ON GOVERNMENT-CREATED CREDIT MARKETS FOR EDUCATION AND ENDOGENOUS GROWTH

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Abstract

Interest in public loans to fund (higher) education has been increasing in the last decades. This paper explores the general welfare properties of government-created credit markets for education in a three-period overlapping generations model with physical and human capital. It shows that the mere existence of public credit markets is second-best in nature, and cannot decentralize the optimum. Achieving the first-best "Golden Rule" balanced growth path requires a government loan system that lends the amounts required for optimal investments in education and an optimally chosen pure pay-as-you-go social security system. Student loans and pensions thus appear as two inseparable elements of the policy that maximizes social welfare.

Keywords: endogenous growth, human capital, intergenerational transfers, public policy, credit markets for education

JEL Classification: D90, H21, H52, H55

1 Introduction

It is generally acknowledged that the impossibility to use human capital as collateral for education investments prevents the existence of complete credit markets to fund these investments. Around the world, there is an increasing interest in funding (higher) education through government student loans. Among the countries that already use them feature UK, Australia, New Zealand, USA, Canada, Colombia, Brazil, Norway, Sweden, the Netherlands, and Japan. Even in Germany, where people have been traditionally reluctant to this idea, a recent contribution shows that deferring higher education tuition fees (i.e. letting students pay later or implicitly lending them to pay tuition) creates a strong majority in favor of tuition versus free education for the young (Lergetporer and Woessmann, 2019). In the debate over whether funding for deferred payments should be private or public, having a comprehensive income tax payment administration in place is one of the arguments often put forward in favor of government versus private loan programs (see, e.g., Chapman, 2016). The government's "power to tax" allows not only to provide funding but also to optimally design the program.

The aim of this paper is to discuss the welfare effects of a government-created credit market for education in an overlapping generations framework where three generations coexist. In this set-up, it is only natural to search for an institution that, by transferring resources from the middle-aged and/or the old-aged to the young for them to invest in education, might increase welfare. In a seminal contribution, Boldrin and Montes (2005) propose a policy scheme that restores the laissez-faire complete market allocation by requiring middle-aged agents to subsidize the education of the young and receive in return a pension when they are old. Andolfatto and Gervais (2006), and Wang (2014) explore the conditions under which these results carry on to the case where borrowing constraints are endogenous.[1]

However, it is well known that, in overlapping generation economies with life-cycle saving, the decentralized complete market allocation will generally fail to achieve optimality. Acknowledging the limits to the normative appeal of the laissez-faire complete market allocation, an alternative approach is to search for public policies that maximize some definition of social welfare. Docquier et al. (2007) and Del Rey and Lopez-Garcia (2013) characterize optimal education subsidies and intergenerational transfers under alternative notions of social welfare, but they assume nonetheless the perfect functioning of credit markets for education. While the former aim to maximize a discounted sum of present and future utilities, the latter focus on the optimal balanced growth path that maximizes the lifetime welfare of a representative generation subject to the constraint that everyone else's welfare is fixed at the same level. The optimal policies in both settings, obviously, differ. Docquier et al. (2007) characterize an optimal education subsidy, and while they find that transfers to the elderly cannot be signed in general, they argue that for realistic private and social discount rates, "the case for generous public pensions is rather weak" (p. 363). In contrast, Del Rey and Lopez-Garcia (2013) show that the optimal policy is characterized by positive pensions, but also by taxes on education investments. It is worth emphasizing that the reason why two instruments are required in either case to decentralize the optimum, is that there are two individual decisions (savings and education investments) that may fail to be optimal.

The assumption that individuals have access to perfect credit markets for education is, admittedly, an extreme one. However, it can be acceptable when the purpose is to emphasize that, even in a framework where the usual sources of inefficiency (i.e., public goods, imperfect competition, missing markets, etc.) are assumed away, a mix of public policies related to education decisions

[1]Other works have justified the connection between education finance and pensions as the outcome of a voting equilibrium (e.g. Konrad, 1995; Kemnitz, 2000; Poutvaara, 2006; Kaganovich and Zilcha, 2012).

and intergenerational transfers can reach optimality.[2] But, be it as it may, the absence in the real world of a credit market to finance education investment emerges as an obvious reason for the price system to fail to attain any normative criterion. Hence the reason for discussing the role that can be played by the public sector in such an environment.

In this paper, we thus remove the complete credit market assumption and explore the welfare properties of a government-created credit market in which any difference between the issue of new credits by younger individuals and the repayment of previous ones by the middle-aged is covered by lump-sum taxes on both the middle-aged and the old-aged. These intergenerational transfers play a double role. On the one hand, they allow to balance the budget whenever the amount of funding demanded by the young is larger or smaller than the amounts repaid by the middle aged for the loans issued the previous period. On the other hand, they help steer savings and thus potentially increase welfare. Notice that assuming lump-sum taxes allows to capture the income effects (but not, of course, the substitution effects) generated by taxes levied on income, consumption and wealth in the real world.

We adopt the three-period lived overlapping generations model used in Boldrin and Montes (2005). The young borrow to finance their education, the middle-aged work, consume, save, return their loans with interest and pay taxes/receive subsidies. Finally, the old-aged consume and pay taxes/receive subsidies. The focus is on the effects along a balanced growth path, and the social welfare criterion against which we evaluate the government-created credit market is the "Golden Rule" characterized in Del Rey and Lopez-Garcia (2013).

It is shown, first, that the mere existence of a government-created credit market is, in itself,

[2] And, needless to say, even if public policy fails to achieve optimality, it can entail welfare improvements. Del Rey and Lopez-Garcia (2016) analyze the comparative dynamics in terms of physical and human capital accumulation on the one hand, and on welfare on the other, associated with changes in the tax parameter addressed to education decisions and the intergenerational distribution of taxes.

second-best in nature, and cannot decentralize the optimum. From the point of view of the design of public policy, this is not surprising. As pointed out above, two policy instruments are required to steer individual decisions and decentralize the first-best. When the government simply provides the funds demanded by the individuals (and later collects repayments), it does not provide them with the precise (marginal) incentives to choose the optimal amount of education. In fact, there is not a single government-sponsored credit market, but infinitely many, each of them associated with a precise intergenerational distribution of taxes. In this context, an increase in the lump-sum tax paid by the old-aged (coupled with the required adjustment in the tax paid by the middle-aged) can either increase or decrease welfare. Importantly, this result holds regardless of whether the marginal product of physical capital is greater or less than the economy's growth rate, and is in sharp contrast with its counterpart in exogenous growth models. Even if welfare reaches a (local) maximum, this cannot possibly be the Golden Rule optimum.

Second, achieving the optimal Golden Rule balanced growth path by means of a government-created credit market requires adding an instrument specifically addressed to achieve the appropriate amount of education. When individuals borrow to fund their education investments, this instrument could be a subsidy on the repayment of the loans, or an amount to be lent. In this paper, we have chosen for the government to determine the amounts to be lent.[3] As shown below, taken together, the government's lending of the amounts needed to attain the optimal investment in education *and* an optimally chosen pure pay-as-you-go social security system allow to decentralize the *first best.* At this Golden Rule balanced growth path, individuals allocate their life-cycle savings obtaining the same return by investing in the market for physical capital or through the pension system. This no-arbitrage condition guarantees that the "intergenerational state" maximizes social welfare.

