Equilibrium Vertical Differentiation in a Bertrand model with Capacity Precommitment*

N. Boccard† and X. Y. Wauthy‡

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Abstract

Both quality differentiation and capacity commitment have been shown to relax price competition. However, their joint influence on the outcome of price competition has not yet been assessed. In this article, we consider a three stage game in which firms choose quality, then commit to capacity and, finally, compete in price. When the cost of quality is negligible, we show that firms do not differentiate their products in a subgame perfect equilibrium, in other words, *capacity precommitment completely eliminates the incentive to differentiate by quality.*

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1 Introduction

It is well-known since Gabszewicz and Thisse (1979)'s seminal contribution that quality differentiation offers a powerful way out of the Bertrand paradox. Many scholars have elaborated on their pioneering work and today a robust "principle of differentiation" prevails in the literature on vertically differentiated industries. As nicely summarized in Shaked and Sutton (1982), firms are indeed likely to "relax price competition through product differentiation".

Interestingly, capacity commitment also has the virtue of relaxing price competition. The seminal contribution in this area is Kreps and Scheinkman (1983). They show how capacity commitment may be instrumental in sustaining Cournot outcomes in pricing games. Since Kreps and Scheinkman (1983) , the strategic value of capacities has been widely studied, though almost exclusively in

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†Departament d'Economia, FCEE, UdG, 17071 Girona, Spain. Financial support from Generalitat de Catalunya (2005SGR213 and Xarxa de referència d'R+D+I en Economia i Polítiques Públiques) and Ministerio de Educación y Ciencia (SEJ2007-60671) are gratefully acknowledged.

‡Facultés universitaires Saint-Louis, CEREC and Université catholique Louvain, CORE. Financial support from the IAP 6/09 contract from the SSTC, Belgian Federal Government is gratefully acknowledged.

markets for non-differentiated goods. For instance, Brock and Scheinkman (1985), Lambson (1994), Compte et al. (2002), Davidson and Deneckere (1990) and Benoît and Krishna (1987) study the role of limited capacities in a repeated game of price competition. Deneckere and Kovenock (1992) rely on capacity constraints to provide a model where, in equilibrium, the dominant firm *chooses* to be the price leader. More recently, Allen et al. (2000) show that capacity precommitment may act as a barrier to entry when price competition takes place post-entry.

Casual observation suggests that in many industries firms sell products differing by quality while being limited by their production capacities. In those industries, it is hard to see a priori whether strategic behavior at the price competition stage is mainly determined by the quality dimension, the capacity restrictions or both. More generally, to what extent are firms' quality choices dependent on the possibility of committing to capacities? How does the strategic value of capacities depend on the degree of differentiation? Despite their relevance, these questions do not seem to have been addressed in the literature, either theoretically or empirically. Our paper takes a first step in this direction.

We study a three stage game of complete information where firms first decide on quality. Then, when the specifications of the product are known, firms build production capacities and, finally, they compete in price on the consumer market. The possibility of committing to capacities before price competition takes place sheds new light on vertical differentiation issues. We show indeed that within the standard model of vertical differentiation, *capacity commitment may supplant quality differentiation in relaxing price competition.* The possibility of committing to capacities before price competition tends to destroy much of the incentive to choose different qualities in the first stage. In particular, if quality costs are sufficiently low, *firms sell homogeneous products in equilibrium.*

This "no-differentiation" result may seem surprising at first sight because it runs against the well-established "principle of differentiation". According to this principle, firms always differentiate their products in order to relax price competition. In fact our finding is quite intuitive. Eaton and Harrald (1992) have already shown that under *quantity* competition, firms are not inclined to differentiate in quality unless this allows a reduction in sunk costs. In particular, under quantity competition, choosing the best available quality is a dominant strategy for all firms when there are no costs to quality upgrading. In the present paper, we show how capacity commitment may transform the initial pricing game into a quantity game. More specifically, in a duopoly game, when production costs are symmetric and products are differentiated, the reduced form of firms' payoffs at the quality stage are the Cournot payoffs. The no-differentiation outcome then naturally follows if quality costs are low.

Our result does not invalidate vertical differentiation as such; instead, it underlines that in a duopoly framework, quality differentiation is more crucially rooted in asymmetries of costs than in a desire to relax competition. In this last respect, indeed, quality differentiation is supplanted by capacity commitment.

Like the present analysis, the literature on multidimensional differentiation can be regarded as dealing with models where firms are endowed with multiple commitment tools, aimed at relaxing competition. In a setting of multidimensional horizontal differentiation, Irmen and Thisse (1998) show that firms always differentiate in equilibrium, but along one dimension only.[1] Neven and Thisse (1990) deal with a two-dimensional model where firms may differentiate their product by quality and (or) variety. They also show that firms differentiate along one dimension only. Furthermore, maximal differentiation obtains (in equilibrium) either in quality or variety depending on the distribution of consumers' tastes. Even closer to our present analysis is Economides (1989)'s setting where quality and variety can be combined. He shows that minimal quality differentiation and maximal variety differentiation are likely outcomes.[2] However, unlike the current paper, he does not consider a population of consumers whose preferences are heterogeneous with respect to quality. All in all, these papers suggest that firms tend to concentrate on one instrument (one dimension of differentiation) in order to relax competition. Our paper does so as well, with the difference that it is differentiation itself which turns out not to be retained as an equilibrium strategy.

Our findings might also be interpreted to suggest that the standard result of Kreps and Scheinkman (1983) could be obtained within a more general game since Cournot outcomes[3] for homogeneous goods can be sustained as subgame perfect equilibrium outcomes of our three-stage game. We will show however that this is not the case for two main reasons. On the one hand, our model formally differs in a crucial respect from that of Kreps and Scheinkman (1983) and, on the other hand, Cournot outcomes do not obtain as the unique subgame perfect equilibrium outcome of our game. Many other outcomes, including the joint profit maximizing one,[4] are sustainable as well.

The paper is organized as follows. Section 2 introduces the model and review properties of the equilibrium of a quality-price game when production capacities are assumed to be arbitrarily large. In section 3, we analyze the class of subgames where products are differentiated and show that these subgames cannot belong to the equilibrium path. We then turn in section 4 to the class of subgames where firms sell homogeneous products and establish the existence of a subgame perfect equilibrium in which firms enjoy equilibrium payoffs equal to the collusive ones. Section 5 concludes.

2 Quality, Capacity, Price : a Three Stage Game

2.1 Model

Consumers' preferences are set according to the simplified framework of Mussa and Rosen (1978), as popularized by Tirole (1988). The good with label i has a quality s_i drawn from the interval $[0,1]$.[5] Consumers have unit demand for the good and are characterized by a "taste for quality" x uniformly distributed on $[0,1]$. The indirect utility function of a consumer with taste for quality x is $u(i,x) = xs_i - p_i$ for $i = 1,2$. Not consuming yields a normalized nil utility. In case of a tie among the two

[1] See Palma et al. (1995) and Anderson and Palma (1988) for early results pointing in the same direction.

[2] Ireland (1987) reports comparable results.

[3] Namely, firms sell Cournot quantities at the Cournot price.

[4] In this outcome each firm sells half of the monopoly quantity at the monopoly price.

[5] We assume the existence of a normalized upper bound for quality. This bound is best understood as having been determined by the current state of technology. We show later that it is not a severe restriction. It is however necessary to ensure that firms' payoffs are bounded from above in the case where quality costs are negligible (see Baye and Morgan (2002)).

products, the consumer randomly chooses among the two with equal probability.

We consider the three-stage game G developing as follows. In stage 1, firms $i = 1,2$ simultaneously choose quality levels s_i. Since we are essentially interested in analyzing the implications of capacity commitment on the intensity of competition, we concentrate on the cases where quality costs are negligible. This way, the presence of quality differentiation must result from strategic concerns and not from costs saving concerns (we address positive cost for quality in Appendix II). In stage 2, the subgame is denoted $G(s_1, s_2)$. Firms have the opportunity to simultaneously commit to capacities k_1 and k_2 at a nearly zero positive unit cost δ. In stage 3, the subgame is denoted $G(s_1, s_2, k_1, k_2)$. Firms simultaneously compete in price. The analysis will be conducted with the concept of subgame perfect equilibrium (hereafter SPE).

In $G(s_1, s_2, k_1, k_2)$, the installed capacity k_i allows firm $i = 1,2$ to produce up to k_i units at constant unit cost c; producing beyond capacity is feasible but at a constant unit cost $c+\theta$, with $\theta > 0$. The marginal cost function is therefore discontinuous at k_i. This cost framework was originally proposed by Dixit (1980) within a quantity competition model and was used by Bulow et al. (1985) and Maggi (1996) under price competition.[6] We assume in the following that $c = 0$ and $\theta = 1$ to capture the notion of limited production capacity; under this assumption, there exist no prices for which it is profitable to produce beyond capacity.

Given costs, firms produce to satisfy demand, i.e. firms cannot turn consumers away once they have named their prices. We follow in this respect the definition of Bertrand competition used for instance by Bulow et al. (1985), Vives (1989, 1990), Kuhn (1994), Dastidar (1995, 1997) and Maggi (1996). This assumption is best viewed as a black-box for complex reputation or regulation effects that are not modeled here.[7] Note that this assumption of *no rationing* considerably simplifies the formal analysis of the capacity game. Yet, it should be mentioned that it is at odds with the more standard literature on capacity commitment such as Kreps and Scheinkman (1983) . Indeed, this literature assumes Bertrand-Edgeworth competition. The nature of the restriction induced by the *no rationing* assumption will become clear in the analysis of the price-setting subgames.

2.2 Pure Bertrand Competition

Having defined our game completely, we now review the standard quality–price game where capacity commitment is not possible. This will provide a suitable benchmark for the analysis of the full game.[8] We denote G^B the benchmark game where firms cannot commit to capacities. Formally, we restrict the analysis to the class of subgames $G(s_1, s_2, k_1, k_2)$ with $k_1, k_2 \geq 1$. When $s_1 \neq s_2$ we may relabel firm l for low quality and h for high quality with $s_h > s_l$.

Lemma 1 *Whatever the quality chocies, $G^B(s_1, s_2)$ has a unique price equilibrium.*

- *If firms sell homogeneous products, the equilibrium is $p_1^* = p_2^* = 0$.*

[6] In contrast to our approach, Maggi (1996) implicitly concentrates on small θ.

[7] For instance car makers could often ration consumers but tend to avoid it and instead engage in costly supplementary production.

