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Computational thinking and repetition patterns in early childhood education: Longitudinal analysis of representation and justification

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Abstract

This paper provides a longitudinal analysis of the understanding of repetition patterns by 24 Spanish children ages 3, 4 and 5, through representation and the type of justification. A mixed quantitative and qualitative study is conducted to establish bridges between algebraic thinking and computational thinking by teaching repetition patterns in technological contexts. The data are obtained using: a) participant observations; b) audio-visual and photographic records; and c) written representations, in drawing format, from the students. The analysis involves, on the one hand, a statistical analysis of the representations of patterns, and on the other, an interpretive analysis to describe the type of justification that children use in technological contexts: "elaboration", "validation", "inference" and "prediction or decision-making". The results show that: a) with respect to the representation of patterns, errors decreased by 27.3% in 3-to-5-year-olds, with understanding and correct representation of repetition patterns gaining prominence in more than 50% of the sample from the age of 4; b) on the type of justification used, it is evident that in 3-and-4-year-olds, "elaboration" predominates, and at 5, progress is made towards "validation". We conclude that it is necessary to design learning sequences connected with theory and upheld through practice, and that foster the active role of the teacher as a promoter of teaching situations that help spur the beginning of computational and algebraic thinking.

Keywords Computational thinking · Algebraic thinking · Repetition patterns · Representation · Justification

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1 Introduction

Addressing the teaching–learning process by using a textbook as the only instructional resource is no longer sufficient in a society based on knowledge that is evolving at an accelerated pace, in which digital technologies are part of daily life and alter our way of being, being and relating (Castañeda et al., 2020).

In a school environment, technology has been defined as a tool that the student can use to learn and grow (Sharapan, 2012). However, the use of technology must be a means to support learning, one that is not isolated and decontextualised. For this reason, the Technology Policy Statement of the National Association for the Education of Young Children (NAEYC) & Fred Rogers Center for Early Learning and Children's Media (2012) provides a guide for early childhood education professionals on the balanced and appropriate use of interactive digital technologies from birth to age eight. The purpose of this statement is to understand, evaluate and integrate suitable technologies for development in the classroom and thus promote digital literacy and computational thinking.

This study assumes that computational thinking is an ability of human reasoning that, through analytical and algorithmic approaches, formulates, analyses and solves problems (Bocconi et al., 2016). Accordingly, the International Society for Technology in Education [ISTE] and the Computer Science Teachers Association [CSTA] (2011) describe the following essential traits of this type of thinking in order to promote and implement it in education: a) organise data logically; b) represent them through models and simulations; c) automate solutions by sequencing in ordered steps; d) identify and analyse solutions and be able to implement the most efficient one; and e) have the ability and attitude to communicate and work as a team to achieve a common goal. Years later, ISTE (2016) set out four computational thinking skills, and urged that proposals be implemented that encompass them: 1) decomposition; 2) abstraction; 3) pattern recognition; and 4) algorithm design.

Within this framework of connections, various studies affirm that technological resources such as robots provide students with the opportunity to engage interactively, multidisciplinarily and cooperatively in the teaching–learning process (Keren & Fridin, 2014; La Paglia et al., 2017; Seckel et al., 2022). Gomoll et al. (2016) view robotics as an efficient tool for promoting basic learning. Rhine and Martin (2008) state that experience with robots facilitates the abstract transfer of mathematics to the practical reality of everyday situations.

Despite this relevance, few studies analyse the relationship between mathematical education and technological contexts in early childhood education (ages 3, 4 and 5). Zhong and Xia (2020) state, for example, that without evidence teachers perceive a limited understanding of the opportunities of robotics, and therefore, without teacher acceptance, it is difficult for technological resources to play a significant role in mathematical education (Keren & Fridin, 2014).

The tangible and materialisable nature of robots provides a concrete, motivational and authentic way to access and consolidate abstract mathematical knowledge, as well as to develop computational thinking and promote understanding, reasoning and justification when tasks are posed that are intended to challenge and connect.

Based on these considerations, the purpose of this study is to analyse the understanding of repetition patterns in students of 3, 4 and 5 years of age when they carry out assisted teaching in technological contexts, and thus provide evidence that motivates teachers to rely on this type of mathematical content through technological resources, while promoting one of the four skills proposed by ISTE (2016): pattern recognition. It is important to note that we will focus on this skill since pattern exploration can be regarded as a springboard to promote generalisation (Vanluydt et al., 2021), anticipation, guesswork, justification, representation, and the precise use of mathematical language. Authors such as Mulligan et al. (2020) argue that a lack of knowledge of patterns and their structure can be a predictor of future mathematical difficulties. Some of the reasons for promoting the teaching of patterns in early childhood education is that patterns provide an essential foundation for the development of mathematical thinking and contribute to the overall process of representation and abstraction in mathematics (Lüken & Kampmann, 2018; McGarvey, 2012; NCTM, 2000). With this in mind, empirical and longitudinal studies have been able to demonstrate how patterns effectively contribute to the mathematical performance of students up to 11 years of age (Nguyen et al., 2016; Rittle-Johnson et al., 2017). Considering the above, Wijns et al. (2019) point out that an open area of research is to describe patterned tasks that exploit the potential of patterns and show how to implement them in early childhood education classrooms.

From this perspective, we define the study problem based on the following research question:

How do technological resources support the understanding of tasks with repetition patterns, and what kind of justification do 3-, 4-, and 5-year-olds use in their representation?

This question leads to the following research objectives:

1. Analyse the understanding of repetition patterns through their representation, as part of an assisted teaching method that relies on technological resources (programmable robots and online games), longitudinally during a three-year intervention with 24 early childhood education students (3, 4 and 5 years old).
2. Describe the type of justification that 3-, 4-, and 5-year-olds use to demonstrate an understanding of repetition patterns in technological contexts.

2 Literature review

This section considers computational thinking and educational robots as scenarios to promote the teaching of repetition patterns, and their representation and justification as processes that promote understanding.