[3] In reality, there are both interest subsidies in public student loan programs and limits to the amounts to be borrowed, but the former are under scrutiny (Barr et al., 2017)

The structure of the paper is as follows. Section 2 presents the model. Section 3 describes the decentralized equilibrium in the presence of government-created credit markets for education, and characterizes the balanced growth paths, providing expressions for the ratios of physical and human capital per unit of efficient labour. It also discusses the indirect utility function that provides the basis for welfare analysis. Section 4 works out the comparative dynamics in terms of physical and human capital accumulation and in terms of welfare. Section 5 focuses on the configuration of public policy required in order to achieve the optimal Golden Rule balanced growth path. Section 6 concludes with some final comments.

2 The Model

Consider the overlapping generations model discussed in Boldrin and Montes (2005). At period t L_{t+1} individuals are born, and they coexist with L_t middle-aged and L_{t-1} old-aged. Population grows at an exogenous rate n ($n > -1$), so that $L_t = (1+n)L_{t-1}$. The endogenous growth nature of the model stems from the fact that individuals born at period $t-1$ inherit the level of human capital (i.e., units of efficient labour per unit of natural labour) of their parents, h_{t-1}. Human capital in period t will result from the interaction of the output invested in education in period $t-1$, d_{t-1}, and the inherited human capital h_{t-1} according to the production function $h_t = E(d_{t-1}, h_{t-1})$. With constant returns to scale, this can be written as $h_t/h_{t-1} = e(\tilde{d}_{t-1})$, where $e(.)$ satisfies the Inada conditions and $\tilde{d}_{t-1} = d_{t-1}/h_{t-1}$ is the amount of output devoted to education per unit of human capital. It then follows that $h_t/h_{t-1} = e(\tilde{d}_{t-1}) = 1 + g_t$, where g_t is the growth rate of productivity.

There is a single good, Y_t, produced with physical capital, K_t, and human capital, H_t, according to a constant returns to scale production function, $Y_t = F(K_t, H_t)$. The middle-aged are the only ones who work, and inelastically supply one unit of natural labour, so that $H_t = h_t L_t$. Physical capital fully depreciates each period. Letting $k_t = K_t/L_t$ and $\tilde{k}_t = K_t/H_t = k_t/h_t$, the technology can be described as $Y_t/H_t = f(\tilde{k}_t)$, where $\tilde{k}_t$ is the ratio of physical capital per unit of efficient labour, and $f(.)$ also satisfies the Inada conditions.

Under perfect competition factor prices will determined by their marginal products, so that, if $1+r_t$ and w_t are, respectively, the interest factor and the wage rate per unit of efficient labour,

$$1 + r_t = f'(\tilde{k}_t) \equiv R(\tilde{k}_t), \tag{1}$$

$$w_t = f(\tilde{k}_t) - \tilde{k}_t f'(\tilde{k}_t) \equiv w(\tilde{k}_t). \tag{2}$$

As in Boldrin and Montes (2005) we assume that there are no private credit markets for education. However, young individuals are allowed to borrow from the public sector the amount they decide in period $t-1$, d_{t-1}, to finance their investment in education. In period t, when they are middle-aged, they have to repay principal plus interest, $(1 + r_t)d_{t-1}$. Also in this period, each member of the new younger generation borrows d_t from the government to finance her education. Surely, the government could borrow each period to lend the young and repay the loans when the middle aged repay their debt. This would very simply replicate the laissez-faire equilibrium with private credit markets without requiring any additional tax. We know, however, that no normative appeal can be attributed to the the laissez-faire in overlapping generations models with physical and human capital (Docquier et al. (2007), Del Rey and Lopez-Garcia (2013)). For this reason, and taking advantage of the government power to tax, we assume that, each period, the government gives out loans to the young and receives loan repayments of the middle-aged. This is also in accordance with what governments do in the real world when providing student loans. Any difference between the repayment of previous credits, $(1 + r_t)d_{t-1}L_t$, and the issue of new ones, d_tL_{t+1}, is covered by lump-sum taxes or transfers. For the sake of generality, we allow taxes/transfers both on the middle-aged, z_t^m, and the elderly, z_t^o. Thus, the government budget constraint in period t is:

$$z_t^m L_t + z_t^o L_{t-1} + (1 + r_t)d_{t-1}L_t = d_t L_{t+1}, \tag{3}$$

which can be rewritten, denoting $\tilde{z}_t^m = z_t^m/h_t$ and $\tilde{z}_t^o = z_t^o/h_{t-1}$, as:

$$\tilde{z}_t^m + \frac{\tilde{z}_t^o}{(1+g_t)(1+n)} + \frac{(1+r_t)}{(1+g_t)}\tilde{d}_{t-1} = (1+n)\tilde{d}_t. \tag{4}$$

This government budget constraint requires some justification. An alternative institutional setting would have two separate programs, one addressed to finance education investment and another one used to achieve the appropriate intergenerational redistribution (i.e. savings). In our view, however, there is no normative reason to require both programs to be independently balanced.[4] Modelling public accounts with a single budget constraint is tantamount to assuming an implicit "inter-program lump-sum transfer" that is optimally chosen. Moreover, these inter-program transfers are commonplace in the real world.[5] As will be made clear below, however, along the *optimal balanced growth path* the "credit system side" of the government budget will be balanced, and the "intergenerational tax-transfer side" of the government budget will, in fact, be a pure pay-as-you-go social security system.

Savings are the consequence of a pure life-cycle motive. Individuals consume in their second and third period only. The lifetime utility function of an individual born at period $t-1$ is $U_t = U(c_t^m, c_{t+1}^o)$, where c_t^m and c_{t+1}^o denote her consumption levels as middle-aged and old-aged in periods t and $t+1$ respectively. This utility function is assumed to be strictly quasi-concave and homogeneous of degree $j > 0$. Individuals born at $t-1$ borrow from the government the amount d_{t-1} that maximizes their lifetime resources. In their middle age they work and obtain labour

[4]Even more, from a normative point of view, having two separate and financially balanced programs is, in general, a worse second-best institutional design than integrating both of them into a single government budget constraint. For in the presence of two budget constraints, one for each program, the "social marginal cost of public funds" (i.e., the effect on social welfare as a consequence of marginally relaxing the program's budget constraints) will in general be different across programs.

