



A naïve HMO study of the casimir effect

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Received: 12 November 2021 / Accepted: 16 December 2021 / Published online: 31 January 2022
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Abstract

The Casimir effect is discussed via an HMO treatment. At this schematic theoretical level, the Casimir effect might be considered as the result of the general quantum mechanical interaction behavior of two sets of particles.

Keywords Casimir effect · HMO · Quantum mechanical interaction

1 Introduction

In the present author's first paper [1] a model to naively study chemisorption was proposed, which, with the appropriate modifications, happens that can be applied to study, in a naïve way too, the Casimir effect [2].

Casimir effect is still nowadays explained via a more physical way using the nature of vacuum fluctuations, see for example reference [3], or via van der Waals forces [4]. There is a large amount of literature on the subject, see for a very small set of references [5–7], for general reviews on the subject nature and applications [8, 9], and finally, a contemporary comprehensive discussion [10] shall be recommended. The present work is having to be considered nearer to the second option than to the first.

2 Problem set up

Suppose two equal sets of N atoms are separated by an arbitrary distance.

Each atomic set might be associated with some HMO ($N \times N$) matrix \mathbb{M}_o .

The structure of such matrix is irrelevant, except knowing that there exists an orthogonal matrix \mathbb{U} of the same dimension fulfilling:

$$\mathbb{U}^T \mathbb{U} = \mathbb{U} \mathbb{U}^T = \mathbb{I},$$

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and such that diagonalizes M_o :

$$U^T M_o U = D_o$$

with:

$$D_o = \text{diag}(d_{oI} | I = 1, N)$$

holding the eigenvalues of M_o , its spectrum.

One can also suppose that the pair of non-interacting atomic sets correspond to an essentially doubly degenerate spectrum.

3 Casimir interaction

Casimir-like interaction between the two atomic sets can be associated with an HMO matrix ($2N \times 2N$), which can be partitioned preserving the Hückel topological matrix style as:

$$A_o = \begin{pmatrix} M_o & \sigma \mathbb{1} \\ \sigma \mathbb{1} & M_o \end{pmatrix}$$

where σ is a scalar and $\mathbb{1}$ a unit matrix of the appropriate dimension ($N \times N$). The scalar σ , which can be called the Casimir resonance integral, corresponds to a factor that makes the usual HMO resonance integral β smaller, such that to explicitly consider a large distance separating both atomic sets in a given Casimir experiment; that is: $\sigma \ll \beta$.

4 Transformation of the casimir interaction matrix

Now one can scale the HMO composite matrix A_o to obtain a new equivalent matrix:

$$A = \sigma^{-1} A_o = \begin{pmatrix} \sigma^{-1} M_o & \mathbb{1} \\ \mathbb{1} & \sigma^{-1} M_o \end{pmatrix} = \begin{pmatrix} M & \mathbb{1} \\ \mathbb{1} & M \end{pmatrix}.$$

And one can define a composite orthogonal hyperdiagonal matrix:

$$V = \begin{pmatrix} U & \mathbb{0} \\ \mathbb{0} & U \end{pmatrix} \Rightarrow V^T V = V V^T = \begin{pmatrix} \mathbb{1} & \mathbb{0} \\ \mathbb{0} & \mathbb{1} \end{pmatrix},$$

where $\mathbb{0}$ is the zero matrix of the appropriate dimension. With such a definition one can easily see that a new transformed matrix Z can be computed:

$$Z = V^T A V = \begin{pmatrix} D & \mathbb{1} \\ \mathbb{1} & D \end{pmatrix}$$

where D is a diagonal matrix containing the eigenvalue spectrum of the matrix A ; in fact, it is trivial to consider:

$$D = \sigma^{-1} D_o.$$

The matrix Z is some kind of symmetric pseudotridiagonal matrix, where the subdiagonals are at the positions of the upper triangle:

$$\forall I = 1, N : (I, I + N)$$

and the associated symmetrical lower triangle is constructed accordingly.

5 Eigensystem spectrum of the casimir interaction

The Casimir interaction pseudotridiagonal matrix Z can be easily transformed into a diagonal matrix by the orthogonal transformation matrix:

$$E = \frac{1}{\sqrt{2}} \begin{pmatrix} \mathbb{1} & -\mathbb{1} \\ \mathbb{1} & \mathbb{1} \end{pmatrix},$$

considering that the matrix E is orthogonal because:

$$E^T E = \frac{1}{2} \begin{pmatrix} \mathbb{1} & \mathbb{1} \\ -\mathbb{1} & \mathbb{1} \end{pmatrix} \begin{pmatrix} \mathbb{1} & -\mathbb{1} \\ \mathbb{1} & \mathbb{1} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 2\mathbb{1} & 0 \\ 0 & 2\mathbb{1} \end{pmatrix} = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix} = E E^T$$

such that acting over Z , yields an hyperdiagonal matrix:

$$T = E^T Z E = \begin{pmatrix} D + \mathbb{1} & 0 \\ 0 & D - \mathbb{1} \end{pmatrix}$$

The meaning of this result indicates that from the initial non-interacting double degenerate spectrum of the two sets of atomic structures, one can obtain a total Casimir interaction spectrum, which is split into two spectral sets, with a gap of 2 between the respective eigenvalues:

$$T = \text{diag}(t_{(+I)} = d_I + 1; t_{(-I)} = d_I - 1 | I = 1, N)$$

6 Possible fluctuation of the casimir interaction energy

The total energy of the Casimir HMO interaction will reflect the eigenvalue gap discussed in the previous section. This is so because the non-interacting energy associated with the two sets can be expressed as:

$$E_{oT} = 2E_o,$$

while the interacting Casimir atomic sets will present total energy in the ground state, expressible with the equation:

$$E_T^{(+)} = E_{oT} + 2N\sigma$$

where the term $\lambda = 2N\sigma$ might be taken as a parameter, considering the number of doubly occupied monoelectronic levels in units of the Casimir resonance integral.

However, one shall also take into consideration the possibility that another state can be attained by the Casimir system, that is:

$$E_T^{(-)} = E_{oT} - 2N\sigma$$

It seems plausible that, due to the small nature of the Casimir interaction parameter σ , the correction $\pm\lambda$ becomes also sufficiently small, to allow some fluctuation between the two states $\{E_T^{(+)}, E_T^{(-)}\}$.

7 Conclusion

It has been shown that the schematic HMO theory can provide a purely theoretical reason for a plausible explanation of the Casimir effect existence.

According to the present discussion and results, which one must be aware of are grounded on a quantum mechanical point of view, two twin sets of atoms, independently of their cardinality, even if separated at large distances, suffer an electronic energy splitting, similar to the one, which is well described in LCAO MO theory, when two hydrogen atoms interact.

Such behavior that can be associated with varied systems, from metallic plates up to swarms of particles, perhaps can be somehow connected with the Einstein-Podolski-Rosen paradox [11], via the present author works [12, 13].

Finally, a recent experimental discussion [14] might be also linked with the present simple theoretical result.

Funding Open Access funding provided thanks to the CRUE-CSIC agreement with Springer Nature.

Declarations

Conflict of interest The author declare that they have no conflict of interest.

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