2.1 Computational thinking and educational robots

Based on the ideas of Papert (1985), Wing (2006, p. 33) introduced and developed the term computational thinking as "a fundamental skill for everyone, not just for computer scientists. To reading, writing, and arithmetic, we should add computational thinking to every child's analytical ability". According to Wing (2006), computational thinking is a set of thinking skills derived from computer sciences that are useful for solving problems in a given context. A few years later, she explained that computational thinking is a type of analytical thinking that supports the development of abstraction and argued that, to provide a basis for understanding and applying computational thinking, these skills have to be presented in early childhood (Wing, 2008). From this perspective, we recognise that computational thinking emerges from computing, but unlike this discipline, computational thinking can be used to transfer skills, such as abstraction, to fields other than programming (Berland & Wilensky, 2015). However, the place that computational thinking occupies in curricula varies from country to country (Nardelli, 2019). As expressed by this author, in some curricula, computational thinking is integrated into every subject, while in others, it is exclusively part of an isolated computer science subject. This varying approach may be the result of the little consensus contained in the literature to define or position itself in a specific model of computational thinking (Nardelli, 2019; Shute et al., 2017).

In this study, we share the vision of the International Society for Technology in Education (ISTE) and the Computer Science Teachers Association (CSTA) (ISTE & CSTA, 2011), who specify that computational thinking allows: a) formulating problems by using technological artifacts to solve them; b) ordering and analysing data in a logical way; c) representing said data with different elements; d) automating solutions through algorithmic thinking, that is, a sequence of ordered steps; e) implementing possible solutions in an effective and efficient way; and f) generalising and transferring this problem-solving to other similar situations or contexts. Thus, computational thinking skills should be taught at an early age in order to trigger early cognitive development in students (Avcı & Deniz, 2022; Buitrago Flórez et al., 2017). It is also necessary to promote digital skills as a way to solve problems creatively by combining abstraction and pragmatism, since such thinking is also based on mathematics (Bråting & Kilhamn, 2021; Valverde-Berrocoso et al., 2015). Our position is consistent with that of Wing (2011), who considers computational thinking as thought processes that are intrinsically related to the formulation of problems and the search for efficiently executable solutions that, as expressed by Shute et al. (2017), are reusable in different contexts. In this context, Wing (2011) emphasises that the main process of computational thinking is abstraction and that this process is very useful for "(…) defining patterns, generalising from specific instances and parameterising" (paragraph 5).

In this scenario, schools must play a crucial role in linking curricular proposals framed in teaching–learning contexts that favour the development of computational thinking (Gutiérrez-Núñez et al., 2022). This is how we should realise that new technologies, in particular robotics, provide a vehicle for different multidisciplinary learning tasks, including new forms of social interaction that facilitate the cognitive, creative and communicative development of the student, as well as to promote multidisciplinary benefits in mathematics (Barker & Ansorge, 2007; Nugent et al., 2009; Schina et al., 2021). Technological environments such as online games, animation programming for children, educational robots, etc., allow learning while applying and understanding abstract concepts in a fun way. Studies carried out in Western countries have concluded that students ages 4 to 6 are able to program simple educational robots (Cejka et al., 2006; Kazakoff et al., 2012). González-González (2019, p. 17–12) chronologically schematises various proposals for children ages 3 to 6 based on a holistic and globalising approach where active and collaborative learning methodologies are executed:

- From 3–4 years old: production and execution of instructions, mainly involving the body itself and action and work with manipulatives (tangible programming).
- Between 4–5: manipulative tangible programming, incorporation of programming through natural tactile interfaces (drag-drop interactions, commands with visual representation, graphic instructions).
- Between 5–6: tangible and tactile programming, possibility of introducing commands with some words (simple instructions).

From this perspective, robotics has been selected as a means for teaching patterns, since, as Bers (2008) states, this type of resource makes abstract ideas more concrete. That is, students can check right away the impact of their programming commands on the robot's actions and understand the logic of instructions, iterative regularities and sequential thinking.

Paris and Paris (2003) argue that the use of sequenced images to tell stories is common in early childhood and that this task requires narrative thinking and understanding of sequences. Computer programming can be regarded as creating a story through sequencing (Kazakoff & Bers, 2014), since students control the behaviour of the robot through scripts. This is how involving children in programming tasks where scripts are executed promotes the externalisation and reflection of their internal thinking processes (Kazakoff & Bers, 2014).

2.2 Teaching repetition patterns: Representation and justification as processes that promote understanding

Algebraic thinking is made up of mental processes that contribute to creating referential meaning for some type of representation, and in turn constructing and expressing generalisations (Cetina-Vázquez & Cabañas-Sánchez, 2022). Through this prism, pattern exploration can be considered a springboard to promote generalisation (Vanluydt et al., 2021) and a component that positively influences early mathematical development (Mulligan et al., 2020; Papic et al., 2011; Wijns et al., 2021), since it promotes the study of regularities, and the connection and representation of relationships through symbols (Radford, 2010). Lüken and Sauzet (2020) affirm that to learn mathematics is to develop the ability to recognise patterns, interpret structures and establish relationships. Therefore, it is necessary for children to have prior experience with pattern tasks to develop their algebraic thinking before they are instructed on the use of algebraic notation and symbology (Carraher & Schliemann, 2018).

Pattern identification involves the observation and recognition of regularities or iterative sequences in objects or data, and for Hsu et al. (2018), recognising patterns and designing algorithms are two possible computational thinking skills that can be introduced early in childhood education.

Learning patterns contributes to the ability of children, from an early age, to recognise, order and organise their world, since it has been shown that pattern recognition, comparison and analysis are factors that determine and promote the intellectual development of children (National Council of Teachers of Mathematics [NCTM], 2000). Several studies demonstrate that pattern recognition is a cognitive ability that influences a wide range of academic skills (Kidd et al., 2013; Pasnak et al., 2015). Mulligan and Mitchelmore (2009) state that, when considering patterns, it is necessary to distinguish between pattern as an ordered sequence or series, and pattern structure, that is, the organisation, rule or core that underlies the pattern. Along these lines, they state that patterns comprise two components: 1) cognitive, related to the knowledge of the structure; and 2) meta-cognitive, linked to the ability to search and analyse patterns. Du Plessis (2018, p. 3) notes that a knowledge of patterns "provides a gateway to the algebraic world of generalised thought". Zippert et al. (2020) state that there is a close relationship between the recognition of patterns and the mathematical capacity of children, given the determining role played by the ability to identify predictable sequences based on underlying rules.