[5]Many of the programs mentioned above are subsidised. Some allow for interest subsidies (e.g. BAfšG loans in Germany), others require repayment only when the debtor's income attains a certain level (e.g. income contingent loans in Australia, England and New Zealand). This means that the programs generate losses that are covered by budgetary contributions (taxpayer money). Were these schemes to generate surpluses, it would only be natural that the surplus would contribute to the general budget.

income $w_t h_t$, pay back the loan $(1 + r_t)d_{t-1}$, pay taxes z_t^m, consume c_t^m, and save s_t. In their old-age, individuals only consume c_{t+1}^o and pay taxes z_{t+1}^o. Thus,

$$c_t^m = w_t h_t - (1 + r_t)d_{t-1} - z_t^m - s_t, \tag{5}$$

$$c_{t+1}^o = (1 + r_{t+1})s_t - z_{t+1}^o. \tag{6}$$

The first-order conditions associated with decision variables d_{t-1}, c_t^m and c_{t+1}^o are:[6]

$$w_t e'(d_{t-1}/h_{t-1}) = 1 + r_t, \tag{7}$$

$$\frac{\partial U(c_t^m, c_{t+1}^o)/\partial c_t^m}{\partial U(c_t^m, c_{t+1}^o)/\partial c_{t+1}^o} = 1 + r_{t+1}. \tag{8}$$

Eq. (7) shows that an individual will invest in education up to the point where the marginal benefit in terms of second period income equals the marginal cost of so doing. It can be interpreted as an arbitrage condition between the return from investing in education and the return from investing in physical capital. Rewriting (7) as $e'(\tilde{d}_{t-1}) = R(\tilde{k}_t)/w(\tilde{k}_t)$, this expression implicitly characterizes the ratio $\tilde{d}_{t-1}$ as a function of $\tilde{k}_t$, i.e., $\tilde{d}_{t-1} = \phi(\tilde{k}_t)$. Since $e'' < 0$, it can readily be shown that $\phi'(\tilde{k}_t) > 0$.

3 Equilibrium with Government Created Credit Markets

We now characterize the decentralized equilibrium with government-created credit markets and lump sum taxes, as well as the ensuing balanced growth paths. Since private credit markets for education do not exist, equilibrium in the market for physical capital will be achieved when the stock available in $t+1$ equals gross savings set aside by the middle-aged in t, i.e., when $K_{t+1} = s_t L_t$.

[6] Notice that these are the same conditions prevailing at the decentralized equilibrium with perfect private credit markets. However, neither the individual lifetime budget constraint nor the government budget constraint are the same. Section 3 characterizes the physical capital equilibrium condition, which also differs when credit markets are government created and not private.

As preferences are homogeneous, the c_{t+1}^o/c_t^m ratio is a function of the interest factor R_{t+1} only. This allows to write $c_t^m = \pi(R_{t+1})\omega_t$, where the function $\pi(.)$ depends on the interest factor and $\omega_t = w_t h_t - R_t d_{t-1} - z_t^m - z_{t+1}^o/R_{t+1}$ is the present value of lifetime income. The equilibrium condition then becomes:

$$(1+n)k_{t+1} = w_t h_t - (1+n)d_t + \frac{z_t^o}{1+n} - \pi(R_{t+1})\left[w_t h_t - (1+n)d_t + \frac{z_t^o}{1+n} - \frac{z_{t+1}^o}{R_{t+1}}\right] \tag{9}$$

where use has been made of (3).[7] The behaviour of the model revolves around that of the $\tilde{k}_t$ ratio. Allowing for the fact that $1 + g_{t+1} = e(\tilde{d}_t)$, (9) translates into:

$$(1+n)e(\tilde{d}_t)\tilde{k}_{t+1} = \Big(1 - \pi(R(\tilde{k}_{t+1}))\Big)\left[w(\tilde{k}_t) - (1+n)\tilde{d}_t + \frac{\tilde{z}_t^o}{(1+n)e(\tilde{d}_{t-1})}\right] + \frac{\pi(R(\tilde{k}_{t+1}))\tilde{z}_{t+1}^o}{R(\tilde{k}_{t+1})} \tag{10}$$

Taking now into account that $\tilde{d}_t = \phi(\tilde{k}_{t+1})$ from (7), this expression implicitly provides $\tilde{k}_{t+1}$ as a function of $\tilde{k}_t$, $\tilde{z}_t^o$ and $\tilde{z}_{t+1}^o$, thus characterizing a first-order non-linear difference equation $\tilde{k}_{t+1} = \Psi(\tilde{k}_t, \tilde{z}_t^o, \tilde{z}_{t+1}^o)$. It is important to emphasize that since investments in education $\tilde{d}_{t-1}$ and $\tilde{d}_t$ are chosen by individuals on a decentralized basis, the government budget constraint (4) will require lump-sum taxes/transfers $(\tilde{z}_t^m, \tilde{z}_t^o)$ in period t and $(\tilde{z}_{t+1}^m, \tilde{z}_{t+1}^o)$ in period $t+1$. Since the next section discusses the balanced growth paths, we can focus on those situations where $\tilde{z}_t^o = \tilde{z}_{t+1}^o = \tilde{z}^o$. The above-mentioned difference equation thus reduces to $\tilde{k}_{t+1} = \Psi(\tilde{k}_t, \tilde{z}^o)$, and any change in $\tilde{z}^o$ will require an adjustment via $\tilde{z}^m$ in the government budget constraint.

A balanced growth path will be a situation where all variables expressed per unit of efficient labour remain constant over time. Thus, the time subscripts can be deleted in the difference equation $\tilde{k}_{t+1} = \Psi(\tilde{k}_t, \tilde{z}^o)$, to ultimately find $\tilde{k} = \Psi(\tilde{k}, \tilde{z}^o)$. An equilibrium ratio $\tilde{k}$ will then be a fixed point of the $\Psi(\tilde{k}, \tilde{z}^o)$ function, and it will be locally stable when $0 < \partial\Psi(\tilde{k}, \tilde{z}^o)/\tilde{k} < 1$ in a neigbourhood

[7] With (9), and making use of (5)-(6) and (3), one can easily obtain the aggregate feasibility constraint, i.e., the equality of total consumption, $c_t^m L_t + c_t^o L_{t-1}$, and the amount of output not devoted to investment in either physical or human capital, $F(K_t, h_t L_t) - (1+n)k_{t+1}L_t - (1+n)d_t L_t$.

of this equilibrium. In what follows, in order to characterize comparative dynamics, we will focus on situations where the equilibrium is unique and (locally) stable.[8] As for the amount of output devoted to education per unit of inherited human capital, we can use (7) to obtain $\tilde{d} = \phi(\tilde{k}(\tilde{z}^o))$, so that the growth rate of productivity, g, will verify $1 + g = e\left[\phi(\tilde{k}(\tilde{z}^o))\right]$. With obvious notation, we can summarize the above discussion with the functions

$$\tilde{k} = \tilde{k}(\tilde{z}^o), \tag{11}$$

$$\tilde{d} = \tilde{d}(\tilde{z}^o). \tag{12}$$

It is well known that in life-cycle models with exogenous growth à la Diamond (1965) the interest factor along a balanced-growth path can be greater or less than (and exceptionally equal to) the growth factor. This property also holds in the present endogenous growth setting both in the laissez-faire (Boldrin and Montes, 2005) and in the presence of government (see Figure 1 below).