[8] Lutz (1997) provides more detailed proofs for this game with unlimited capacities. Since the analysis is rather standard, we refer the interested reader to his paper.

- *If firms sell different qualities, the equilibrium is* $p_l^* = \frac{s_l(s_h-s_l)}{4s_h-s_l}$, $p_h^* = \frac{2s_h(s_h-s_l)}{4s_h-s_l}$.

Proof The first part of Lemma 1 follows directly from the fact that at the no-differentiation limit, our model corresponds to a standard Bertrand model with a linear demand function and zero production costs. Therefore the unique equilibrium is $p_1^* = p_2^* = 0$.

For $s_h > s_l$, at the price stage the demands resulting from consumers' choices, given prices p_h and p_l, are

$$D_l(p_l,p_h) = \begin{cases} 1-\frac{p_l}{s_l} & \text{if} \quad p_l \le p_h - s_h + s_l \\ \frac{p_h s_l - p_l s_h}{s_l(s_h-s_l)} & \text{if} \quad p_h - s_h + s_l \le p_l \le p_h \frac{s_l}{s_h} \\ 0 & \text{if} \quad p_l \ge p_h \frac{s_l}{s_h} \end{cases} \tag{1}$$

$$D_h(p_l,p_h) = \begin{cases} 1-\frac{p_h}{s_h} & \text{if} \quad p_h \le \frac{s_h}{s_l} p_l \\ 1-\frac{p_h-p_l}{s_h-s_l} & \text{if} \quad \frac{s_h}{s_l} p_l \le p_h \le p_l + s_h - s_l \\ 0 & \text{if} \quad p_h \ge p_l + s_h - s_l \end{cases} \tag{2}$$

Straightforward computations yield the following best response functions:

$$\psi^l(p_h) = \begin{cases} p_h \frac{s_l}{2s_h} & \text{if} \quad p_h \le \frac{2s_h(s_h-s_l)}{2s_h-s_l} \\ p_h + s_h - s_l & \text{if} \quad \frac{2s_h(s_h-s_l)}{2s_h-s_l} \le p_h \le s_h - \frac{s_l}{2} \\ \frac{s_l}{2} & \text{if} \quad p_h \ge s_h - \frac{s_l}{2} \end{cases} \tag{3}$$

$$\psi^h(p_l) = \begin{cases} \frac{s_h-s_l+p_l}{2} & \text{if} \quad p_l \le \frac{s_l(s_h-s_l)}{2s_h-s_l} \\ p_l \frac{s_h}{s_l} & \text{if} \quad \frac{s_l(s_h-s_l)}{2s_h-s_l} \le p_l \le \frac{s_l}{2} \\ \frac{s_h}{2} & \text{if} \quad p_l \ge \frac{s_l}{2} \end{cases} \tag{4}$$

The best response functions intersect at $\left(p_l^*, p_h^*\right) = \left(\frac{s_l(s_h-s_l)}{4s_h-s_l}, \frac{2s_h(s_h-s_l)}{4s_h-s_l}\right)$ which is the unique pure strategy price equilibrium.

Because of non-negativity constraints, demands are not concave and we cannot exclude directly the existence of a mixed strategy equilibrium. As we show in Lemma 3, we may rule out the existence of mixed strategy equilibria. ■

Note that quantities demanded at the equilibrium prices are

$$D_l^* = \frac{s_h}{4s_h - s_l} \quad \text{and} \quad D_h^* = \frac{2s_h}{4s_h - s_l} \tag{5}$$

and that the closed form equilibrium payoffs are

$$\Pi_h^B(s_h,s_l) \equiv \frac{4s_h^2(s_h-s_l)}{(4s_h-s_l)^2} \tag{6}$$

$$\Pi_l^B(s_h,s_l) \equiv \frac{s_l s_h (s_h-s_l)}{(4s_h-s_l)^2} \tag{7}$$

We now turn to the first stage of the game where qualities are chosen.

Lemma 2 *Up to a permutation of players, there is a unique SPE of G^B in which chosen qualities are $s_h^* = 1$ and $s_l^* = \frac{4}{7}$.*

Proof Note first that $s_1 = s_2$ cannot be part of an equilibrium because it yields zero profits to both firms while any deviation in quality leads to a price subgame where products are differentiated, so that payoffs resulting from this deviation are strictly positive. Therefore, product differentiation must prevail in any SPE. Standard computations using (6) and (7) enable to show the existence of a unique SPE (up to a permutation of players) where one firm chooses the best available quality $s_h = 1$ and the other one optimally differentiates to the lower quality $s_h = \frac{4}{7}$. ■

3 Differentiated Goods

A key assumption of our Bertrand competition model is that firms are not allowed to ration consumers. Therefore, raising one's price in order to increase the competitor's demand beyond installed capacity is not profitable since it does not generate spillovers. In other words the lack of quasi-concavity associated with Bertrand-Edgeworth competition is not present in this model. On the other hand, since the extra marginal cost of producing beyond capacity is $\theta = 1$, no firm will find it profitable to name a price such that its demand exceeds capacity.

We build on these observations to identify the nature of the set of equilibria in $G(s_1, s_2, k_1, k_2)$ with $s_1 \neq s_2$. We then go backward to the capacity stage and characterize equilibrium capacity levels. Last, we establish the non-existence of a subgame perfect equilibrium displaying product differentiation.

3.1 Price Competition under Capacity Commitment

The best response of firm i to price p_j is the "classical" best response $\psi^i(p_j)$ as defined in equations (3-4), provided the corresponding demand does not exceed capacity i.e., whenever $D_i\left(\psi^i(p_j), p_j\right) \leq k_i$. Solving this equation for equality defines a critical level $\tilde{p}_j(k_i)$, above which firm i would face a demand that exceeds its capacity if it were to play along $\psi^i(p_j)$. When $p_j > \tilde{p}_j(k_i)$, firm i prefers to respond by selling its capacity at the highest possible price, i.e. the price p_i that solves $D_i(p_i, p_j) = k_i$. Let us denote this price $p_i^k(p_j)$. The best response functions are therefore piecewise linear with a "classical" branch, $\psi^i(p_j)$, where firms fight for market shares and a strategic branch, $p_i^k(p_j)$ where they exactly sell their capacity. In equilibrium it may be the case that two, one or zero firms are capacity-constrained. The formal analysis (developed in the Appendix) reveals that there are four possible equilibrium configurations in the space of capacities, as displayed on Figure 1.

In region A, installed capacities are sufficiently large to sustain the Nash equilibrium in prices characterized in Lemma 1; hence the lower left-hand corner of region A is the pair of quantities (D_l^*, D_h^*) sold at the equilibrium of G^B given by equation (5). For a smaller k_h, we move to area B where firm h is capacity-constrained in the price equilibrium. Likewise if k_l is smaller we pass from area A to C where firm l is capacity-constrained in the price equilibrium. Finally, in region D, the Nash equilibrium is the pair of prices which equate each firm's demand to its capacity. We prove the following theorem in the Appendix .

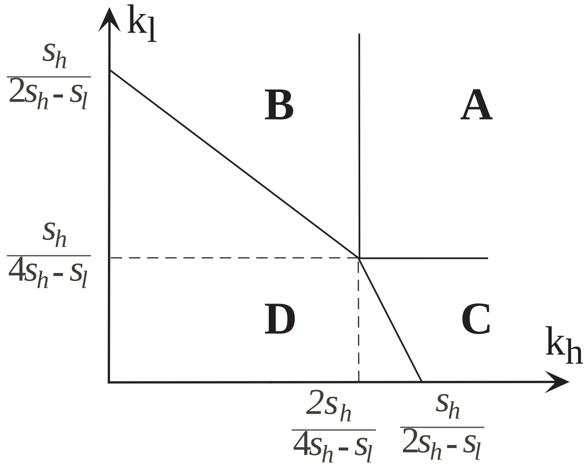

Figure 1: The capacity space

Theorem 1 *Consider $k_1, k_2 \leq 1$ and $s_1 \neq s_2$, $G(s_1, s_2, k_1, k_2)$ has a unique price equilibrium. Four different formulas apply according to the combination of capacities:*

$$
\begin{array}{lll}
[\mathbf{A}] & p_l^A = \frac{s_l(s_h-s_l)}{4s_h-s_l}, p_h^A = \frac{2s_h(s_h-s_l)}{4s_h-s_l} & \textit{if } k_l \geq \frac{s_h}{4s_h-s_l} \textit{ and } k_h \geq \frac{2s_h}{4s_h-s_l} \\
[\mathbf{B}] & p_l^B = \frac{(1-k_h)s_l(s_h-s_l)}{2s_h-s_l}, p_h^B = \frac{2(1-k_h)s_h(s_h-s_l)}{2s_h-s_l} & \textit{if } k_l \geq \frac{(1-k_h)s_h}{2s_h-s_l} \textit{ and } k_h \leq \frac{2s_h}{4s_h-s_l} \\
[\mathbf{C}] & p_l^C = \frac{(1-2k_l)s_l(s_h-s_l)}{2s_h-s_l}, p_h^C = \frac{(s_h-k_l s_l)(s_h-s_l)}{2s_h-s_l} & \textit{if } k_l \leq \frac{s_h}{4s_h-s_l} \textit{ and } k_h \geq \frac{s_h-k_l s_l}{2s_h-s_l} \\
[\mathbf{D}] & p_l^D = (1-k_h-k_l)s_l, p_h^D = (1-k_h)s_h - k_l s_l & \textit{if } k_l \leq \frac{(1-k_h)s_h}{2s_h-s_l} \textit{ and } k_h \leq \frac{s_h-k_l s_l}{2s_h-s_l}
\end{array}
\tag{8}
$$

3.2 Capacity Choice

Going backward in the game tree, we analyze firms' strategic incentives with respect to capacity levels. The intuition is captured by referring to Figure 2 and by relying on the intermediate results established in Theorem 1.