McGarvey (2012) believes that when patterns are presented to children, they are more likely to develop the skills needed to understand relationships within a pattern and begin to use symbols to represent those relationships. Alsina et al. (2021) point out that the representation of mathematical ideas and procedures is an indispensable process for learning, and therefore, if there is no representation there is no understanding, and without understanding there can be no mathematical learning. Therefore, it is assumed that, from an early age, children have to represent in order to learn patterns (Acosta et al., under review). When children engage in programming activities and create scripts, they are externalising and reflecting on their internal thought processes.

From this point of view, Mulligan et al. (2004) affirm that the external images of a child reflect the structural characteristics of his/her internal representations, which allows capturing the conceptual understanding of the child. Therefore, representing also refers to the act of externalising an internal mental abstraction (Goldin, 2020). Against this backdrop, we conceptualise representation in mathematics as an interconnected process that provides specific evidence, through the use of different signs, graphs and/or natural language, of the mathematical knowledge and procedures that students have.

Additionally, promoting communication in the math classroom helps students to express themselves more clearly, which in turn allows teachers to understand what students are thinking and thus make assertive pedagogical decisions (Ingram et al., 2019). In this regard, Chua (2017, p.115) notes that "To probe into the mathematical reasoning of students, another tool is needed to make such reasoning visible: justification". Ball and Bass (2003) point out that justification is a necessary skill to discover and understand new mathematical concepts, from a flexible perspective that allows transferring mathematical procedures, to other situations and restructuring previous knowledge by generating new arguments. Cornejo-Morales et al. (2021)

indicate that this reconstruction manifests itself between children's interpersonal and intrapersonal discourses when they share and justify their views. Staples et al. (2012) consider justification as a learning practice whereby children "(…) improve their understanding of mathematics and their proficiency in doing mathematics; it is a means of learning and doing mathematics" (p. 447). Cox et al. (2017) thus argue that it is necessary to listen to the mathematics of students, in order to monitor, select, sequence, discuss and share the knowledge generated among peers.

From this perspective, Chua (2017) believes that justification tasks are an integral element for learning mathematics with understanding, pointing out four types of justification: 1) "elaboration", which requires presenting the strategy or approach used; 2) "validation", which implies using arguments to support or refute a mathematical conclusion; 3) "inference", which requires connecting and interpreting the mathematical result; and 4) "prediction or decision-making", which requires finding evidence and generalisations to support a mathematical statement. This makes it possible to organise, understand, communicate and justify the mathematical nature of actions previously carried out on the educational, social, creative and technological level. Because of this, educational proposals must be constructed for the purpose of increasing the codification of the structural characteristics that comprise the pattern, to facilitate the representation and its justification.

3 Method

In keeping with our goals, we have designed a mixed study to test the opportunities for understanding provided by technological contexts when teaching repetition patterns to Early Childhood Education students (ages 3, 4 and 5). Creswell and Plano Clark (2018) state that this type of research intentionally combines the perspectives, approaches, data forms and analyses associated with a quantitative and qualitative design in order to provide a more complete and nuanced understanding of the object of study. The research design relies on quantitative + qualitative methods with the intention of emphasising the quantitative phase and the complementary role of the qualitative phase in the design of mixed methods (Creswell & Plano Clark, 2018).

From this perspective, our design facilitates a descriptive and interpretive analysis that seeks to show an understanding of repetition patterns through their representation and justification, within the framework of a teaching process that relies on the use of technological resources.

3.1 Participants

The program was implemented longitudinally over three consecutive years with 24 children, all belonging to the same class of a public school in Spain. The sample consisted of 12 boys and 12 girls. In the first year of the intervention, the average age of the participants was 3 years and 9 months, then 4 years and 9 months and 5 years and 9 months (SD = 5 months) for the second and third years of the intervention, respectively.

This group was selected through a non-probabilistic sampling of an accidental or causal nature (Fernández et al., 2014), since the selection criteria were determined by the possibility of having access to this group; by the continuity and longitudinal monitoring of the tutor; and by virtue of being a school with low enrolment mobility in preschool courses.

Each year, before starting the implementation of the teaching tasks, the Test of Early Mathematics Ability 3 (TEMA3), designed by Ginsburg and Baroody (2003). This test has been created for the purpose of: a) identifying students who are significantly above or below their peers; b) identifying strengths and weaknesses of early mathematical competence; c) documenting students' progress; and d) facilitating a measure of students' mathematical competence validated for use in research projects. In our case, it was administered in order to determine the children's existing mathematical knowledge and have a validated data point for our study.

Considering the interpretative scale of the TEMA3, which indicates that a score below 70 is very poor and one above 130 is very good, we note that the group generally obtained an average score of 93, 81 and 89 at the ages of 3, 4, and 5, respectively.

The participants had previous knowledge of working with patterns. During the three years of the intervention, cross-cutting tasks with patterns were implemented through different educational contexts (real situations, manipulatives, playful, literary and graphic resources) following the Approach of the Mathematics Teaching Itineraries (EIEM) (Alsina, 2019, 2020a, 2022). This approach shuns the teaching of mathematics based on repetition and memorisation, and instead proposes that mathematics be taught as a journey from the concrete to the abstract through deliberate teaching sequences that consider different contexts (Acosta & Alsina, 2021; Acosta et al., 2022).

3.2 Ethical considerations

Before starting the field work, and in an effort to provide transparency, and based on ethical principles, the families were informed of the following aspects: purpose of the research; goals and timeline; procedure; and the need to record audio and video of the sessions. As a result, all the families involved in the study provided their informed consent and approval. In the same way, the children's desire to participate or not in the activities and to be recorded and photographed was respected throughout the intervention.

3.3 Design and procedure

During the three school years, three tasks were designed in technological contexts that relied on different resources. In keeping with the implementation in González-González (2019), the first year, the intervention involved the Bee-bots programmable robots; the second year, the students worked on patterns using an iPad and an online game; and finally, in the third year they solved challenges with the Cubetto robot. All the tasks included three intervention sessions: 1) introduction; 2) performance of the challenge, and 3) conclusion, dialogue and confirmation of the content learned. In order to ensure personalised and individual instruction, the group was divided into two randomly. As a result, over the course of three years, 9 sessions lasting 50 min each were held, prioritising tasks with repetition patterns in technological contexts.