To discuss the welfare implications of public policy, we follow the approach in Del Rey and Lopez-Garcia (2013). This entails adopting the balanced-growth-path version of the individual utility function that results from re-scaling consumptions in each period to express them in terms of output per unit of *efficient* labour, i.e., $\tilde{c}_t^m = c_t^m/h_t$ and $\tilde{c}_{t+1}^o = c_{t+1}^o/h_t$. With c_t^m, c_{t+1}^o and h_t growing at the same rate g, the ratios $\tilde{c}_t^m$ and $\tilde{c}_{t+1}^o$ will be constant and well defined, giving rise to

[8] In overlapping-generations models with exogenous growth it is difficult to ensure existence, uniqueness, and (local) stability of balanced growth paths. In his seminal contribution, Diamond (1965) only provides a proof of these issues in the laissez-faire equilibrium for the double Cobb-Douglas case. De la Croix and Michel (2002) reflect these inherent difficulties, which of course apply to (and are enhanced in) more complicated models like the current one. It can easily be shown that with logarithmic utility and Cobb.Doublas production functions, the laissez-faire balanced growth path with a positive value of $\tilde{k}$(and $\tilde{d}$) is unique and globally stable. This is precisely the version of the model which gives rise to the simulations presented in Figure 1 below.

a finite welfare level. This procedure allows to write the following utility function:[9]

$$\tilde{U} = U(\tilde{c}^m, \tilde{c}^o). \tag{13}$$

Along a balanced growth path, the government budget constraint (4) becomes:

$$\tilde{z}^m + \frac{\tilde{z}^o}{(1+g)(1+n)} = \frac{[(1+g)(1+n)-(1+r)]\,\tilde{d}}{(1+g)}, \tag{14}$$

where $1+g = e(\tilde{d})$. Using (5) and (6), the lifetime budget constraint can be written:

$$\tilde{c}^m + \frac{\tilde{c}^o}{1+r} = w - (1+n)\tilde{d} + \frac{\tilde{z}^o[(1+r)-(1+g)(1+n)]}{(1+r)(1+g)(1+n)} \equiv \tilde{\omega}, \tag{15}$$

where $\tilde{\omega}$ is the present value of lifetime income expressed in terms of output per unit of efficient labour. And (8) and the equilibrium condition (10) are, respectively:

$$\frac{\partial U(\tilde{c}^m, \tilde{c}^o)/\partial \tilde{c}^m}{\partial U(\tilde{c}^m, \tilde{c}^o)/\partial \tilde{c}^o} = 1+r, \tag{16}$$

$$w - (1+n)\tilde{d} + \frac{\tilde{z}^o}{(1+g)(1+n)} - \tilde{c}^m = (1+g)(1+n)\tilde{k}. \tag{17}$$

In order to analyze the welfare effects from changes in $\tilde{z}^o$, we need a function relating utility to this tax parameter. Using (15) and (16), the demands for consumption along a balanced growth path will be $\tilde{c}^m = \tilde{c}^m(\tilde{\omega}, R)$ and $\tilde{c}^o = \tilde{c}^o(\tilde{\omega}, R)$. Thus, an indirect utility function can be obtained that provides the maximum level of $\tilde{U}$ as a function of $\tilde{\omega}$ and R:

$$\tilde{U} = U[\tilde{c}^m(\tilde{\omega}, R), \tilde{c}^o(\tilde{\omega}, R)] = V(\tilde{\omega}, R) \tag{18}$$

[9]Notice that, since $U(c_t^m, c_{t+1}^o)$ is homogeneous of degree j, we can write:

$$\tilde{U}_t = U(\tilde{c}_t^m, \tilde{c}_{t+1}^o) = U(c_t^m/h_t, c_{t+1}^o/h_t) = (1/h_t^j)U(c_t^m, c_{t+1}^o) = (1/h_t^j)U_t,$$

In words, a "new" utility function is obtained by means of a monotonic transformation of the original one, thus ensuring that *ordinal* preferences are respected. Notice that both have the same functional form, and are homogeneous of degree j. Also, the slope, curvature and higher derivatives of indifference curves in $(\tilde{c}_t^m, \tilde{c}_{t+1}^o)$ space are the same as those of in (c_t^m, c_{t+1}^o) space. For further discussion and a rationale for adhereing to (13) see Del Rey and Lopez-Garcia (2013).

For later use, we can obtain the partial derivatives of V with respect to its arguments. Using (16):

$$\frac{\partial V}{\partial \tilde{\omega}} = \frac{\partial U}{\partial \tilde{c}^m}\left(\frac{\partial \tilde{c}^m}{\partial \tilde{\omega}} + \frac{1}{R}\frac{\partial \tilde{c}^o}{\partial \tilde{\omega}}\right) = \frac{\partial U}{\partial \tilde{c}^m}, \tag{19}$$

$$\frac{\partial V}{\partial R} = \frac{\partial U}{\partial \tilde{c}^m}\left(\frac{\partial \tilde{c}^m}{\partial R} + \frac{1}{R}\frac{\partial \tilde{c}^o}{\partial R}\right) = \frac{\partial U}{\partial \tilde{c}^m}\frac{\tilde{c}^o}{R^2} \tag{20}$$

where the last equality follows in both cases from differentiation of (15).[10]

Finally, taking into account that $\tilde{\omega}$ in (15) is a function of $\tilde{k}$, $\tilde{d}$ and $\tilde{z}^o$, that R depends on $\tilde{k}$, and that both $\tilde{k}$ and $\tilde{d}$ were found above to depend on $\tilde{z}^o$, we find a new indirect utility function that depends on $\tilde{z}^o$ only, that is, $\tilde{U} = V\left[\tilde{\omega}\left(\tilde{k}(\tilde{z}^o), \tilde{d}(\tilde{z}^o), \tilde{z}^o\right), R\left(\tilde{k}(\tilde{z}^o)\right)\right]$. For short:

$$\tilde{U} = \tilde{V}(\tilde{z}^o), \tag{21}$$

explicitly relating $\tilde{U}$ to the lump-sum tax paid by the members of the older generation when the government creates and operates a credit market for education. This expression, together with (11) and (12), are the relevant functions to undertake the comparative dynamics of changes in $\tilde{z}^o$.

4 Welfare Effects of the Intergenerational Transfers Associated with Government Created Credit Markets

Before discussing the analytics associated with comparative dynamics, it is important to emphasize that, in our setup, the creation by the government of a credit market for education is consistent with an *infinite* number of combinations of lump-sum taxes z_t^o (and the associated z_t^m in (4)). As stated above, at a given period t, the repayment of previous credits by the middle-aged, $(1 + r_t)d_{t-1}L_t$, and the new loans taken by the young, $d_t L_{t+1}$, need not generally coincide, as both d_{t-1} and d_t are chosen by different individuals in a decentralized way. The difference between these sums has to be

[10]When obtaining $\partial V/\partial R$ in (20), $\tilde{\omega}$ is taken as given at this stage. The effect of R on the present value of lifetime resources $\tilde{\omega}$ will come forth when we differentiate $\tilde{\omega}$ with respect to $\tilde{k}$.

met, and hence the need for introducing lump-sum taxes z_t^m and z_t^o in the same budget constraint. In other words, in our setting, the operation of a government-created market for education loans *cannot possibly* be separated from the very existence of the intergenerational transfers induced by z_t^m and z_t^o, which, of course, will also influence savings.

We can now ask: in the presence of a government-created credit market for education, and for a given distribution of the ensuing tax burden (i.e., a value of $\tilde{z}^o$ and the associated $\tilde{z}^m$) what will be, first, the impact on both physical and human capital accumulation, and, second, on social welfare, of a slight variation in the tax distribution? This amounts to working out the derivatives $d\tilde{k}(\tilde{z}^o)/d\tilde{z}^o$ and $d\tilde{d}(\tilde{z}^o)/d\tilde{z}^o$ in (11)-(12) and $d\tilde{V}(\tilde{z}^o)/d\tilde{z}^o$ in (21).