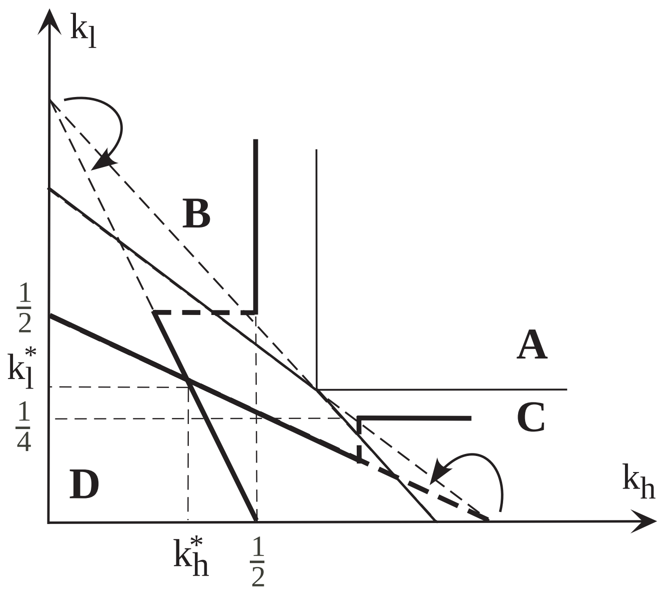

Figure 2: Capacity best responses

If the pair (k_h, k_l) lies in area A or B, then firm l is never capacity constrained i.e., neither the price nor its demand depend on its capacity. As capacity is costly, it is in the interest of firm l to reduce it. Thus, her capacity best response cannot lie in the interior of A or B. In region B, firm h's payoffs depend only on its own capacity; it is thus possible to identify a constant capacity best response in the interior of this region. Likewise, firm h avoids areas A and C by reducing its capacity, and firm l has a constant best response candidate in region C. Accordingly, there exists no equilibrium in the interior of regions A, B or C.

In area D, both firms have "small" capacities and sell their full capacity in the corresponding price equilibrium. If an equilibrium exists, it must lie in region D. Using the characterization of equilibrium prices in $G(s_1, s_2, k_1, k_2)$ given in (8), we formally define payoffs in region D by $k_l p_l^D$ and $k_h p_h^D$ for firm l and h respectively. The best responses candidates in region D are easily computed as $\kappa^l(k_h) = \frac{1-k_h}{2}$ and $\kappa^h(k_l) = \frac{s_h - k_l s_l}{2s_h}$. Since the two best reply lines are obtained from the frontier lines by rotation at their common axis point,[9] their intersection

$$k_l^* = \frac{s_h}{4s_h - s_l} \quad \text{and} \quad k_h^* = \frac{2s_h - s_l}{4s_h - s_l} \tag{9}$$

lies within area D. By comparing each firm's best response candidate in region D and B or C respectively, we can characterize capacity best responses; they are shown in bold face on Figure 2. Both correspondences jump up when facing a competitor with a large capacity. These jumps occur for capacity levels which exceed the candidate equilibrium values so that the existence of a pure strategy equilibrium is not called into question. This is why (k_l^*, k_h^*) is the unique SPE of $G(s_h, s_l)$.

To see that the capacity equilibrium replicates Cournot outcomes, observe that the demand system (1-2) is invertible from quantities to prices and yields exactly the market clearing prices obtained in (8-D): p_l^D and p_h^D. The payoffs in the corresponding quantity game are

$$\hat{\Pi}_l(q_l, q_h) = q_l p_l^D = q_l(1 - q_h - q_l)s_l \tag{10}$$

$$\hat{\Pi}_h(q_h, q_l) = q_h p_h^D = q_h\big((1 - q_h)s_h - q_l s_l\big) \tag{11}$$

The best responses (in this quantity game) are easily characterized as $q_l = \frac{1-q_h}{2}$ and $q_h = \frac{s_h - s_l q_l}{2s_h}$. Solving for a Nash equilibrium, we immediately obtain $q_l^* = k_l^*$ and $q_h^* = k_h^*$, as given by (9). We can therefore claim that in our duopoly framework, *under Bertrand competition and vertical differentiation, capacity precommitment yields Cournot outcomes.* This claim is summarized in the next theorem which is formally proved in the Appendix

Theorem 2 *For $s_1 \neq s_2$, $G(s_1, s_2)$ has a unique SPE, replicating the Cournot outcome of the corresponding quantity setting game with product differentiation.*

Some comments are in order at this step. Theorem 2 states that Cournot outcomes are subgame perfect equilibrium outcomes of a game where capacity commitment precedes price competition in vertically differentiated markets. This is strongly reminiscent of the Kreps and Scheinkman (1983) result. Let us stress however that the present analysis *cannot* be viewed as an extension of their analysis

[9] The best reply $\kappa^l(k_h) = \frac{1}{2}(1 - k_h)$ involves a factor $\frac{1}{2}$ whereas the frontier $k_l = \frac{s_h}{2s_h - s_l}(1 - k_h)$ involves a greater coefficient i.e., flatter on Figure 2. Likewise, the best reply $\kappa^h(k_l) = \frac{1}{2s_h}(s_h - k_l s_l)$ involves a factor $\frac{1}{2s_h}$ whereas the frontier $k_h = \frac{1}{2s_h - s_l}(s_h - k_l s_l)$ involves a greater coefficient i.e., steeper on Figure 2.

to the case of a differentiated market. Indeed, the rules of the pricing game are quite different, since we consider Bertrand competition whereas they deal with Bertrand-Edgeworth competition. Studying the behavior of our model at the no-differentiation limit unambiguously reveals this difference. While Kreps and Scheinkman (1983) obtain a unique subgame perfect equilibrium (replicating the Cournot outcome), we indeed obtain multiple subgame perfect equilibria which may entail lower installed aggregate capacity than the Cournot ones, and therefore higher prices.

3.3 Quality Competition

Recall that in the first stage of G, qualities are chosen at no cost. We limit ourselves in this section to quality choices $s_1 \neq s_2$. Thanks to Theorem 2, we can compute the firms' gross payoffs arising from the subgame perfect capacity equilibrium. Using capacities (9) and the price equilibrium associated to region D as defined in (8), we obtain:

$$\Pi_h(s_h, s_l) \equiv \pi_h^D(k_l^*, k_h^*) = \frac{s_h(2s_h - s_l)^2}{(4s_h - s_l)^2} \quad (12)$$

$$\Pi_l(s_h, s_l) \equiv \pi_l^D(k_l^*, k_h^*) = \frac{s_l s_h^2}{(4s_h - s_l)^2} \quad (13)$$

In the benchmark case G^B where firms have *unlimited capacities* (so called Bertrand competition), Lemma 2 shows that the best response of the low quality firm is to set $s_l = \frac{4s_h}{7}$ and thus remain the low quality firm. The ability to commit in capacity alters the price competition landscape. In G, the low quality firm's payoff is monotonically increasing in its own quality (as is the case under Cournot competition). Therefore, the low quality firm tends to *imitate* the high quality one and we reach the no-differentiation limit.

Proposition 1 *In game G, there exists no SPE where firms choose different qualities.*

Proof Observe that $\frac{\partial \Pi_h}{\partial s_h} = \frac{(2s_h - s_l)}{(4s_h - s_l)^3}\left(7s_h^2 + (s_h - s_l)^2\right) > 0$ and $\frac{\partial \Pi_l}{\partial s_l} = \frac{(4s_h + s_l)s_h^2}{(4s_h - s_l)^3} > 0$. We restrict our attention to pure strategies. If $s_1 = s_h > s_l = s_2$ was true in a SPE, then the high quality firm would surely choose the highest possible quality $s_h = 1$. Then no choice $s_l < 1$ can be optimal since $\hat{s}_l = \frac{1+s_l}{2} \in (s_l; 1)$ would be a better choice than s_l. If firm #1 plays a mixed strategy F_1, the payoff for firm #2 outside F_1's atoms is given by a weighted average of Π_h and Π_l. Since both terms are increasing with quality, π_2 is increasing. If top quality is not an atom of F_1, then firm #2 must be playing top quality; we are back to the pure strategies case. If top quality is an atom of F_1, then firm #2 has no best reply (using the previous argument) which means that F_1 cannot be part of an equilibrium. ■

Corollary 1 *If there exists a SPE of G, firms must chose the same quality and earn at least the Cournot payoff associated to maximal quality.*

Proof The first statement is a logical consequence of the previous proposition. Let s^* be the common choice in a SPE. If $s^* < 1$, a firm can deviate to $s = 1$ and earn $\Pi_h(1, s^*) = \frac{(2-s^*)^2}{(4-s^*)^2} > \frac{1}{9}$. If $s^* = 1$, a firm can deviate to $s < 1$ and earn $\Pi_l(s, 1) = \frac{s}{(4-s)^2} \leq \frac{1}{9}$. Since the limit of this deviation payoff is $\frac{1}{9}$ at $s = 1$, it cannot be the case that the equilibrium payoff is lesser than $\frac{1}{9}$. ■

4 Homogeneous Goods

Having ruled out the presence of product differentiation in a SPE of the overall game G, we must tackle the case of identical qualities $s_1 = s_2 = s$. Our model then simplifies to a linear demand $D(p) = \max\{0, 1 - \frac{p}{s}\}$. In the subgame $G(s, s, k_1, k_2)$, firms simultaneously name prices and produce to satisfy demand. We assume that demand is split equally between the two firms in case of a tie.

In the presence of capacity constraints, a firm may typically end up facing a demand level which exceeds installed capacity. Since rationing is not allowed, this firm meets demand even if it exceeds capacity and therefore sells at a loss those units beyond capacity. When prices are low, individual demand addressed to each firm may exceed capacity even when the firms share the market. These two configurations add to those usually prevailing under Bertrand competition. Accordingly, the profit function for $i = 1, 2$ in $G(s, s, k_1, k_2)$, assuming prices are chosen in $[0, s]$, is defined by relying on five branches.

$$\Pi_i(p_i, p_j) = \begin{cases} k_i - (1 - p_i)\left(1 - \frac{p_i}{s}\right) & \text{if } p_i < p_j \text{ and } p_i < (1 - k_i)s \quad (a) \\ p_i(1 - \frac{p_i}{s}) & \text{if } (1 - k_i)s \le p_i < p_j \quad (b) \\ \frac{p_i}{2}(1 - \frac{p_i}{s}) & \text{if } p_i = p_j \ge (1 - 2k_i)s \quad (c) \\ k_i - \frac{1}{2}(1 - p_i)\left(1 - \frac{p_i}{s}\right) & \text{if } p_i = p_j < (1 - 2k_i)s \quad (d) \\ 0 & \text{if } p_i > p_j \quad (e) \end{cases} \tag{14}$$

Branch (a) defines the firm's payoff when firm i is a price leader which faces a demand exceeding installed capacity. Branch (b) corresponds to the standard Bertrand price leader. Branch (c) defines payoffs in case of tie where the firm is unconstrained. Branch (d) corresponds to a tie at a low constraining price. Lastly, branch (e) corresponds to the case where firm i's price is strictly larger than j's.