The implementation of the intervention considered the phases proposed in the Mathematical Literacy Model for Children (Alsina, 2017). The phases, described in Fig. 1, promote the students' mental autonomy, the formulation of hypotheses, as well as the design and application of creative problem-solving strategies, all by relying on a verified and negotiated debate to reach a joint construction of solutions (Alsina, 2020b).

Below, we specify each of the phases based on the design, planning and intervention of this study:

Phase 1. Mathematisation of the teaching context: As part of a longitudinal study, the context of technological resources is planned to convey the teaching of repetition patterns. The content is approached keeping the literature in mind, and an itinerary of increasing difficulty is set up according to the pattern types and the age at which they are taught, as shown in Fig. 2.

Phase 2. Prior mathematical knowledge: The Ginsburg & Baroody TEMA-3 Test is applied every year (2003) and, through proper questions such as strategies to bring out the students' knowledge, reflexive processes are induced that generate the co-construction of knowledge within the framework of teaching for understanding (Anijovich & Mora, 2021).

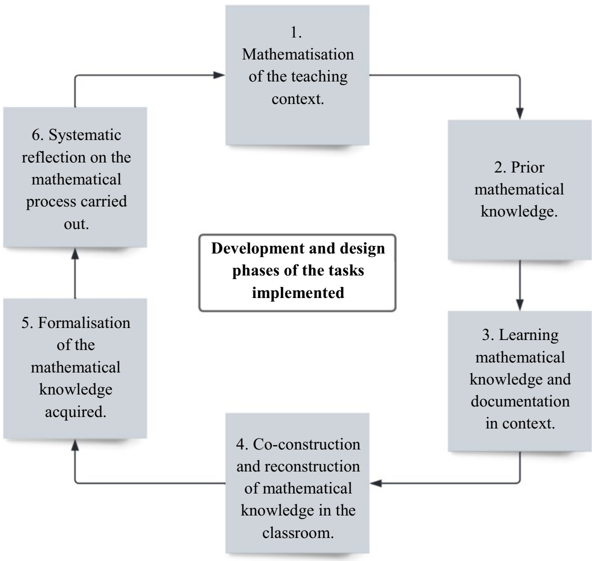


Fig. 1 Mathematical Literacy Model for Children, (Alsina, 2017)

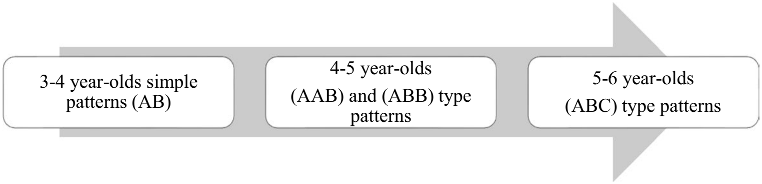


Fig. 2 Types of patterns introduced at each age

Phase 3. Learning mathematical knowledge and documentation in context: We opted for tasks that engage skills, such as copying, interpolating, extending, translating or abstracting, and identifying the unit of repetition, providing the opportunity to create sequences in order to solve a challenge or problem conveyed in a technological context (Fig. 3). The tasks designed underwent a validation process involving the judgment of eight experts from the Working Group "Research in Early Childhood Mathematics Education" (IEMI in Spanish) of the Spanish Society for Research in Mathematics Education (SEIEM). These experts evaluated the following aspects using a questionnaire: a) the degree of correspondence; b) the formulation; and c) the relevance. All the actions carried out by the students based on the tasks proposed were documented.

Phase 4. Co-construction and reconstruction of mathematical knowledge in the classroom: The learning produced is communicated in context, prioritising a precise and adequate mathematical language. In this phase, "the new co-built knowledge

Age	Task	Sessions
3	The students are prompted to enter the correct sequence of commands into the programmable educational robots (Bee-bots) in order to achieve the challenge proposed: to help the bee collect as much pollen as possible.	1. We are robots. 2. Execution of the challenge. 3. Space for reflection, representation and formalisation.
4	Copy, extend and interpolate patterns through the online game "In series", accessible at: https://clic.xtec.cat/projects/enserie/jclic.js/index.html	1. Familiarisation with the iPad. 2. Execution of tasks. 3. Space for reflection, representation and formalisation.
5	Patterns are created with Lego Duplo pieces, and based on the interaction with the programming sheets of the "Cubetto" robot, sequences of actions are associated with the series created with the manipulatives. Next, the route designed is checked and the finished building, which follows the same pattern that was used with the programming sheets, is placed at the end of the route.	1. We are robots. 2. Execution of the challenge. 3. Space for reflection, representation and formalisation.

Fig. 3 Tasks designed and implemented longitudinally

is contrasted with prior knowledge, leading to the reconstruction of mathematical knowledge" (Alsina, 2020b. p. 174).

Phase 5. Formalisation of the mathematical knowledge acquired: The process of representation is used to materialise the formalisation of the understanding of the repetition patterns addressed, in order to start on the path to justification.

Phase 6. Systematic reflection on the mathematical process carried out: Introspection on the practice itself is promoted by creating iterative cycles of longitudinal reflection that question certainties and explore new perspectives in order to reflexively recover what happened and think about improving future actions (Anijovich & Mora, 2021).

3.4 Information-gathering techniques

Data were collected at three levels (Fig. 4).

As shown in Fig. 4, we opted to use: a) ethnographic methodological schemes of participant observation that rely on a field diary as a tool to record the children's spontaneous expressions during the performance of the tasks; b) pedagogical documentation through a fixed and mobile audio-visual record of all the sessions; and c) written representations, in drawing format, of all the children's output as a sample of the formalisation of the knowledge acquired.

Kawulich (2005) believes that observation and active participation promotes a direct interaction that allows researchers to learn and reflect on the activities that are implemented with the participants in a natural setting. In this scenario, the pedagogical documentation gives a voice to the child's thinking and makes it possible to interpret the knowledge and skills of younger children through verbal and non-verbal expressions (Björklund et al., 2020). We agree with Mitchelmore (2018) when he states that, "the processes of pedagogical documentation inherently support the interaction of the collection and generation of concurrent data" (p.190), where the observer acts as an active agent who co-constructs meaning in a reflective, active and reciprocal way in order to create a plural and transformative space (Mitchelmore, 2018).