4.1 Effects on Physical and Human Capital Accumulation

The balanced growth path version of (10) in (11), can be rewritten for convenience as:

$$\tilde{k} = \frac{1-\pi(R(\tilde{k}))}{(1+n)e(\phi(\tilde{k}))}\left[w(\tilde{k}) - (1+n)\phi(\tilde{k}) + \frac{\tilde{z}^o}{(1+n)e(\phi(\tilde{k}))}\right] + \frac{\pi(R(\tilde{k}))\tilde{z}^o}{R(\tilde{k})(1+n)e(\phi(\tilde{k}))} \tag{22}$$

As (22) is implicitly given by $\tilde{k} = \Psi(\tilde{k}, \tilde{z}^o)$, we obtain:

$$\frac{\partial\tilde{k}(\tilde{z}^o)}{\partial\tilde{z}^o} = \frac{\partial\Psi(.)}{\partial\tilde{k}}\frac{\partial\tilde{k}(\tilde{z}^o)}{\partial\tilde{z}^o} + \frac{\partial\Psi(.)}{\partial\tilde{z}^o} = \frac{1}{\Omega}\left[\frac{1-\pi(R(\tilde{k}))}{(1+n)e(\phi(\tilde{k}))} + \frac{\pi(R(\tilde{k}))}{R(\tilde{k})}\right] \tag{23}$$

where $0 < \Omega = 1 - \partial\Psi(.)/\partial\tilde{k} < 1$ along a locally stable balanced growth path. Since the expression in brackets in (23) is positive, $\partial\tilde{k}(\tilde{z}^o)/\partial\tilde{z}^o > 0$, and in the presence of a government-created credit market for education loans, an increase in the lump-sum tax paid by the old-aged will translate into an increase of the $\tilde{k}$ ratio. Thus, the message emerging from exogenous growth models with a pure life-cycle saving motive à la Diamond (1965) and Bierwag, Grove and Khang (1969) continues to hold in the current framework: the larger the lump-sum tax paid by the old-aged, the greater the tax provision they will set aside in their middle age, and the greater savings and physical capital accumulation will be.

We can now turn to the expression $\tilde{d} = \phi(\tilde{k}(\tilde{z}^o))$ in (12). Since $\phi'(\tilde{k}) > 0$, and using (23), it follows that $d\tilde{d}(\tilde{z}^o)/d\tilde{z}^o = \phi'(\tilde{k})(d\tilde{k}(\tilde{z}^o)/d\tilde{z}^o) > 0$. Therefore, the above result concerning physical capital can be extended to human capital: an increase in the lump-sum tax paid by the old-aged will also increase the amount of resources devoted to education and thus the growth rate of the economy. The inuition is simple: an increase in $\tilde{z}^o$ will lead to a greater $\tilde{k}$, and along with it to a greater wage rate and a lower interest factor. Since $e'(\tilde{d}) = R(\tilde{k})/w(\tilde{k})$ characterizes the education decision in (7) both effects point to a greater education investment. Summarizing,

Proposition 1 *Along a balanced growth path in the presence of a government-created credit market for education, an increase of the lump-sum tax paid by the old-aged (coupled with the required endogenous adjustment of the lump-sum tax paid by the middle-aged), translates into a greater accumulation of both physical and human capital, and an increased growth rate.*

4.2 Welfare Effects

We can now focus on the welfare effects of changing $\tilde{z}^o$ along a balanced growth path in the presence of a government-created credit market for education. The conclusion in overlapping generations models with exogenous growth à la Diamond (1965) is simple: when the marginal product of physical capital is higher [resp. lower] than the economy's growth rate (reflecting under [resp. over] accumulation of physical capital with respect to the Golden Rule), a lump-sum transfer from [resp. to] older to [resp. from] younger generations provides a means to bring the economy closer to the Two-Part Golden Rule. At this path, the lifetime welfare of a representative generation is maximized subject to the feasibility constraint and the additional constraint that everyone else's welfare is fixed at the same level [Diamond (1965), Samuelson (1968, 1975)]. Interestingly, as our analysis will show, in the current model where human capital is the engine of growth, this condition does *not* hold any longer.

Using the indirect utility function $\tilde{U} = V\left[\tilde{\omega}\left(\tilde{k}(\tilde{z}^o), \tilde{d}(\tilde{z}^o), \tilde{z}^o\right), R\left(\tilde{k}(\tilde{z}^o)\right)\right]$ in (21) we have:

$$\frac{d\tilde{V}(\tilde{z}^o)}{d\tilde{z}^o} = \frac{\partial V}{\partial \tilde{\omega}}\left(\frac{\partial \tilde{\omega}}{\partial \tilde{k}}\frac{d\tilde{k}}{d\tilde{z}^o} + \frac{\partial \tilde{\omega}}{\partial \tilde{d}}\frac{d\tilde{d}}{d\tilde{z}^o} + \frac{\partial \tilde{\omega}}{\partial \tilde{z}^o}\right) + \frac{\partial V}{\partial R}\frac{dR}{d\tilde{k}}\frac{d\tilde{k}}{d\tilde{z}^o}. \tag{24}$$

After some manipulation (detailed in the Appendix), (24) can be rewritten, using (19)-(20), (15) and (17), as:

$$\begin{aligned}\frac{d\tilde{V}(\tilde{z}^o)}{d\tilde{z}^o} &= \frac{\partial U}{\partial \tilde{c}^m}\left[\frac{-\tilde{k}f''\left[(1+r)-(1+g)(1+n)\right]}{(1+r)} - (1+n)\phi' - \frac{\tilde{z}^o e'\phi'}{(1+g)^2(1+n)}\right]\frac{d\tilde{k}(\tilde{z}^o)}{d\tilde{z}^o} + \\ &\quad + \frac{\partial U}{\partial \tilde{c}^m}\frac{\left[(1+r)-(1+g)(1+n)\right]}{(1+r)(1+g)(1+n)}.\end{aligned} \tag{25}$$

The second term in (25) can be shown to be equal to $(\partial V/\partial\tilde{\omega})(\partial\tilde{\omega}/\tilde{z}^o)$. Therefore, it captures a pure income effect from modifying $\tilde{z}^o$ for a given value of $\tilde{k}$ (and thus of $\tilde{d}$). It can easily be signed according to whether the interest factor is greater or less than the economy's growth factor. The first term, however, captures the complicated interaction among the income and price effects of the remaining terms in (24), which are ultimately related to $d\tilde{k}/d\tilde{z}^o$. The sign of the first of the three addends therein can again easily be signed, depending again on whether $(1+r)$ is greater or less than $(1+g)(1+n)$. The second one is definitely positive, but it is affected by a minus sign. And the sign of the third one, as both e' and ϕ' are positive, will be the opposite of that of the lump-sum tax paid by the old-aged. As a consequence, we immediately conclude that the sign of $d\tilde{V}/d\tilde{z}^o$ will in general be *indeterminate* regardless of the divergence $[(1+r)-(1+g)(1+n)]$.[11]

[11] Notice that this indeterminacy remains even when $(1+r) = (1+g)(1+n)$, as the bracket affecting $d\tilde{k}(\tilde{z}^o)/d\tilde{z}^o$ cannot be signed even in this case. This is so because although the first summand therein vanishes, the second one is positive (but affected by a minus sign) and the sign of the third one is the opposite to that of $\tilde{z}^o$ (and thus it is ambiguous). It has to be emphasized that, as shown below in Section 5, the mere fact that the interest factor and the economy's growth factor coincide does *not* attach any normative appeal to this balanced growth path. Some further conditions have to be fulfilled in order to attain the "Golden Rule" in the present endogenous growth setting.