The equilibrium analysis starts by observing that three different strategy profiles are relevant: *undercutting*, *pricing above* and *matching* the other firm's price. Introducing quantitative restrictions while preventing rationing has two direct effects. Because the "no-rationing" rule prevents the existence of demand spillovers, the kind of high price strategic deviation that generates price instability in Bertrand-Edgeworth models is not at work here. However "pricing above" may be a relevant strategy because it allows a firm to avoid losses by securing zero sales. Since demand is discontinuous (goods are homogeneous), undercutting the other firm's price may lead to losses if one's capacity is low relative to the demand that has to be served (recall indeed that $p_i \le s \le 1$ implies that the second term in (14:a) is negative).[10] Best responses then conform to intuition i.e., one should price above an aggressive price, match an intermediate one and undercut a large one.

Theorem 3 in the Appendix shows there is a multiplicity of equilibria in the pricing subgames. If capacities are not too dissimilar, matching the other's price is a best reply for both; there is thus a continuum of equilibria featuring positive payoffs for both firms. If capacities are too dissimilar, there are no pure strategies equilibria. A priori, the multiplicity of equilibria prevents the straightforward application of backward induction. Nevertheless, we are able to construct a SPE of G where firms

[10]Care must be taken though that undercutting is not a properly defined optimal response.

play almost collusively by choosing top quality, the collusive capacity and the monopoly price; they are deterred from large capacity deviation by the credible threat of Bertrand cutthroat competition with arbitrary small payoff.

Proposition 2 *There exists a symmetric SPE of G where firms select top quality ($s = 1$) and share the monopoly profits.*

Proof On the equilibrium path, firms play $s = 1, k = \frac{1}{4}$ and $p = \frac{1}{2}$. Firms earn $\frac{1}{8}$ in equilibrium. If a firm deviates to $s \neq 1$, she earns $\Pi_l(s, 1)$ as defined by equation (13). Straightforward computations indicate that $\Pi_l(s, 1) \leq \frac{1}{9} < \frac{1}{8}$; this is thus a dominated choice. We now tackle capacity deviations in $G(1, 1)$. If a firm deviates to $k < \frac{1}{4}$, the best she can do is sell exactly her capacity at monopoly price, thus earn $\frac{k}{2} < \frac{1}{8}$; this is a dominated choice. For a deviation to $k > \frac{1}{4}$, we construct in Lemma 5 of the Appendix, an equilibrium of $G(1, 1, \frac{1}{4}, k)$ where the deviant firm earns an arbitrary small payoff. This particular continuation price equilibrium allows to prevent upwards capacity deviations. ■

Three comments are appropriate at this step. First, Proposition 2 establishes the existence of a SPE where firms choose the best available quality. Notice that other SPE displaying no-differentiation and a lower quality level exist as well in game G. However, lower qualities are associated with lower profits and since a firm can jump over her competitor to earn a high-quality differentiated payoff, there is a lower bound to the quality level that can be sustained in a SPE. To show that common quality must be large in any SPE of G, let us define the critical quality level $\bar{s}$ making a top-quality firm indifferent between i) enjoying duopoly profits obtained by relying on capacity commitment and top quality level and ii) half of the monopoly payoff obtained when matching the other's quality s (and colluding afterwards). Formally, With this definition in hand we may state:

Proposition 3 *In every SPE of G, the common quality is* $s^* \geq \bar{s} \simeq 0.95$.

Proof Proposition 1 has shown that differentiation cannot take place while Proposition 2 has shown existence of an equilibrium, hence there is a common quality s^* in every SPE. We first prove that the equilibrium payoff in $G(s^*, s^*)$ is bounded by $\frac{1}{8}s^*$.

Observe that $\frac{1}{8}s^*$ is the maximum payoff for a firm, conditional on both firms naming the same price with probability one. To earn more than this, a firm, say #2, must succeed to undercut her opponent with positive probability. Since the equilibrium payoff can be computed at any price in the support of the strategy, firm 2 must undercut firm 1 with positive probability at her own maximum price $\bar{p}_2$ which means that firm 1's equilibrium strategy puts mass above $\bar{p}_2$. Since the equilibrium payoff of firm 1 can be computed at her top price, she would get zero demand, thus zero profit, a contradiction with the minimum payoff bound established in Corollary 1.

By switching to the top quality, a deviant firm earns $\Pi_h(1, s^*)$. In a SPE, this cannot be greater than $\frac{1}{8}s^*$ i.e., $s \geq \bar{s} \simeq 0.95$, the solution to the cubic equation $\Pi_h(1, s) = \frac{s}{8}$ over the interval $[0; 1]$ (see eq. (12)).[11] ■

[11] The exact value of $\bar{s}$ is as follows: $\bar{s} \equiv \frac{1}{3}\left(16 - 4\sqrt{7}\left(\cos\left(\frac{1}{3}\arctan\left(\frac{3\sqrt{111}}{67}\right)\right) - \sin\left(\frac{1}{3}\arctan\left(\frac{3\sqrt{111}}{67}\right)\right)\right)\right)$.

Notice that the maximum payoff associated to $\bar{s}$ is $\frac{\bar{s}}{8} \simeq 0.118$ whereas in $G(1,1)$ (top quality), the equilibrium payoff is in the range $\left[\frac{1}{9};\frac{1}{8}\right] \simeq [0.111;0.125]$.

Notice also that Proposition 2 extends to costly quality. The no-differentiation result is exactly preserved whenever the cost of quality is small, otherwise product differentiation prevails in a SPE of G but to a lesser degree than in the no-commitment situation. The argument is straightforward. One can compute the equilibrium quality choices under Bertrand and Cournot without capacity commitment. In the former case, product differentiation always prevail. In the latter case, we show in Lemma 7 that when quality costs is defined by $c(s) = \frac{s^2}{F}$, there exists a treshold $\tilde{F}$ below which firms choose identical qualities under Cournot competition (cf. Appendix II). Last, although the time sequence we assumed seems natural, the robustness of our result to the ordering of strategic choices can be questioned. Appendix III studies the alternative sequence where firms commit to capacities and then choose qualities; it is shown that if there is no cost for quality, firms choose identical qualities in a SPE. More generaly, equilibrium product differentiation is systematically lower under Cournot competition than under Bertrand competition without capacity commitment.

5 Conclusion

In this article, we have shown that quality differentiation *as a tool for relaxing price competition* is not a robust principle once capacity commitment is allowed. More precisely, our analysis concludes that *capacity commitment and Bertrand competition systematically induce less product differentiation relative to the game where capacity commitment is not possible.* Furthermore, if the cost of quality is low enough, then the ability to commit to capacities before Bertrand competition leaves no room for quality differentiation as a strategic decision aimed at relaxing competition.

Considering a richer game where capacity precommitment is possible, we shed new light on quality choice as well as on price competition. In our setting, capacity commitment relaxes price competition so effectively that differentiation may become unprofitable. More generally, the residual incentive to differentiate by quality is the one that prevails under quantity competition. It is well-known in this respect that quantity competition induces less differentiation than price competition (cf. Motta (1993)). Our analysis therefore leads us to concur that quality differentiation may rely more heavily on quality costs considerations than on a desire to relax competition per se.

As we make apparent in the analysis of the capacity-setting subgame, Bertrand competition (as opposed to Bertrand-Edgeworth competition) is central in obtaining our minimum-differentiation result so easily. Allowing for rationing severely complicates the analysis because the non-existence of pure strategy equilibria is endemic in the pricing subgames where product differentiation prevails. Moreover, the computation of mixed strategy equilibria in such games is not straightforward. Preliminary results obtained in a more simple setting (Boccard and Wauthy (2000)) suggest that our present findings could generalize to Bertrand-Edgeworth games. At this step however, this remains an open conjecture.

Finally, from an empirical point of view, our analysis suggests that in industries whose technology exhibits rigid production capacities, quality differentiation should basically reflect cost differentials;

if upgrading quality is not too costly, less product differentiation should be observed.

Appendix

I Proofs

Proof of Theorem 1 *If $s_h > s_l$, then $G(s_h, s_l, k_h, k_l)$ has a unique pure strategy equilibrium.*
When firm h names a price p_h, the demand addressed to firm l is $D_l(p_l, p_h)$ since rationing is not allowed (equation (1)). Firm l's profit is therefore

$$\Pi_l(p_l, p_h) = \begin{cases} p_l D_l(p_l, p_h) & \text{if} \quad D_l(p_l, p_h) \le k_l \\ (p_l - \theta) D_l(p_l, p_h) + \theta k_l & \text{if} \quad D_l(p_l, p_h) \ge k_l \end{cases}$$

Given p_h, firm l always has the opportunity to set its price so as maintain the equality $D_l(p_l, p_h) = k_l$; this particular price is

$$p_l^k(p_h) = \begin{cases} \frac{s_l}{s_h}\left(p_h - k_l(s_h - s_l)\right) & \text{if} \quad p_h \le s_h - k_l s_l \\ (1 - k_l) s_l & \text{if} \quad p_h \ge s_h - k_l s_l \end{cases}$$

Observe now that since $\theta = 1$, firm l is better off serving exactly its capacity by raising price if necessary than meeting excess demand. Thus, there are only two best response candidates against any p_h: the "classical" best response $\psi^l(p_h)$ or the "strategic" $p_l^k(p_h)$. It is immediate to see that the best response is $\sigma^l(p_h) = \max\{\psi^l(p_h), p_l^k(p_h)\}$ and since the maximum operator is applied to a pair of continuous and piecewise linear functions, σ^l is likewise continuous and piecewise linear. We now proceed to derive its exact formulation. Observe firstly that since $p_l^k(0) < 0 = \psi^l(0)$, $\sigma^l = \psi^l$ in a neighborhood of 0. More precisely,

- if $k_l \ge \frac{s_h}{2s_h - s_l}$ then,

$$\sigma^l(p_h) = \psi^l(p_h) = \begin{cases} p_h \frac{s_l}{2s_h} & \text{if} \quad p_h \le \frac{2s_h(s_h - s_l)}{2s_h - s_l} \\ p_h - s_h + s_l & \text{if} \quad \frac{2s_h(s_h - s_l)}{2s_h - s_l} \le p_h \le s_h - \frac{s_l}{2} \\ \frac{s_l}{2} & \text{if} \quad p_h \ge s_h - \frac{s_l}{2} \end{cases}$$