3.5 Analysis of the data obtained

We conducted a descriptive analysis with a dual focus on the responses recorded during the intervention (Esterberg, 2002). On the one hand, the students'

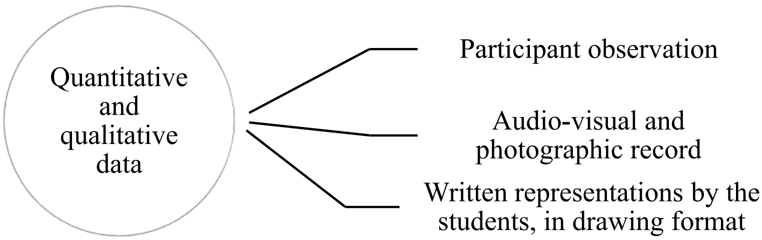


Fig. 4 Methods of obtaining qualitative and quantitative data

representations are categorised by age into two case groups: 1) correct, when the written production exhibits no errors; and 2) incorrect, when it does exhibit errors. An invalid case occurs when the child is absent on the day of the intervention. The quantitative analysis was carried out as shown in Fig. 5.

On the other hand, the children's contributions are analysed in an interpretive way to describe the type of justification they use, based on the Chua model (2017) as an instrument (Fig. 6). This analysis involves cyclical and deductive reviews, where categorisation disagreements are triangulated and discussed until a consensus is established.

With the intention of complementing the above data, audiovisual evidence is transcribed and discussed after the fact by analysing the discourse used by the student to justify their actions. Figure 7 shows the scheme that guided the reduction of the data and the assignment of the categories described by Chua (2017) through the *Atlas.ti* program.

At this point in the qualitative analysis, it is necessary to highlight that the detection of the most relevant fragments allows us to review the relationships between the texts and reality, revealing the discourse used by the child, its point of origin, how it flows, and what accompanies it (Leeuwen, 2008).

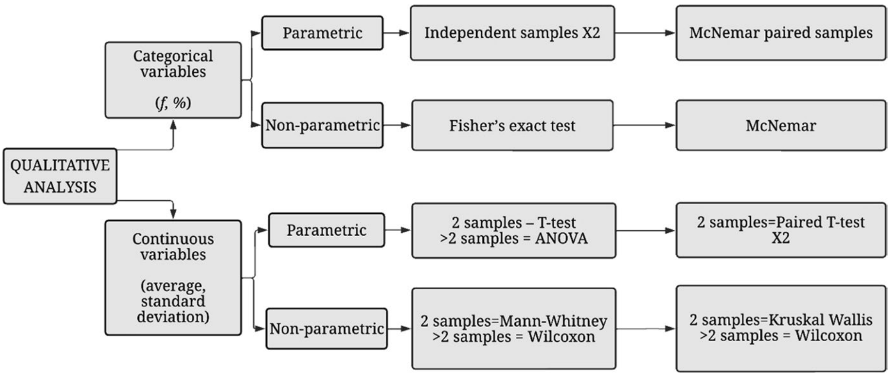


Fig. 5 Flow diagram showing the statistical analysis employed

Type of justification	Purpose of the justification	Supplemental elements
Elaboration	Explain how…	A description of what was done
Validation	Explain why…	Evidence to accept or refute a claim
Inference	Explain what…	Use of mathematical discourse with keywords from the task or challenge addressed
Prediction/decision-making	Explain whether…	A generalised decision with evidence to support or refute the mathematical claim

Fig. 6 Type of justification based on the purpose and elements that supplement it Source: Adapted from Chua (2017)

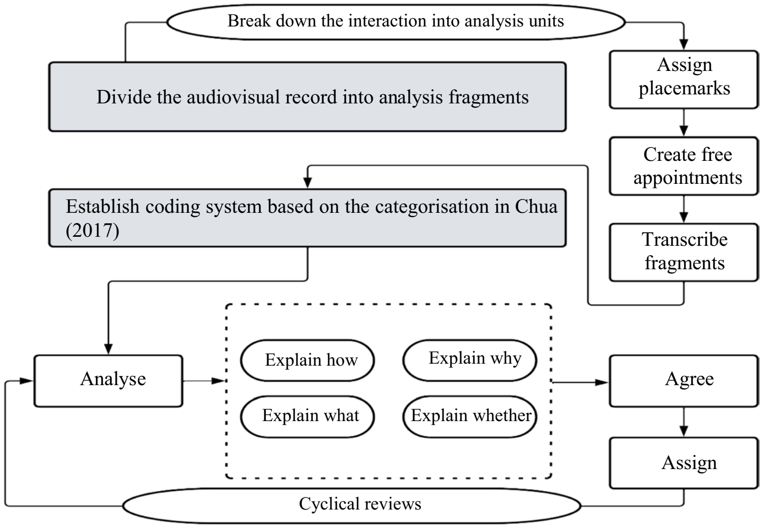


Fig. 7 Data reduction processes and categorisation of the students' justification

4 Results

In keeping with the objectives of our study, we analyse, on the one hand, the understanding of repetition patterns in a context of technological resources, through their representation; and on the other, we describe the type of justification used by children aged 3, 4 and 5 to demonstrate their understanding of how said repetition patterns are taught. Table 1 shows the valid cases that comprise the final analysis sample.

As the table above shows, two participants were lost in the first year and two in the third year, since they did not attend school on the day of the intervention.

4.1 Understanding repetition patterns through their representation

The results of Fig. 8 show an increasing trend of correct representations that are evidenced longitudinally.

In general, we see that 40.9% of 3-year-olds represented without errors the repetition pattern proposed through tasks with Bee-bot; 58.3% of 4-year-olds did the same with the patterns practiced using online games; and 68.2% of 5-year-olds with the sequence executed using Cubettos. We also see that errors decreased by 27.3% between the first year of the intervention and the last.

Table 1 Description of valid cases during the longitudinal intervention

3		4		5	
f	%	*f*	%	*f*	%
22	91.7	24	100	22	91.7

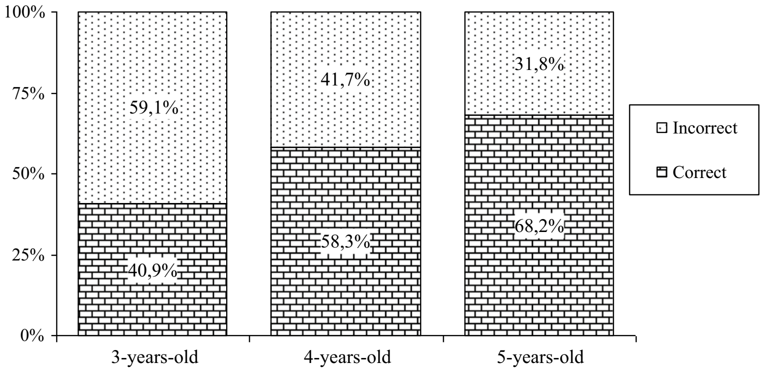


Fig. 8 Categorisation of longitudinal results

Examples of representations categorised as correct are presented in Table 2. Due to space constraints, one example is chosen for each age.