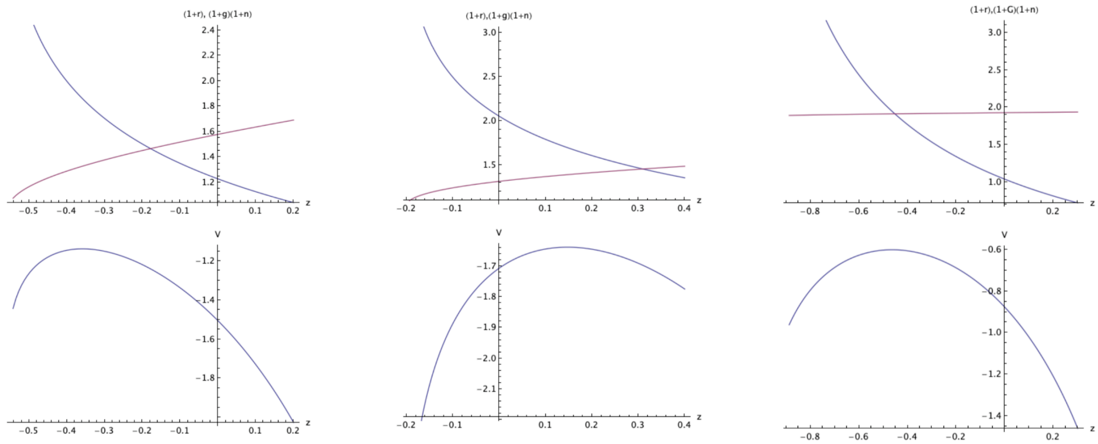
$(1+r) > (1+g)(1+n)$	$\tilde{z}^o \gtrless 0$	$d\tilde{V}/d\tilde{z}^o \gtrless 0$
$(1+r) < (1+g)(1+n)$	$\tilde{z}^o \geqslant 0$	$d\tilde{V}/d\tilde{z}^o < 0$
$(1+r) < (1+g)(1+n)$	$\tilde{z}^o < 0$	$d\tilde{V}/d\tilde{z}^o \gtrless 0$
$(1+r) = (1+g)(1+n)$	$\tilde{z}^o \geqslant 0$	$d\tilde{V}/d\tilde{z}^o < 0$
$(1+r) = (1+g)(1+n)$	$\tilde{z}^o < 0$	$d\tilde{V}/d\tilde{z}^o \gtrless 0$

Table 1: The sign of $d\tilde{V}(\tilde{z}^o)/d\tilde{z}^o$ as a consequence of the interaction of the terms in (25).

Table 1 portrays the results concerning the sign of $d\tilde{V}/d\tilde{z}^o$ arising from the interaction of the different terms in (25). If $(1+r) > (1+g)(1+n)$, the sign of the term in the square brackets therein is ambiguous regardless of that of $\tilde{z}^o$, and along with it $d\tilde{V}/d\tilde{z}^o$ may either be positive or negative. If $(1+r) < (1+g)(1+n)$, the value taken by $\tilde{z}^o$ becomes crucial. If $\tilde{z}^o \geqslant 0$ [resp. < 0], the term in the square brackets will be negative [resp. indeterminate] and thus $d\tilde{V}/d\tilde{z}^o < 0$ [resp. $\gtrless 0$]. Finally, if it happens that $(1+r) = (1+g)(1+n)$ and $\tilde{z}^o \geqslant 0$ [resp. < 0], the whole term in the square brackets will be negative [resp. will have an indeterminate sign] and therefore $d\tilde{V}/d\tilde{z}^o < 0$ [resp. $\gtrless 0$]. Summarizing:

Proposition 2 *Along a balanced growth path in the presence of government-created credit markets for education, the effect on welfare of an increase of the lump-sum tax paid by the old-aged (coupled with the required endogenous adjustment of the lump-sum tax paid by the middle-aged), may either be positive or negative regardless of whether the marginal product of capital is greater or less than the economy's growth factor.*

As will be more apparent in Section 5, the ambiguous message from Propositon 1 reflects the *second-best* nature of the normative problem posited in this section. Indeed, in a setting where there are not private credit markets for education, public policy pursues two targets. These targets are: (i) creating a credit market with which not only allow for investment in education but also to



(a) $\alpha = .2, \beta = .9, \lambda = .25, A = B = 2, n = 0$ (b) $\alpha = .4, \beta = .9, \lambda = .15, A = B = 2, n = 0$ (c) $\alpha = .2, \beta = .9, \lambda = .01, A = B = 2, n = 0$

Figure 1: Examples with logarithmic utility and Cobb-Douglas production functions

attain the appropriate (i.e., *optimal)* amount of *education*; and (ii) implementing the (also) *optimal* investment in *physical* capital. The mere existence of a government-managed credit market for education investment achieves (i) only partially. Clearly, investment in human capital now becomes feasible, but it need not coincide with the optimal one. In a very real sense, to achieve two targets we are left with a single instrument: the lump-sum tax on the old-aged, $\tilde{z}^o$, with which to induce intergenerational transfers. In a economy where saving is the result of a life-cycle motive, this tax instrument can redistribute resources between individuals in different stages of their life-cycle and affect the accumulation of physical capital in the desired way. But it is a poor instrument with which to indirectly achieve (i.e., through the $\tilde{d} = \tilde{d}(\tilde{z}^o)$ function) the appropriate amount of investment in education.

Figure 1 illustrates the results contained in Table 1 with the aid of three examples associated with the case of logarithmic utility, $U = log(c^m) + \beta log(c^o)$, $\beta > 0$, and Cobb-Douglas production functions, $f(\tilde{k}) = A\tilde{k}^\alpha$ and $(1+g) = B\tilde{d}^\lambda$, $A, B > 0$, $0 < \alpha, \lambda < 1$. The downward sloping functions in the upper part of both panels represent, using (1) and (11), the interest factor as $R(\tilde{z}^o) = f'\left[\tilde{k}(\tilde{z}^o)\right]$, where $R'(\tilde{z}^o) = f''(d\tilde{k}/d\tilde{z}^o) < 0$. Their upward sloping counterparts, using (12), portray the economy's growth factor as $G(\tilde{z}^o) = (1+n)e\left[\tilde{d}(\tilde{z}^o)\right]$, with $G'(\tilde{z}^o) = (1+n)e'(d\tilde{d}/d\tilde{z}^o) > 0$. The lower panels allow to interpret (25) and show that the sign of the derivative $\partial\tilde{V}(\tilde{z}^o)/\partial\tilde{z}^o$ can be positive, negative or zero for different combinations of the parameters. Notice, in passing, that the origins of coordinates in Figure 1 do *not* represent the laissez-faire balanced growth path, but the situation where $\tilde{z}^o = 0$, and thus, using (14), the only lump-sum tax is that on the middle-aged. Also, as shown, there is no reason for the balanced growth path verifying $(1+r) = (1+g)(1+n)$ to be associated with a positive or a negative value of $\tilde{z}^o$. But be it as it may, when the functions $R(\tilde{z}^o)$ and $G(\tilde{z}^o)$ intersect, it will be the case from the government budget constraint (14) that $\tilde{z}^m + \tilde{z}^o/(1+g)(1+n) = 0$, and the government is, in fact, operating a pay-as-you-go or a reverse pay-as-you-go social security system, according to the sign of $\tilde{z}^m$ and $\tilde{z}^o$. Notice also that in all the situations depicted in Figure 1 the curves $V(\tilde{z}^o)$ reach a maximum, so there is a value of the tax on the elderly that, in this *second-best* setup, maximizes social welfare. However, it has to be emphasized that, as discussed in the next section, those maxima cannot possibly decentralize the first-best Golden Rule balanced growth path.