- if $\frac{s_h}{2s_h - s_l} \ge k_l \ge \frac{1}{2}$ then,

$$\sigma^l(p_h) = \begin{cases} p_h \frac{s_l}{2s_h} & \text{if} \quad p_h \le 2k_l(s_h - s_l) \\ \frac{s_l}{s_h}\left(p_h - k_l(s_h - s_l)\right) & \text{if} \quad 2k_l(s_h - s_l) \le p_h \le s_h - k_l s_l \\ p_h - s_h + s_l & \text{if} \quad s_h - k_l s_l \le p_h \le s_h - \frac{s_l}{2} \\ \frac{s_l}{2} & \text{if} \quad p_h \ge s_h - \frac{s_l}{2} \end{cases}$$

- if $k_l \le \frac{1}{2}$ then,

$$\sigma^l(p_h) = \begin{cases} p_h \frac{s_l}{2s_h} & \text{if} \quad p_h \le 2k_l(s_h - s_l) \\ \frac{s_l}{s_h}\left(p_h - k_l(s_h - s_l)\right) & \text{if} \quad 2k_l(s_h - s_l) \le p_h \le s_h - k_l s_l \\ (1 - k_l) s_l & \text{if} \quad p_h \ge s_h - k_l s_l \end{cases}$$

For firm h, a similar analysis takes place; the price solving $D_h(p_l, p_h) = k_h$ is

$$p_h^k(p_l) = \begin{cases} p_l + (1-k_h)(s_h - s_l) & \text{if} \quad p_l \leq s_l(1-k_h) \\ (1-k_h)s_h & \text{if} \quad p_l \geq s_l(1-k_h) \end{cases}$$

and as above the best response $\sigma^h(p_l)$ is the maximum of $p_h^k(p_l)$ and

$$\psi^h(p_l) = \begin{cases} \frac{s_h - s_l + p_l}{2} & \text{if} \quad p_l \leq \frac{s_l(s_h - s_l)}{2s_h - s_l} \\ p_l \frac{s_h}{s_l} & \text{if} \quad \frac{s_l(s_h - s_l)}{2s_h - s_l} \leq p_l \leq \frac{s_l}{2} \\ \frac{s_h}{2} & \text{if} \quad p_l \geq \frac{s_l}{2} \end{cases}.$$

More precisely,

- if $k_h \leq \frac{1}{2}$ then $\sigma^h(p_l) = p_h^k(p_l)$
- if $k_h > \frac{1}{2}$, then

$$\sigma^h(p_l) = \begin{cases} \frac{s_h - s_l + p_l}{2} & \text{if} \quad p_l \leq (2k_h - 1)(s_h - s_l) \\ p_l + (1-k_h)(s_h - s_l) & \text{if} \quad (2k_h - 1)(s_h - s_l) \leq p_l \leq s_l(1-k_h) \\ p_l \frac{s_h}{s_l} & \text{if} \quad s_l(1-k_h) \leq p_l \leq \frac{s_l}{2} \\ \frac{s_h}{2} & \text{if} \quad p_l \geq \frac{s_l}{2} \end{cases}$$

We have seen that best response functions are continuous and piecewise linear, hence a pure strategy equilibrium must be at their intersection. Notice that when a firm is in a monopoly situation (σ^h or σ^l is constant) the other firm faces a zero demand. This latter firm will therefore decrease her price to secure a positive demand.

Accordingly, only branches

$$\begin{cases} p_h \frac{s_l}{2s_h} & (l1) \\ p_h - s_h + s_l & (l2) \\ \frac{s_l}{s_h}\left(p_h - k_l(s_h - s_l)\right) & (l3) \end{cases} \quad \text{and} \quad \begin{cases} \frac{s_h - s_l + p_l}{2} & (h1) \\ p_l + (1-k_h)(s_h - s_l) & (h2) \\ p_l \frac{s_h}{s_l} & (h3) \end{cases}$$

for firm l and h can arise in an equilibrium. We have thus 9 possible but mutually exclusive configurations for candidate equilibria. We rule out 5 of them.

- $(l1 - h1)$: the solution is denoted $[A]$ with $p_l^A = \frac{s_l(s_h - s_l)}{4s_h - s_l}$, $p_h^A = \frac{2s_h(s_h - s_l)}{4s_h - s_l}$.
- $(l1 - h2)$: the solution is denoted $[B]$ with $p_l^B = \frac{(1-k_h)s_l(s_h - s_l)}{2s_h - s_l}$, $p_h^B = \frac{2(1-k_h)s_h(s_h - s_l)}{2s_h - s_l}$.
- $(l1 - h3)$: leads to $\frac{p_l}{p_h} = \frac{s_l}{2s_h} = \frac{s_l}{s_h}$, a contradiction.
- $(l2 - h1)$: leads to $2p_h = p_h$, a contradiction.
- $(l2 - h2)$: leads to $p_h - p_l = s_h - s_l = (1-k_h)(s_h - s_l)$, a contradiction.
- $(l2 - h3)$: leads to $p_l = p_l \frac{s_l}{s_h} - s_h + s_l < p_l$, a contradiction.
- $(l3 - h1)$: the solution is denoted $[C]$ with $p_l^C = \frac{(1-2k_l)s_l(s_h - s_l)}{2s_h - s_l}$, $p_h^C = \frac{(s_h - k_l s_l)(s_h - s_l)}{2s_h - s_l}$.
- $(l3 - h2)$: the solution is denoted $[D]$ with $p_l^D = (1-k_h-k_l)s_l$, $p_h^D = (1-k_h)s_h - k_l s_l$.
- $(l3 - h3)$: leads to $\frac{s_l}{s_h}\left(p_h - k_l(s_h - s_l)\right) = \frac{s_l}{s_h} p_h$, a contradiction.

It is easily verified that the four regions A, B, C and D form a partition of the capacity space (see Figure 1). Thus, we have identified the unique pure strategy equilibrium for all configurations of parameters.

- In region $[A]$ where installed capacities are large, the solution is valid if $p_l^A \le (2k_h - 1)(s_h - s_l) \Leftrightarrow k_h \ge \frac{2s_h}{4s_h - s_l}$ and if $p_h^A \le 2k_l(s_h - s_l) \Leftrightarrow k_l \ge \frac{s_h}{4s_h - s_l}$.
- In region $[B]$, the high quality firm is capacity constrained, the solution is valid if $p_l^B > (2k_h - 1)(s_h - s_l) \Leftrightarrow k_h \le \frac{2s_h}{4s_h - s_l}$ and if $p_h^B \le 2k_l(s_h - s_l) \Leftrightarrow k_l \ge \frac{(1-k_h)s_h}{2s_h - s_l}$.
- In region $[C]$, the low quality firm is capacity constrained, the solution is valid only if $p_l^C \le (2k_h - 1)(s_h - s_l) \Leftrightarrow k_h \ge \frac{s_h - k_l s_l}{2s_h - s_l}$ and if $p_h^C \ge 2k_l(s_h - s_l) \Leftrightarrow k_l \le \frac{s_h}{4s_h - s_l}$.
- In region $[D]$, both firms are capacity constrained, the solution is valid only if $p_l^D \ge (2k_h - 1)(s_h - s_l) \Leftrightarrow k_h \ge \frac{s_h - k_l s_l}{2s_h - s_l}$ and if $p_h^D \ge 2k_l(s_h - s_l) \Leftrightarrow k_l \le \frac{(1-k_h)s_h}{2s_h - s_l}$.

It is readily observed that the above set of inequalities covers the whole range of capacities; we have thus characterized the whole class of equilibria and shown that for any pair (k_l, k_h) there exists a unique price equilibrium. Lemma 3 takes care of mixed strategies. ■

Lemma 3 *The pricing game $G(s_h, s_l, k_h, k_l)$ has no non degenerate mixed strategy equilibrium.*

Proof We limit ourselves to probability distributions, composed of a density function and a denumerable number of atoms. In a mixed strategy equilibrium (F_l, F_h), s_l and s_h are upper bounds on prices. As D_h is zero for $p_l < p_h - s_h + s_l$, we can write $\pi_h(p_h, F_l) = \int_{p_h - s_h + s_l}^{s_l} g(p_h, p_l)\,dF_l(p_l)$ where

$$g(p_h, p_l) \equiv \begin{cases} p_h D_h(p_h, p_l) & \text{if} \quad D_h(p_h, p_l) \le k_h \\ p_h k_h + (p_h - 1)(D_h(p_h, p_l) - k_h) & \text{if} \quad D_h(p_h, p_l) > k_h \end{cases}$$

Notice that g is concave in p_h and almost everywhere (a.e.) twice differentiable in both variables. The distribution F_l is composed of a density f_l and atoms $\left(\alpha_m, p_m\right)_{m \in M}$. We have

$$\pi_h(p_h, F_l) = \int_{p_h - s_h + s_l}^{s_l} g(p_h, x) f_l(x)\,dx + \sum_{m \in M} \alpha_m g(p_h, p_m)$$

thus, using "$\dot{}$" to denote derivation w.r.t. p_h

$$\begin{aligned} \dot{\pi}_h \underset{a.e.}{=} & \int_{p_h - s_h + s_l}^{s_l} \dot{g}(p_h, x) f_l(x)\,dx + g(p_h, p_h - s_h + s_l) f_l(p_h - s_h + s_l) + \sum_{m \in M} \alpha_m \dot{g}(p_h, p_m) \\ = & \int_{p_h - s_h + s_l}^{s_l} \dot{g}(p_h, p_l)\,dF_l(p_l) + \sum_{m \in M} \alpha_m \dot{g}(p_h, p_m) \end{aligned}$$

since $D_h(p_h, p_h - s_h + s_l) = 0$. We then derive

$$\ddot{\pi}_h \underset{a.e.}{=} \int_{p_h - s_h + s_l}^{s_l} \ddot{g}(p_h, p_l)\,dF_l(p_l) + \dot{g}(p_h, p_h - s_h + s_l) f_l(p_h - s_h + s_l) + \sum_{m \in M} \alpha_m \ddot{g}(p_h, p_m) < 0$$

as $\ddot{g} \underset{a.e.}{<} 0$ and $\dot{g}(p_h, p_h - s_h + s_l) = -\frac{p_h}{s_h - s_l} < 0$.

Since the derivative of the profit is a.e. decreasing and may eventually jump down, profit which is continuous must be concave; hence the best response to F_l is a pure strategy p_h. Since the best response to a singleton p_h for firm l is a pure strategy, the equilibrium has to be in pure strategies.