The information presented in Table 2 shows how children represent patterns with different repetition units.

Table 3 shows the correlation between the score provided by the TEMA-3 of Ginsburg and Baroody (2003) and the categorisation of the participants' written productions.

Table 3 shows a direct relationship between the TEMA-3 score and the correct representation of the repetition patterns for 3- and 4-year-olds, yielding a significant P-value of 0.080 and 0.039, respectively. For 5-year-olds, no statistically significant differences were observed, despite a percentage of 93.53% correct versus 81.71% incorrect.

Table 2 Examples of correct representations for 3-, 4- and 5-year-olds

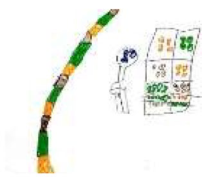
	3	4	5
Correct representations	 Representation of a pattern with an AAB unit, despite not havi ng been introduced directly, but agreed between equals.	 Alphanumeric representation of one of the proposals of the online game that suggested expanding the series: ■■1■■1	 Representation of a patt ern with an ABC unit executed using Cubettos.

Table 3 Correlation TEMA-3 score and written representations in drawing format

Age	3		4		5	
	Correct	Incorrect	Correct	Incorrect	Correct	Incorrect
Categories	13	9	14	10	15	7
Average	88.92	104.56	83.14	71.10	93.53	81.71
Standard deviation	19.02	20.29	17.475	20.404	19.364	18.355
P-value	0.080		0.039		0.191	

P-value calculated using the parametric T-Student statistic for two independent samples with a 95% confidence level

4.2 Type of justification that 3-, 4-, and 5-year-olds use to demonstrate an understanding of repetition patterns in technological contexts

The results obtained from the qualitative analysis carried out through the *Atlas.ti* program are presented below. This analysis breaks down all the recorded sessions into analysis units, focusing on the justification provided by each child over the course of the task. Once the relevant fragment was detected, it was transcribed and coded as per the deductive categorisation in Chua (2017). The coding system adopted presents an increasing level of sophistication depending on the categories proposed by Chua (2017): 1) Elaboration; 2) Validation; 3) Inference; and 4) Prediction/decision-making. Our data are summarised in Table 4.

In general, the data in Table 4 show how the "elaboration" category is present in ages 3, 4 and 5, showing a relevant presence of 63.6% and 50% for ages 3 and 4, respectively. It should be noted that "validation" also predominates longitudinally, being the type of justification most used by 5-year-old children (45.5%). The "inference" does not appear until age 4, occupying a presence of 20.8% and showing a slight increase of 11% at age 5. Finally, "prediction/decision-making" remains absent at ages 3 and 4, and only two 2 children (9.1%) exhibit this type of justification to argue—with evidence tailored to their age—the underlying rule that governs the repetition pattern in the tasks carried out using Cubettos.

Table 4 Longitudinal distribution of frequencies and percentages by the type of justification used by the children

Purpose of the justification	Age 3		Age 4		Age 5	
	f	%	*f*	%	*f*	%
1. Elaboration (Explain how)	14	63.6	12	50	3	13.6
2. Validation (Explain why)	8	36.4	7	29.2	10	45.5
3. Inference (Explain what)	0		5	20.8	7	31.8
4. Prediction/decision-making (Explain whether)	0		0		2	9.1

During the longitudinal qualitative analysis, some recurring questions were observed by the teacher, which drove a specific type of justification. By way of example, some of them are shown in Fig. 9.

As Fig. 9 shows, in general, the questions prompt a more elaborate response in an effort to check the children's level of understanding.

Next, Table 5 establishes a statistical correlation between the categorised results (Chua, 2017) and the TEMA-3 score (Ginsburg & Baroody, 2003).

Table 5 shows that, for 3-year-olds, the "validation" subgroup has a higher TEMA-3 score than the "elaboration" subgroup, with a gap between averages of 29.75 points. In the 4-year-olds, five responses from the children emerge involving "inference" tasks. This subgroup, with respect to the previous types of justification, shows a difference between averages of 45.9 and 22 points in relation to the "elaboration" and "validation" subgroups, respectively. Finally, in 5-year-olds, two responses are categorised in the "prediction/decision-making" subgroup, revealing a gap between averages of 58.3, 41.5 and 25.9 points with the "elaboration", "validation" and "inference" subgroups, respectively. Longitudinally, a statistically significant relationship is described between the TEMA-3 score and the type of justification, with the P-value for 3-year-olds being 0.000.

Examples of the most prominent types of justification for each age are provided in Table 6.

Based on the findings shown in Table 6, we see how the children describe the task they performed and propose evidence to validate their answers.