5 Optimal Credit Markets and Intergenerational Transfers

In the endogenous growth literature it has become standard to associate the notion of optimality with the maximization of a infinite, discounted sum of utility levels. Docquier et al. (2007) are an example of this procedure in the framework of the present model. Discounting of future utilities is crucial when productivity growth translates into consumption growth and then into continuously increasing utility indices: it ensures that the infinite discounted sum is well defined. But it is also liable to some normative weaknesses. First, the choice of a particular social discount rate is inherently arbitrary. Second, more importantly, the precise cardinalization of the utility function (i.e., its degree of homogeneity) affects in a fundamental way the optimal balanced growth path and, thus, modifies both the social optimum and the public policies supporting it (Del Rey and Lopez-Garcia, 2012).

In order to avoid these uncomfortable issues, Del Rey and Lopez-Garcia (2013) associate optimality with the maximization of the utility function (13), defined over consumption levels per unit of *efficient* labour, subject to the constraint that the welfare of the representative individual of every other generation, defined in the same way, is fixed at the same level. This procedure gives rise to the counterpart in the current framework of the Two-Part Golden Rule in exogenous growth models. This choice can be rationalized as a social planner wanting to treat all individuals alike while being respectful of their preferences.

The *Golden Rule* balanced growth path in this model, where the accumulation of human capital is the engine of growth, is the constellation of values $(\tilde{c}_*^m, \tilde{c}_*^o, \tilde{k}_*, \tilde{d}_*)$ that maximize $\tilde{U} = U(\tilde{c}^m, \tilde{c}^o)$ subject to the balanced growth path version of the feasibility constraint (see footnote 6), i.e.:

$$\tilde{c}^m + \frac{\tilde{c}^o}{(1+n)e(\tilde{d})} = f(\tilde{k}) - (1+n)\tilde{d} - (1+n)e(\tilde{d})\tilde{k}, \tag{26}$$

The optimality conditions, derived in Del Rey and Lopez-Garcia (2013), are:

$$f'(\tilde{k}_*) = (1+n)e(\tilde{d}_*), \tag{27}$$

$$\frac{\partial U(\tilde{c}_*^m, \tilde{c}_*^o)/\partial \tilde{c}^m}{\partial U(\tilde{c}_*^m, \tilde{c}_*^o)/\partial \tilde{c}^o} = (1+n)e(\tilde{d}_*), \tag{28}$$

$$e'(\tilde{d}_*)\left(\frac{\tilde{c}_*^o}{\left((1+n)e(\tilde{d}_*)\right)^2} - \tilde{k}_*\right) = 1 \tag{29}$$

which, together with (26), provide four equations to be solved in the four variables $(\tilde{c}^m, \tilde{c}^o, \tilde{k}, \tilde{d})$.[12]

[12]Eq. (27) is the equality of the marginal product of physical capital and the growth factor of labour expressed

We are now in a position to ascertain the *first-best* optimal policy, i.e., the one that allows to decentralize the resource allocation that achieves the Golden Rule and maximizes social welfare. As we mentioned in Section 4, the mere fact that the government operates a credit market for education (with the associated intergenerational transfers) is insufficient to attain the first best. The reason is that although individuals may choose their education investment, they are not given the marginal incentives for their choice to be optimal. Indeed, using (26), the optimality condition (29) can be rewritten as

$$w(\tilde{k}_*)e'(\tilde{d}_*) = R(\tilde{k}_*)\left(1 + \frac{e'(\tilde{d}_*)\Lambda_*}{R(\tilde{k}_*)}\right), \tag{30}$$

where $\Lambda_* = (1+n)e(\tilde{d}_*)\tilde{k}_* + (1+n)\tilde{d}_* + \tilde{c}^m_* > 0$. Comparison of (30) and (7) makes it clear that when choosing how much to invest in education, and if they had access to credit and were confronted with the optimal wage and interest factor, $w(\tilde{k}_*)$ and $R(\tilde{k}_*)$, individuals would fail to take into account the term $e'(\tilde{d}_*)\Lambda_*/R(\tilde{k}_*) > 0$. As a consequence, they would over-invest in education. Hence, the mere existence of a goverment-created credit system and its associated intergenerational transfers prove insufficient to attain the first best. There are different possible ways to lead the individuals to their optimal investment in human capital: education can be subsidised, or publicly provided, to the young; also, education loan repayments can be subsidised. Since our aim is to design the optimal government created credit market, we are going to assume that the government decides how much individuals can borrow.[13] Note that our aim in not to prove that the *first best* can be decentralised, but to identify the intergenerational arrangements required for both investments in

in efficiency units. Eq. (28) is the equality of the marginal rate of substitution between second and third period consumption and the economy's growth rate, and provides the counterpart in the current model of the so-called "biological" interest rate [Samuelson (1958)]. Finally, (29) is the condition that equates marginal benefit and marginal cost associated with changes in $\tilde{d}$.

[13] In England, where the student loan is an entitlement, not a mandate, 90% of students take the maximum loan for fees repayment (SLC, 2019). In Australia, where fees can be paid upfront or deferred, 85% defer (ATO, 2019). Thus, an overwhelming majority of students borrow the amounts predetermined by the government in these two cases. We are indebted to Nicolas Barr and Bruce Chapman for their help in obtaining this information.

human and physical capital to be optimal when the government manages the credit market for education and can decide how much individuals borrow.

To do so, consider that at period $t-1$ the government operates a credit market for education where it lends younger individuals $\tilde{m}$ units of output per unit of efficient labour to be repaid with interest $(1+r_t)\tilde{m}$ when they are middle-aged in period t.[14] With investment in education being *exogenous* for them, the only decisions taken by individuals will be those associated with their consumption. As a consequence, the function $\tilde{d}_t = \phi(\tilde{k}_{t+1})$ in (7) will not be relevant any longer, but the remaining behavioural assumptions and relationships continue to be as in Section 2.