This proof can be modified to prove that the Bertrand pricing game $G^B(s_1, s_2, k_1, k_2)$ has no non-degenerate mixed strategy equilibrium. ■

Proof of Theorem 2 *The capacity pair (k_l^*, k_h^*) defines the unique equilibrium of $G(s_h, s_l)$.* Since demand is bounded by unity, sales are also bounded by unity, thus there is nothing that a firm could do with excessive capacity that she could not do with unit capacity. We may thus restrict capacities to $[0;1]$.

The frontiers of areas A, B, C and D derived in Theorem 1 are the thin plain lines of Figure 2. Let us consider first the optimal k_l against k_h. The equilibrium payoff of firm l resulting from price competition does not depend on her own capacity level in region A and B. Thus a point interior to $A \cup B$ cannot be optimal due to the positive cost δ for capacity. Since the equilibrium prices are continuous with respect to capacities (cf. equation (8)) so are demands and profits in the capacity game. Hence we can search for the best response of firm l in $C \cup D$.

Using the equilibrium characterization of Theorem 1, we derive the payoff of firm l in region C as $\pi_l^C(k_l, k_h) = k_l(1-2k_l)\frac{s_l(s_h-s_l)}{2s_h-s_l}$, so that the best response against k_h is $\frac{1}{4}$. In region D, the payoff is $\pi_l^D(k_l, k_h) = k_l(1-k_h-k_l)s_l$, leading to the best response $\frac{1-k_h}{2}$. We thus have to compare the profits associated to those two best response candidates; the solution of $\pi_l^D\left(\frac{1-k_h}{2}, k_h\right) = \pi_l^C\left(\frac{1}{4}, k_h\right)$ in k_h is $\tilde{k_h} = 1-\sqrt{\frac{s_h-s_l}{2(2s_h-s_l)}}$. We obtain the best response

$$\kappa^l(k_h) = \begin{cases} \frac{1-k_h}{2} & \text{if } k_h \le \tilde{k_h} \\ \frac{1}{4} & \text{if } k_h \ge \tilde{k_h} \end{cases} \tag{15}$$

A similar analysis shows that the best response of firm h lies in area $B \cup D$. As $\pi_h^B(k_l, k_h) = k_h(1-k_h)\frac{2s_h(s_h-s_l)}{2s_h-s_l}$ the best response is $\frac{1}{2}$ in area B while $\pi_h^D(k_l, k_h) = k_h((1-k_h)s_h - k_l s_l)$ yields the best response $\frac{s_h-k_l s_l}{2s_h}$ for area D. Hence firm h's best response is

$$\kappa^h(k_l) = \begin{cases} \frac{s_h-k_l s_l}{2s_h} & \text{if } k_l \le \tilde{k_l} \\ \frac{1}{2} & \text{if } k_l \ge \tilde{k_l} \end{cases} \tag{16}$$

where $\tilde{k_l} = \frac{s_h}{s_l}\left(1-\sqrt{\frac{2(s_h-s_l)}{2s_h-s_l}}\right)$ is the solution of $\pi_h^D\left(\frac{s_h-k_l s_l}{2s_h}, k_l\right) = \pi_h^B\left(\frac{1}{2}, k_l\right)$ in the domain where $k_l \le 1$.

Using Figure 2, it is clear that the first branches of (15) and (16) are obtained from the frontier lines by rotation at their common axis point, thus their intersection $k_l^* = \frac{s_h}{4s_h-s_l}$ and $k_h^* = \frac{2s_h-s_l}{4s_h-s_l}$ is within area D. To prove that this candidate is the equilibrium, we only need to check that $k_i^* < \tilde{k_i}$ for $i = h, l$. Algebraic manipulations show that $k_l^* < \tilde{k_l} \Leftrightarrow 2s_l^2(3s_h - s_l) > 0$ which is true over the relevant domain $(s_l < s_h)$. Likewise, $k_h^* < \tilde{k_h} \Leftrightarrow \frac{1}{2}s_l\left(s_h(16s_h - 9s_l) + s_l^2\right) > 0$.

In order to establish the equivalence of this equilibrium with Cournot outcomes, observe that the demand system defined by equations (1) and (2) is invertible and yields the system characterizing the price equilibrium of region D i.e., p_l^D and p_h^D as functions of quantity variables k_l and k_h. Solving for a Nash equilibrium of this new quantity game we immediately obtain (k_l^*, k_h^*). Lemma 4 takes care of mixed strategies. ■

Lemma 4 *Equilibria of $G(s_l, s_h)$ are in pure strategies.*

Proof Assume that in equilibrium firm $i = 1, 2$ plays a capacity distribution F_i over $[0, 1]$ whose lower and upper bounds are denoted $\underline{k}_i$ and $\bar{k}_i$. Observe that $\underline{k}_h \ge \kappa^h\left(\frac{1}{2}\right) = \frac{1}{2} - \frac{s_l}{4s_h} > 0$ because π_h is increasing in k_h for all k_l whenever $k_h < \frac{1}{2} - \frac{s_l}{4s_h}$ so that $\pi_h(k_h, F_l)$ is increasing over this range. Since

π_h is decreasing over areas A and C (see figure 3 below) and decreasing down to $\frac{1}{2}$ (or less) in areas D and B, $\bar{k}_h \leq \frac{1}{2}$. Hence the analysis of $\pi_l(k_l, F_k)$ will concentrate on areas B and D because the vertical limit of A is greater than $\frac{1}{2}$.

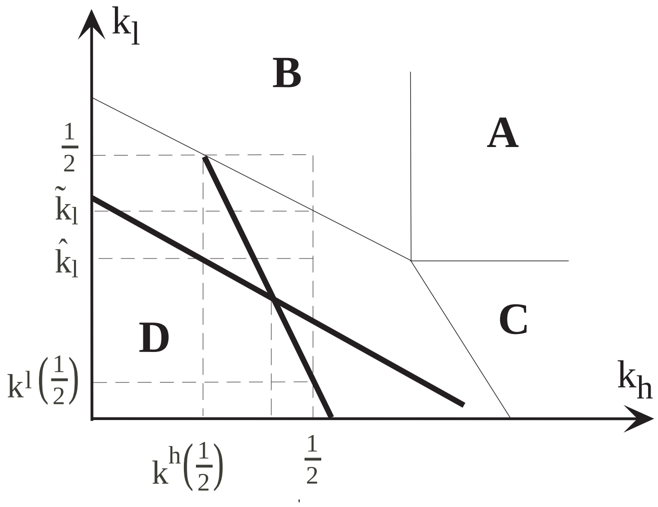

Figure 3: Elimination of mixed strategy equilibria

We claim that $\bar{k}_l \leq \hat{k}_l \equiv \kappa^l\left(\kappa^h\left(\frac{1}{2}\right)\right) = \frac{1}{4} + \frac{s_l}{8s_h}$. Indeed π_l is decreasing in k_l over area B and over area A down to the κ^l line, thus for $k_l > \hat{k}_l$, $\pi_l(k_l, F_k)$ is also decreasing. The frontier BD intersects with the vertical at $k_h = \frac{1}{2}$ for $\tilde{k}_l = \frac{s_h}{2(2s_h - s_l)}$ and we can check that $\tilde{k}_l - \hat{k}_l = \frac{s_l^2}{8(2s_h - s_l)s_h} > 0$ which means that for any k_h in $\left[\underline{k}_h, \bar{k}_h\right]$ and any k_l in $\left[\underline{k}_l, \bar{k}_l\right]$, the pair (k_h, k_l) lies in area D so that $\pi_h(k_h, F_l)$ is concave and has a unique maximizer. Since the best response of firm l to a pure strategy k_h is another pure strategy we have indeed shown that $\left(k_l^*, k_h^*\right)$ characterized earlier is the unique equilibrium. ■

Theorem 3 *The price game $G(s, s, k_i, k_j)$ has a multiplicity of equilibria.*

- *If capacities are similar, a continuum of equilibria exists, in which firms name identical prices.*
- *Otherwise, there exists no pure strategy equilibrium.*

Proof

Step 1: Pseudo best replies

In order to characterize the best reply of firm i, we denote $p = p_j$ the competitor's price and adopt the shorthands $p^-, p^=, p^+$ in order to identify respectively undercutting, matching and pricing-above strategies. Unless the domain of admissible prices is finite, undercutting is not properly defined so that we speak of pseudo best responses.

We first identify the critical price levels which define the relevant payoff regimes. Equating (14:a) to zero, we use the relevant root to define

$$\nu_s(k_i) \equiv \frac{1}{2}\max\left\{0, 1 + s - \sqrt{(1-s)^2 + 4k_i s}\right\}, \tag{17}$$

The threshold $\nu_s(k_i)$ defines the critical price for which undercutting yields so much demand that the profit made over inframarginal units is exactly compensated by the losses made over the units beyond the current capacity. This cutoff plays a role comparable to that of the marginal cost in the standard Bertrand competition: no firm would ever undercut the other's price below this treshold. Notice that $p \leq \nu_s(k_i) \Rightarrow \Pi_i(p^-, p) \leq 0 = \Pi_i(p^+, p)$.

Next, we define the critical price level for which a firm is indifferent between matching and pricing above (the other's price), i.e. is indifferent beween sharing the market and facing no demand. To this end, we equate (14:d) to (14:e) and use the relevant root to define

$$\phi_s(k_i) = \frac{1}{2}\max\left\{0; 1+s-\sqrt{(1-s)^2+8k_i s}\right\}. \tag{18}$$

We may state that $p \leq \phi_s(k_i) \Rightarrow \Pi_i(p,p) \leq 0 = \Pi_i(p^+, p)$.

Last, equating (14:a) and (14:c), we define $\rho_s(k_i)$ the treshold at which a firm is indifferent between serving full demand beyond capacity and matching the other's price while selling below capacity.

$$\rho_s(k_i) \equiv \frac{1}{2}\max\left\{0; s+2-\sqrt{(2-s)^2+8k_i s}\right\} \tag{19}$$

Notice that $p \lesseqgtr \rho_s(k_i) \Rightarrow \Pi_i(p^-, p) \lesseqgtr \Pi_i(p,p)$.

Bringing together this information, we may characterize the pseudo best reply:

- If $p < \phi_s(k_i)(< \nu_s(k_i))$, both undercutting and matching yield negative profits so that the best response is pricing above (p^+) to guarantee zero losses.
- If $\phi_s(k_i) \leq p \leq \max\{0; (1-2k_i)s\}$, the first inequality means that firm i can secure positive profits by matching so that matching dominates pricing above. The second inequality means that matching leads to a constrained capacity (14:d) and as one can see from (14:a), undercutting leads to an even more stringent constraint (see the unit weight instead of $\frac{1}{2}$ over negative term).
- If $\max\{0; (1-2k_i)s\} \leq p \leq \rho_s(k_i)$ then matching is optimal. Since $p \geq \phi_s(k_i)$ remains true, matching keeps dominating pricing above and by construction of $\rho_s(k_i)$, matching dominates undercutting.
- If $p > \rho_s(k_i)$, the reversal in (19) makes undercutting the optimal strategy.