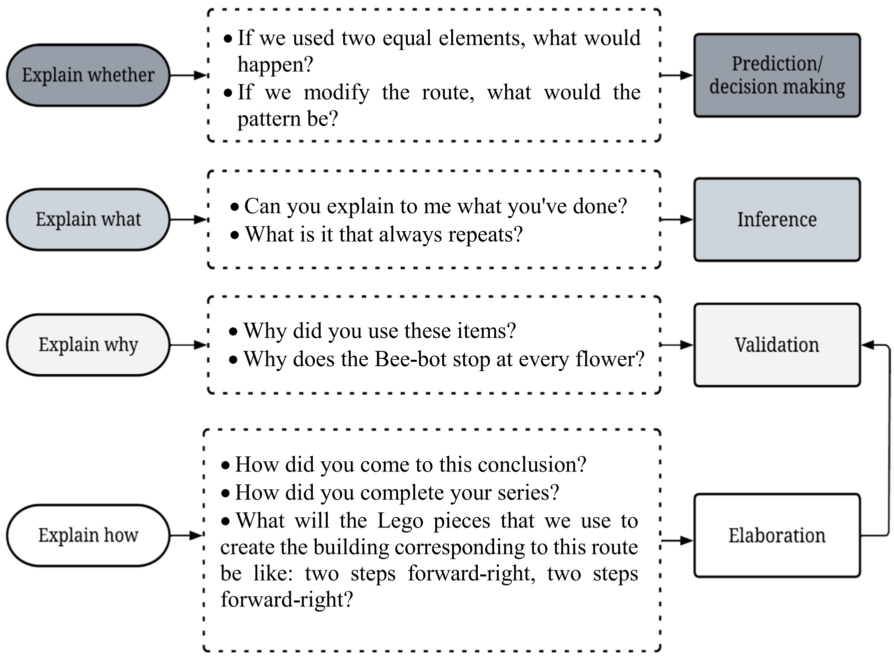


Fig. 9 Examples of questions used by the teacher over the course of the tasks

Table 5 Correlation TEMA-3 score vs justification

Age 3	*N*	Average	Standard deviation	*P*-value
1. Elaboration	14	85.00	13.485	0.000
2. Validation	8	114.75	15.229	
3. Inference				
4. Prediction/decision-making				
Age 4	*N*	Average	Standard deviation	*P*-value
1. Elaboration	12	64.5	8.713	0.000
2. Validation	7	88.4	7.480	
3. Inference	5	110.4	8.142	
4. Prediction/decision-making				
Age 5	*N*	Average	Standard deviation	*P*-value
1. Elaboration	3	66.7	1.155	0.000
2. Validation	10	83.5	10.047	
3. Inference	7	99.1	16.965	
4. Prediction/decision-making	2	125.0	7.071	

P-value calculated using the Bonferroni statistic with a 95% confidence level

Table 6 Examples of the predominant type of justification for 3-, -4 and 5-year-olds

Age	Type of justification	Documentation	Field transcriptions
3	Elaboration		Teacher: How many stops will the bee have to make? Student: 1, 2, and 3. There are 3 flowers. Teacher: Very good, continue with the challenge.
4	Elaboration		Teacher: How did you complete the series? Student: By putting a large triangle and two small ones, a large triangle and two small ones. Teacher: So what did you put between the two large triangles? Student: Two small triangles.
5	Validation	 	Student 1: The Cubetto moves up, turns to one side, then the other, moves up, turns to one side and then the other. Student 2: And the blue chip? Student 1: The blue one repeats everything. Teacher: So, how many colours will we use for the Lego building? Student 1: three colours. Teacher: Why three colours? Student 1: Because it moves forward, turns to one side, and turns to the other. They are three separate things. My building will be yellow-green-blue, yellow-green-blue.

5 Discussion and conclusions

In this study, we analysed the opportunities for understanding provided by technological contexts when teaching repetition patterns to students ages 3, 4 and 5. In keeping with the objectives of the study, we analysed, on the one hand, the understanding of repetition patterns through their representation and, on the other, the type of justification used by children ages 3, 4 and 5 to demonstrate their understanding of said patterns. From this perspective, one of the main contributions of this paper is our initial findings that show a relationship between computational thinking and algebraic thinking in early childhood education. Using technological resources as a teaching context, pattern recognition is adopted as a tangential skill that permeates both types of thinking. In this relational framework between computational thinking and algebraic thinking, pattern recognition provides common ground for the two types of thinking in question. From the context of algebraic thinking, pattern recognition establishes a boundary that provides a consolidated path forward toward functional thinking (Acosta et al., 2022; Lüken & Sauzet, 2020; Pincheira et al., 2022; Warren & Cooper, 2006). The same thing happens from the field of computational thinking, where pattern recognition plays an essential role in problem solving (ISTE, 2016). Thus, based on the assumption that technological resources, like robotics, provide a concrete means for manipulating abstract concepts (Bers, 2008), our results support the contributions of Bråting and Kilhamn (2021) in viewing the tangent between computational thinking and algebraic thinking as occurring at the study of the structure, decomposition and recognition of patterns. As posited in Ye et al. (2023), the integration between computational thinking and mathematical domains makes abstract ideas more accessible to young children, thus promoting learning through understanding.

More specifically, in relation to the understanding of repetition patterns through their representation, we observed an upward longitudinal trend of success by the children, that is, a gradual increase in the number of correct representations. From ages 3 to 5 year, the errors decreased by 27.3%, since, starting from the age of 4, over 50% of the sample understood and correctly represented repetition patterns. It is thus evident that technological contexts can positively influence the understanding of repetition patterns by allowing the abstract structure of a series to be manipulated in a concrete manner. In this sense, we acknowledge the finding in Lüken and Sauzet (2020) when they explain that to learn mathematics is to develop the ability to recognise patterns, interpret structures and establish relationships. As specified by NCTM (2000), in Pre-K-2 (3 to 8 years old), the goal is to "recognise, describe and extend patterns, such as sequences of sounds and shapes or simple numerical patterns, and translate from one representation to another [and] analyse how repetitive and increasing patterns are generated (NCTM, 2000, p.90)". This is how we begin to recognise that the teaching of patterns in K-2 (5 to 8 years old) is related to sequences, functions and relationships (Carraher & Schliemann, 2019). Later, in grades 3–5 (8–12 years old), students are expected to be able to use multiple representations (words, tables, graphs) to describe, extend, and make generalisations about geometric and numerical patterns.

Authors such as Carpenter et al. (2005) regard students' intuitive knowledge of patterns as a foundation that helps in the transition to early algebraic thinking. Engaging children in algebraic thinking early involves designing tasks and learning opportunities that promote abstraction and generalisation while fostering the ability to think structurally (Stephens et al., 2015). Precisely for Kieran (2004), algebraic thinking implies "(…) analysing relationships between quantities, noticing the structure, studying change, generalising, problem solving, modelling, justification, testing and prediction" (p.149). The aim is thus to take advantage of the opportunity provided by technological contexts when teaching repetition patterns with early childhood education students (3, 4 and 5 years old) to build a solid foundation of learning and concrete processes that aid in the acquisition and processing of more sophisticated knowledge in later stages of teaching–learning. In this regard, the study conducted has shown that children, with proper support from a teacher, begin to perceive a series not as the alternation of elements, but as a structure governed by a rule of repetition.

