study of variables and processes]. Comares.

Segovia, I., Castro, E., & Rico, L. (1989). *Estimación en cálculo y medida* [Estimation in calculation and measurement]. Síntesis.

Shulman, L. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*, 1-23. https://doi.org/10.17763/haer.57.1.j463w79r56455411

Siegel, A. W., Goldsmith, L. T., & Madson, C. R. (1982). Skill in estimation problems of extent and numerosity. *Journal for Research in Mathematics Education, 13*(3), 211-232. https://doi.org/10.5951/jresematheduc.13.3.0211