Step 2: Equilibrium characterization

Assume w.l.o.g. $k_i \leq k_j$ and observe that

$$\rho_s(k_j) < \phi_s(k_i) \Leftrightarrow \left\{k_i < \frac{1}{2} \text{ and } k_j > \gamma_s(k_i) \equiv k_i + \tfrac{s-1+\sqrt{(1-s)^2+8k_i s}}{4s}\right\}$$

The analysis of Step 1 can now be used fairly easily. When $\rho_s(k_j) \geq \phi_s(k_i)$, any pair (p,p) where $p \in [\phi_s(k_i), \rho_s(k_j)]$ is a symmetric equilibrium since both undercutting and pricing above are dominated by matching. Observe that there are no other pure strategy equilibria. In such equilibria, firms earns $\frac{p}{2}(1-\frac{p}{s}) \leq \frac{s}{8}$. When $\rho_s(k_j) > \phi_s(k_i)$, the previous equilibria cease to exist and given the nature of pseudo best replies, no pair of single prices may define a pure strategy equilibrium. Equilibria, if they exist, are fully mixed. Lemma 5 constructs a mixed strategy equilibria of $G(1,1,k_1,k_2)$; the same arguments can be used to construct a mixed strategy equilibria of $G(s,s,k_1,k_2)$ for $s < 1$.■

Lemma 5 *For $k_1 < k_2$, there exists an equilibrium of* $G(1,1,k_1,k_2)$ *where firm 2 earns an arbitrary small payoff.*

Proof The proof is by construction. Let firm 2 play the pure strategy $\underline{p}$ while firm 1 plays a distribution F_1 over the support $\left[\underline{p};\bar{p}\right]$. Since $k_2 \leq 1$, $\forall \epsilon \in (0;k_2)$, it is true that $\underline{p} \equiv v_1(k_2-\epsilon) = 1-\sqrt{k_2-\epsilon} < \bar{p} \equiv 1-k_2+\epsilon$ (cf. eq. (17)). On the equilibrium path, firm 1 has zero demand and zero payoff while firm 2 is a monopoly earning $\pi_2(\underline{p},F_1) = \epsilon > 0$ as (14:a) applies. If firms 2 chooses $p_2 < \underline{p}$, she becomes an even more constrained monopoly and thus make a lower profit. If she picks $p_2 > \bar{p}$, she has zero demand and zero payoff. The case $p \in \left[\underline{p};\bar{p}\right]$ forces us to taylor F_1.

- For $p \leq 1-k_2$, (14:a) applies, thus $\bar{\pi}_2(p_2,F_1) = \left(1-F_1(p_2)\right)\left(k_2-(1-p)^2\right)$. A sufficient condition to make $\underline{p}$ optimal is

$$\forall p \in \left[\underline{p};1-k_2\right], \qquad \frac{\partial \bar{\pi}_2}{\partial p_2} \leq 0 \Leftrightarrow \frac{f_1(p)}{1-F_1(p)} \geq \frac{2(p-1)}{k_2-(1-p)^2} \tag{20}$$

- For $p \geq 1-k_2$, (14:b) applies, profit is $\hat{\pi}_2(p_2,F_1) = \left(1-F_1(p_2)\right)\left(p(1-p)\right)$. A sufficient condition to make $\underline{p}$ optimal is

$$\forall p \in \left(1-k_2;\bar{p}\right], \qquad \frac{\partial \hat{\pi}_2}{\partial p_2} \leq 0 \Leftrightarrow \frac{f_1(p)}{1-F_1(p)} \geq \frac{1-2p}{p(1-p)} \tag{21}$$

Solving $\bar{\pi}_2(p,.) = \epsilon = \hat{\pi}_2(p,.)$, we find solutions $\bar{F}(p) = 1-\frac{\epsilon}{k_2-(1-p)^2}$ and $\hat{F}(p) = 1-\frac{\epsilon}{p(1-p)}$ with associated densities $\bar{f}$ and $\hat{f}$. Notice that $\hat{F}(1-k_2) = \bar{F}(1-k_2)$. For a small ϵ, we define $f_1(p) \equiv \bar{f}(p)+\epsilon$ over $\left[\underline{p};1-k_2\right]$ and $f_1(p) \equiv \hat{f}(p)+\epsilon$ over $\left(1-k_2;\bar{p}\right]$. Observe that $\forall p \in \left[\underline{p};1-k_2\right]$,

$$f_1 > \bar{f} \Rightarrow F_1(p) = \int_{\underline{p}}^{p} f_1(x)dx > \bar{F}(p) \Rightarrow \frac{f_1(p)}{1-F_1(p)} > \frac{\bar{f}(p)}{1-\bar{F}(p)}$$

Likewise, $\forall p \in \left(1-k_2;\bar{p}\right]$, since $f_1 > \hat{f}$, we have

$$F_1(p) = F_1(1-k_2) + \int_{1-k_2}^{p} f_1(x)dx > \hat{F}(1-k_2) + \int_{1-k_2}^{p} \hat{f}(x)dx = \hat{F}(p)$$

hence $\frac{f_1(p)}{1-F_1(p)} > \frac{\hat{f}(p)}{1-\hat{F}(p)}$. The distortions applied to $\bar{F}$ and $\hat{F}$ guarantee that (20) and (21) are satisfied. It remains to make sure that F_1 is a probability distribution by setting an atom $\alpha \equiv 1-\lim_{\bar{p}} F_1$ at $\bar{p}$. We have

$$\lim_{\bar{p}} F_1 = \hat{F}(1-k_2) + \epsilon(1-k_2-\underline{p}) + \int_{1-k_2}^{\bar{p}} \left(\hat{f}(x)+\epsilon\right)dx = \hat{F}(\bar{p}) + \epsilon(\bar{p}-\underline{p})$$

thus $\alpha = \frac{\epsilon}{\bar{p}(1-\bar{p})} - \epsilon(\bar{p}-\underline{p}) > 0 \Leftrightarrow 1 > \bar{p}(1-\bar{p})(\bar{p}-\underline{p})$ which is true.

Lastly, we must check that firm 1 has no incentive to undercut $\underline{p}$. Since $k_1 < k_2$, we have $v_1(k_1) > v_1(k_2)$. Hence, whatever the difference k_2-k_1, there is a ϵ small enough making $v_1(k_1) > v_1(k_2-\epsilon) = \underline{p}$ which proves that firm 1 would be a constrained monopolist making losses, if she was to price below $\underline{p}$. ■

II Costly quality

In this Appendix, we extend the analysis to the case where quality is costly. We treat Bertrand price without capacity commitment and Cournot quantity competition. Recall that the later is the synthesis of capacity choice and price competition under commitment i.e., $G(s_1, s_2)$.

Lemma 6 *In a SPE of G^B, firms choose different qualities. The ratio of optimal qualities $\frac{s_l^*}{s_h^*}$ increases towards $\frac{4}{7}$ as quality costs become negligible.*

Proof In the game G^B with unlimited capacities, a pure strategy equilibrium for the choice of qualities is characterized by the first order conditions

$$\frac{\partial \Pi_h^B(s_h, s_l)}{\partial s_h} = \frac{2s_h}{F} \quad \text{and} \quad \frac{\partial \Pi_l^B(s_h, s_l)}{\partial s_l} = \frac{2s_l}{F} \tag{22}$$

Both equations have a unique analytical real solution under the relevant quality hierarchy, which we denote by $s_h(F, s_l)$ and $s_l(F, s_h)$. Direct computations also show that the second order conditions are satisfied for a maximum. Next, we have $\frac{\partial^2 \Pi_h^B(s_h,s_l)}{\partial s_h^2} = -\frac{8s_l^2(5s_h+s_l)}{(4s_h-s_l)^4} < 0$ and $\frac{\partial^2 \Pi_h^B(s_h,s_l)}{\partial s_h \partial s_l} = \frac{8s_h s_l(5s_h+s_l)}{(4s_h-s_l)^4} > 0$ while $\frac{\partial^2 \Pi_h^B(s_h,s_l)}{\partial s_h^2} = -\frac{2s_l^2(8s_h+7s_l)}{(4s_h-s_l)^4} < 0$ and $\frac{\partial^2 \Pi_h^B(s_h,s_l)}{\partial s_h \partial s_l} = \frac{2s_h s_l(8s_h+7s_l)}{(4s_h-s_l)^4} > 0$. Hence, best responses are positively sloped. As shown by Aoki and Prusa (1997) in a similar setting, the equilibrium quality differential does not depend on F as long as the high quality is not constrained ($s_h^* < 1$). Indeed,

$$
\begin{aligned}
(22) \Leftrightarrow 4\left(4s_h^2 - 3s_h s_l + 2s_l^2\right) &= s_h^2 \tfrac{4s_h - 7s_l}{s_l} = \tfrac{2}{F}(4s_h - s_l)^3 \\
\Leftrightarrow 4\left(4z^2 - 3z + 2\right) &= z^2(4z-7) = \tfrac{2s_l}{F}(4z-1)^3
\end{aligned} \tag{23}
$$

where $z = \frac{s_h}{s_l}$. The solution of the LHS of (23) is[12] $\tilde{z} \simeq 5.25$ leading to a quality ratio $\frac{s_l^*}{s_h^*}$ of $\frac{1}{\tilde{z}} \simeq 19\%$. Plugging $\tilde{z}$ into the RHS of (23), we can single out $s_l^*(F) = \frac{\tilde{z}^2(4\tilde{z}-7)}{2(4\tilde{z}-1)^3}F \simeq 0.024F$ and $s_h^*(F) \simeq 0.127F$. Observe that $s_h^* < 1 \Leftrightarrow F < \frac{2(4\tilde{z}-1)^3}{\tilde{z}^3(4\tilde{z}-7)} \simeq 7.79$.