When students, through Bee-bots, online games or Cubettos, engage in actions that follow a recurring sequence, it provides them an introduction into the abstraction of the underlying rule of a repetition pattern. This fact supports the promotion of one of the four computational thinking skills described by ISTE (2016): pattern recognition. This ability encourages problem solving, since, as stated by Lee et al. (2023),

> without pattern recognition, each problem is new and novel so that problem solving or information processing does not become more systematic and efficient. Further, without pattern recognition, sorting by characteristics or by similarities and differences is especially challenging. Characteristics — those features or qualities belong to a group, thing, or person — are used to make sense of much of the world and to organise our thinking (p. 459).

However, it is necessary to bear in mind that recognition of the repetition structure does not manifest itself at the age of 3 (Acosta & Alsina, 2020; Acosta et al., 2022). Rittle-Johnson et al. (2015) confirmed that this is only the case from the age of four or five, and requires the use of instructional explanations by the teacher.

This study has also provided results that coincide with the findings of authors such as Kidd et al. (2013) and Warren and Miller (2013), who state that there is a clear relationship between the ability to perform tasks with patterns and an individual's mathematical ability. From this perspective, we found a statistically significant correlation between the TEMA-3 score of students aged 3 and 4 and the successful performance of the representation task. One notable result of our study is the finding that the level of sophistication of the mathematical justification was also directly related to the TEMA-3 score, yielding a statistically significant *P*-value for 3-, 4- and 5-year-olds. Our interpretation, then, is that the higher the TEMA-3 score, the more sophisticated the justification.

In relation to the type of justification used by children ages 3, 4 and 5 to demonstrate their understanding of repetition patterns, we were able to show the importance of having the teacher encourage, through questions, reflections typical of computational thinking; that is, explicit arguments focused on pattern recognition (ISTE, 2016) in order to provide a response to a challenge or task articulated in a technological context. Taking into consideration the contributions of NCTM (2014), one of the traits of effective mathematics teaching practices relies on formulating questions to children to elicit explanations of their own reasoning processes, and thus be able to construct a mathematically meaningful discourse. Accordingly, we relied on the use of open questions that seek to promote and mobilise increasingly sophisticated justifications focused on understanding the structure as a replicable rule, thus avoiding teacher questions that are answered with a "yes" or a "no". These questions are intended to promote active participation and externalise the level of understanding of children.

As we have seen, based on the implementation of pattern repetition tasks with technological resources, at the age of 3, more than half of the participants in our study use "elaboration" as a type of justification, using a descriptive type of discourse; that is, they only explain the successive elements of their representation. At the age of 4, "elaboration" continues to be the predominant type of justification in 50% of the sample. However, some students start using mathematical discourse with keywords from the task or challenge presented, with the "inference" subgroup thus accounting for 20.8%. Finally, by the age of 5, the majority type of justification is in the "validation" subgroup at 45.5%, with the "prediction/decision-making" subgroup emerging with 9.1%. It should be noted that the two students who were in this type of justification group scored higher than average on the TEMA-3 by Ginsburg and Baroody (2003). Longitudinally, then, we see diversification in the type of justification, which allows the students to advance toward learning based on comprehension and mathematical literacy. From this perspective, the knowledge shared between equals provides an opportunity to favour the construction of an increasingly sophisticated and significant mathematical discourse, both among those who are involved in the justification, and the audience that is inspired by said justification (Staples et al., 2012).

The limitations of our study are as follows: first, the sample size, which does not allow the results to be generalisable; however, we share the idea that educational phenomena are sensitive to the context, therefore, our purpose is for these real references guide future action through reflection (Radford & Sabena, 2015) and for our conclusions to be a source of inspiration, without claiming to be directly generalisable to other realities; and second, the lack of digital skills in some children may have influenced the performance of the challenge, as may the loss of two participants in the first and last year of implementation. As future lines of research, we propose continuing to explore the relationship that is established between computational thinking and algebraic thinking as a part of mathematical thinking, since as English (2018) argues, these two types of thinking are naturally related. This is why the Organisation for Economic Co-operation and Development [OECD] (2018) includes computational thinking in the PISA test on mathematical competence. In order to further reinforce these links, various authors agree that greater attention is required to ensure a fruitful connection (Bråting & Kilhamn, 2021; Lv et al., 2023; Ye et al., 2023). In the future it will be necessary to extend the study to larger samples and build upon the results obtained in this paper to consolidate the data obtained around the role of technological resources in the variables analysed; likewise, another essential line will be to study the links between computational thinking and algebraic thinking through repetition pattern tasks in other teaching resources that are widely employed in early childhood education, such as real contexts or the use of manipulatives (e.g., Alsina, 2022; Clements & Sarama, 2020). Finally, another line of future research might involve investigating the difficulties and challenges faced by teachers to employ teaching practices that promote links between computational thinking and algebraic thinking.

In summary, we believe it necessary to emphasise: a) the meticulous design of learning sequences connected with theory and supported by practice in order to establish links between the development of computational thinking and the beginnings of algebraic thinking (Acosta et al, 2022; Pincheira et al, 2022); b) the role of the teacher as a guide who accompanies and prompts students to recognise, transfer and represent patterns in technological contexts; c) the use of questions that promote pattern recognition (ISTE, 2016) and prompt children to engage in a significant mathematical discourse.

Our twenty-first century society demands citizens with critical-thinking and decision-making skills to provide creative solutions to contemporary problems. This may be one of the greatest reasons why computational thinking, like algebraic thinking, is starting to make its way into today's schools (Kilhamn et al., 2022). Zhong and Xia (2020) note that young children need opportunities to explore, engage and experiment with interactive media in order to promote learning through enjoyment and a concrete perspective. Although there is still a long way to go, this study shows teachers how technological resources can also contribute to a tangible understanding of abstract mathematical concepts and pattern recognition in order to promote two types of thinking—computational and algebraic—that can coexist in a connected way.

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Data availability Data generated or analysed during this study are available from the authors on request.

Declarations

Ethical considerations Before starting the field work, and in an effort to provide transparency, and based on ethical principles, the families were informed of the following aspects: purpose of the research; goals and timeline; procedure; and the need to record audio and video of the sessions. As a result, all the families involved in the study provided their informed consent and approval. In the same way, the children's desire to participate or not in the activities and to be recorded and photographed was respected throughout the intervention.

Competing interest Authors declare no competing interest.

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