For $F \geq 7.79$, the high quality firm chooses top quality and the equilibrium value $s_l^*(F)$ for the low quality firm ceases to be given by (23); it now solves $\frac{\partial \Pi_l^B(1,s_l)}{\partial s_l} = \frac{4-7s_l}{(4-s_l)^3} = s_l \frac{2}{F} \Leftrightarrow F = g(s_l^*)$ where $g(z) \equiv \frac{2z(4-z)^3}{4-7z}$. Since g is an increasing convex function, $s_l^*(F) = g^{-1}(F)$ is uniquely defined and is an increasing concave function of F with limit $\frac{4}{7} \simeq 57\%$.[13] ■

Lemma 7 *When firms compete a la Cournot,*

- *If $F < \frac{54}{5}$, firms differentiate their products in equilibrium.*
- *If $F \geq \frac{54}{5}$, they do not differentiate in equilibrium.*

[12] $\tilde{z} = \frac{1}{12}\left(\left(8927 - 24\sqrt{39279}\right)^{1/3} + \left(8927 + 24\sqrt{39279}\right)^{-1/3} + 23\right)$.

[13] The apparent arbitrariness of setting a finite upper bound to qualities is now easy to justify: if $F < 7.79$, that is to say, the cost of quality matters, then no firm wishes to choose top quality and the differentiation index $\frac{s_l - s_h}{s_h}$ remains constant and equal to 81%. It is only when F becomes large that there is a problematic tendency to adopt an infinite quality.

Proof Let us consider a pure strategy SPE $\left(s_i^*, s_j^*\right)$ with differentiation i.e., $s_i^* > s_j^*$. We relabel firm h and l. The payoffs (net of quality costs) are $\Pi_h(s_h, s_l) = \frac{s_h(2s_h - s_l)^2}{(4s_h - s_l)^2}$ and $\Pi_l^C(s_h, s_l) = \frac{s_l s_h^2}{(4s_h - s_l)^2}$. The two FOCs for an interior equilibrium $\frac{\partial \Pi_h}{\partial s_h} = \frac{2s_h}{F}$ and $\frac{\partial \Pi_l}{\partial s_l} = \frac{2s_l}{F}$ can be combined to give the following system

$$\frac{(2s_h - s_l)\left(8s_h^2 - 2s_h s_l + s_l^2\right)}{s_h} = \frac{(4s_h + s_l)\, s_h^2}{s_l} = \frac{2}{F}(4s_h - s_l)^3$$

$$\Leftrightarrow (2 - 1/z)\left(8z^2 - 2z + 1\right) = (4z + 1)\, z^2 = \tfrac{2}{F} s_l (4z - 1)^3 \tag{24}$$

where $z = s_h/s_l$. The first equation of (24) has solution $\bar{z} \simeq 2.79$.[14] In our model with bounded maximum quality, the quality leader reaches the top $s_h^* = 1$ as soon as $F \geq 7.75$. Then the equilibrium value $s_l^*(F)$ for the low quality firm solves $F = \frac{2s_l(4 - s_l)^3}{(4 + s_l)}$, which a concave function of s with maximum at $s = \frac{4\sqrt{7} - 8}{3} \simeq 0.86$. We deduce that $s_l^*(F)$ is uniquely defined and is an increasing **convex** function of F i.e., reaches also the upper bound. We may then look for the solution of $\frac{\partial \Pi_l^C(1,1)}{\partial s_l} = 2\frac{1}{F}$ which is exactly $F = \frac{54}{5}$. ■

III Capacity choice before quality choice

The game G is altered into a new game Γ as follows: firms choose first capacities k_i and k_j and then qualities s_i and s_j.

Lemma 8 *In any SPE of Γ, firms choose identical qualities and earn at least the Cournot payoff associated to top quality.*

Proof Notice first that whenever $s_h > s_l$, and whatever capacitiy levels have been selected, the price equilibrium derived in the proof of Theorem 1 still applies in the last stage of the game. Let us consider a SPE $\left(k_i^*, k_j^*\right)$ of Γ. We prove by contradiction that any equilibrium $\left(s_i^*, s_j^*\right)$ of $\Gamma\left(k_i^*, k_j^*\right)$ features identical qualities. If not, we have equilibrium qualities $s_h^* > s_l^*$ in $\Gamma\left(k_i^*, k_j^*\right)$ and the pricing game $\Gamma\left(k_i^*, k_j^*, s_h^*, s_l^*\right)$ has a unique equilibrium. We may then use the areas A, B, C and D of figure 3. The argument of the proof is then the following: first we show that it is impossible to end up in equilibrium in region A, B or C. We are then left with region D. However, in this region, both firms' payoff increase in own quality. Accordingly, we should not expect quality differences to prevail. The only remaining candidate equilibria exhibit no differentiation. The argument developed in game G for the cases where products were homogeneous may then be applied.

If the price equilibrium of $\Gamma\left(k_i^*, k_j^*, s_h^*, s_l^*\right)$ is interior to area A of figure 3 above, then the closed form payoffs in $\Gamma\left(k_i^*, k_j^*\right)$ are

$$\Pi_l^A = \frac{s_h s_l (s_h - s_l)}{(4s_h - s_l)^2} \quad \text{and} \quad \Pi_h^A = \frac{4s_h^2(s_h - s_l)}{(4s_h - s_l)^2}$$

in a neighborhood of (s_h^*, s_l^*). The best responses are $s_l^A = \frac{4s_h}{7}$ and $s_h^A = 1$ hence the equilibrium of $\Gamma\left(k_i^*, k_j^*\right)$ is either $\left(s_i^*, s_j^*\right) = \left(\frac{4}{7}, 1\right)$ or $\left(1, \frac{4}{7}\right)$ and furthermore capacities must satisfy $k_l^* \geq \frac{7}{24}$ and

14 $\frac{(4\bar{z}+1)\bar{z}^2 F}{2(4\bar{z}-1)^3} \simeq \frac{F}{22.47}$ and $s_h^* = \bar{z} s_l^* \simeq \frac{F}{7.75}$.

$k_h^* \geq \frac{7}{12}$ (we evaluate the conditions for interiority of region A for $s_l = \frac{4}{7}$ and $s_h = 1$). Consider the following deviation: $k_l = \frac{7}{24} \leq k_l^*$. Then, the equilibrium of $\Gamma\left(k_l, k_h^*\right)$ must be $\left(s_l^*, s_h^*\right) = \left(\frac{4}{7}, 1\right)$ because firm l has now a too small capacity to adopt the high quality (condition $k_h^* \geq \frac{7}{12}$); neverthelesss it benefits from the capacity cost reduction given that payoffs in area A are capacity independent. Now firm h can also reduce its capacity down to $\frac{7}{12}$ because it is assured of being the high quality firm. We have thus proved that the quality equilibrium of $\Gamma\left(k_l^*, k_h^*\right)$ (if differentiated) cannot be interior to area A.

If the price equilibrium of $\Gamma\left(k_i^*, k_j^*, s_h^*, s_l^*\right)$ is interior to area C, then payoffs are

$$\Pi_h^C = \left(1 - \frac{p_h^C - p_l^C}{s_h - s_l}\right) p_h^C = \frac{(s_h - s_l)(s_h - k_l s_l)^2}{(2s_h - s_l)^2}$$

and

$$\Pi_l^C = k_l p_l^C = k_l(1-2k_l)\frac{s_l(s_h - s_l)}{2s_h - s_l}$$

in a neighborhood of (s_h^*, s_l^*). Since Π_h^C is increasing with s_h, the best response of firm h is $s_h^C = 1$. As $\frac{\partial \Pi_l^C}{\partial s_l} \propto \frac{2s_h^2 - 4s_h s_l + s_l^2}{(2s_h - s_l)^2}$, the best response of firm l is $s_l^C = \left(2-\sqrt{2}\right)s_h$. The equilibrium is thus $\left(s_i^*, s_j^*\right) = \left(2-\sqrt{2}, 1\right)$ or $\left(1, 2-\sqrt{2}\right)$ where capacities must satisfy $k_l^* \leq \frac{1}{2+\sqrt{2}} \simeq 0.29$ and $k_h^* \geq \frac{1-k_l\left(2-\sqrt{2}\right)}{\sqrt{2}}$ (frontiers of C computed for equilibrium values), the latter value being itself greater than 0.59. Whenever firm l plays $k_l < \frac{1}{2+\sqrt{2}}$, the equilibrium of $\Gamma\left(k_l, k_h^*\right)$ must be $\left(2-\sqrt{2}, 1\right)$ because firm l does not have enough capacity to be the quality leader. The low quality firm gets $\Pi_l^C \propto k_l(1-2k_l)$ and optimally chooses $k_l = \frac{1}{4} < \frac{1}{2+\sqrt{2}}$. When facing $k_l < \frac{1}{2+\sqrt{2}}$, the high quality firm h receives a profit independently of its capacity thus she should reduce it down to the frontier value $k_h^* = \frac{1-\frac{1}{4}\left(2-\sqrt{2}\right)}{\sqrt{2}} \simeq 0.60$.

We have thus shown that the quality equilibrium of $\Gamma\left(k_i^*, k_j^*\right)$ (if followed by differentiation) cannot be interior to area C. A symmetric result holds for area B where the high quality firm is capacity constrained in the price equilibrium. The previous steps have shown that the price equilibrium of $\Gamma\left(k_i^*, k_j^*, s_h^*, s_l^*\right)$ lies within area D whose definition can be inverted to fit the order of strategic moves:

$$k_h < \frac{s_h - k_l s_l}{2s_h - s_l} \Leftrightarrow \frac{s_l}{s_h} > \max\left\{\frac{2k_l - 1 + k_h}{k_l}, \frac{2k_h - 1}{k_h + k_l}\right\} \tag{25}$$

In a neighborhood of (s_h^*, s_l^*), payoffs are

$$\Pi_h^D = ((1-k_h)s_h - k_l s_l)\, k_h \quad \text{and} \quad \Pi_l^D = (1 - k_h - k_l)s_l k_l$$

We observe that profits increase with own quality and that small increases by each firm are compatible with (25), hence no differentiated quality equilibrium exists. It must be the case that $s_i^* = s_j^*$ in the quality equilibrium of $\Gamma\left(k_i^*, k_j^*\right)$. Furthermore the common quality s^* is larger than $\bar{s}$ because any firm could deviate to $s = 1$ in $\Gamma\left(k_1^*, k_2^*\right)$ (cf. proof of Proposition 1). Although there is multiplicity of equilibria in the pricing game $\Gamma\left(k_i^*, k_j^*, s^*, s^*\right)$, a perfect equilibrium of $\Gamma\left(k_1^*, k_2^*\right)$ gives each firm at least its Cournot payoff for top quality (an ϵ-deviation guarantees it). ■

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