5.1 Optimal lending and intergenerational transfers

In order to characterize the optimal orthopaedics, $\tilde{m}_*$ and $\tilde{z}^o_*$ (and, thus, $\tilde{z}^m_*$ in (14)), that convert a decentralized equilibrium into the Golden Rule, we can compare the optimal resource allocation in (26)-(29) on the one hand, with the configuration composed by (1), (2), (15), (16), (17) and the relationship $1+g = e(\tilde{d})$ when the government sets $\tilde{m}$ and $\tilde{z}^o$ on the other, forcing $(\tilde{c}^m_*, \tilde{c}^o_*, \tilde{k}_*, \tilde{d}_*)$ to be the same in both cases. As for the amount to be lent to younger individuals, obviously, it must be the case that $\tilde{m}_* = \tilde{d}_*$, so that the optimal growth factor of productivity, $1+g_* = e(\tilde{d}_*)$, is attained. And concerning $\tilde{z}^o_*$, from (15) and (17), evaluated both at the Golden Rule,

$$\frac{\tilde{c}^o_*}{\Big((1+n)e(\tilde{d}_*)\Big)^2} - \tilde{k}_* = \frac{-\tilde{z}^o_*}{\Big((1+n)e(\tilde{d}_*)\Big)^2}, \tag{31}$$

from which, using (29), results:

$$\tilde{z}^o_* = \frac{-\Big((1+n)e(\tilde{d}_*)\Big)^2}{e'(\tilde{d}_*)} < 0. \tag{32}$$

[14] Alternatively, as mentioned before, we could refer to this arrangement as *public education financed with a graduate tax.*

In words, the lump-sum tax paid by the older generation at the Golden Rule has to be *negative*, i.e., the elderly will be receing a positive transfer.

It should be remarked that the results implying $\tilde{m}_* = \tilde{d}_*$ and the value of $\tilde{z}^o_*$ in (32) could have also been found reformulating the framework in Section 2 to obtain, with obvious notation, the functions $\tilde{k} = \breve{k}(\tilde{z}^o, \tilde{m})$ and $\tilde{U} = \breve{V}(\tilde{z}^o, \tilde{m})$, and working out the expressions $\partial \breve{V}(\tilde{z}^o_*, \tilde{m}_*)/\partial \tilde{z}^o = \partial \breve{V}(\tilde{z}^o_*, \tilde{m}_*)/\partial \tilde{m} = 0$. The procedure we have followed, comparing (26)-(29) on the one hand, with (1), (2), (15), (16), (17) on the other, is, without any doubt, simpler. It should also be stressed that the nature of the $\breve{V}(\tilde{z}^o, \tilde{m})$ function just defined and that of $\tilde{V}(\tilde{z}^o)$ in (21) are very different. Indeed, in the former the government directly sets the amount $\tilde{m}$ devoted to education, being able to choose its optimal level. In contrast, in the latter, the individuals are the ones who decide on education investment, and, for this reason, this solution is necessarily second-best.

Importantly, at the Golden Rule, the principal plus interest paid by the middle-aged on their credits, $f'(\tilde{k}_*)\tilde{d}_*$, just equals the issue of new loans to the young, $(1+n)e(\tilde{d}_*)\tilde{d}_*$. Thus, the "credit system side" of the government budget is balanced and the right-hand-side in (14) vanishes, i.e., $\tilde{z}^m_* + \tilde{z}^o_*/(1+n)e(\tilde{d}_*) = 0$. The "tax-transfer side" of the goverment budget is, in fact, a pure *pay-as-you-go* social security system with a tax $\tilde{z}^m_* = (1+n)e(\tilde{d}_*)/e'(\tilde{d}_*) > 0$ on the middle-aged and a pension benefit $-\tilde{z}^o_* = \left((1+n)e(\tilde{d}_*)\right)^2/e'(\tilde{d}_*) > 0$ to the old-aged. Summarizing:

Proposition 3 *When the government creates a credit market for education, the Golden Rule balanced growth path can be achieved: (i) lending individuals when young the amount $\tilde{m}_* = \tilde{d}_*$, and (ii) operating a pure pay-as-you-go social security system where the old-aged receive the pension benefit $-\tilde{z}^o_*$ in (32) and the middle-aged pay a contribution $\tilde{z}^m_* = -\tilde{z}^o_*/((1+n)e(\tilde{d}_*))$.*

To understand the implications of Proposition 3, rewrite (17), and evaluate it at the Golden

Rule balanced growth path:

$$w(\tilde{k}_*) - (1+n)\tilde{m}_* - \tilde{z}_*^m - \tilde{c}_*^m = e(\tilde{m}_*)(1+n)\tilde{k}_* \tag{33}$$

Clearly, if $\tilde{z}_*^m$ were zero or negative, the left-hand side of (33) (i.e. savings) would be too large to attain the physical capital market equilibrium condition, rendering it impossible to achieve the Golden Rule.

It has to be emphasized that the design of social security in Proposition 3 is in sharp contrast with the one that arises in life-cycle models with exogenous growth. As shown in Samuelson (1975), in such models the nature of the optimal social security that supports the Two-Part Golden Rule depends in a crucial way on whether the laissez-faire (physical) capital-labour (in natural units) ratio is higher or lower than the optimum one. In other words, it entails a transfer from [resp. to] the middle-aged to [resp. from] the old-aged through a pay-as-you-go social security system [resp. a "reverse" pay-as-you-go system or a "more-than-fully-funded" one] when along the laissez-faire balanced growth path the marginal product of capital is less [resp. greater] than the economy's growth rate. In contrast, as shown above, when human capital is the engine of growth, the intergenerational effects of the optimal policy are *always* equivalent to those of a pay-as-you-go social security system.[15]

5.2 The Optimal Intergenerational State

Proposition 3 describes a social security system and a government created credit market for education that decentralizes the Golden Rule balanced growth path. An alternative procedure leading to exactly the same allocation would be to maintain the social security system $(\tilde{z}_*^m, -\tilde{z}_*^o)$ in (ii) and to replace the direct lending of $\tilde{m}_*$ in (i) with a different instrument, namely: (iii) a lump-sum tax

[15]Two recent contributions on the welfare effects of pay-as-you-go social security in different frameworks are Chu and Cheng (2019) and Hu (2018).

$\tilde{x}_* = (1+n)\tilde{d}_*$ on the middle aged, whose revenues would be used to directly finance the amount of education $\tilde{m}_*$ received by the young. As the revenue from this tax on the middle-aged at period t, $(1+n)\tilde{d}_* L_t$, would equal the cost of education to the newly-born, $\tilde{d}_* L_{t+1}$, the effects of (i) and (iii) would be indistinguishable from those of (i) and (ii).

This alternative procedure with which to attain the Golden Rule is reminiscent of the "intergenerational state" used in Boldrin and Montes (2005) to replicate the laissez-faire complete market allocation in the absence of private credit markets. It consists of two programs, one with which the middle-aged pay a tax to finance youths' education, and another one with which the same middle-aged pay a social security contribution. When this contribution equals the subsidy the middle-aged received as youngsters plus interest, the private return from investing in education coincides with what would be the cost of financing education via a complete credit market. Therefore, the arbitrage condition (7) between the returns from investing in education (i.e., $\tilde{d}$) and in physical capital (i.e., $\tilde{k}$) at the laissez faire is satisfied. However, in the present endogenous growth framework there is nothing inherently desirable in the laissez faire, and thus no normative appeal should be attached to the result of individual behaviour and interaction in the marketplace. As the laissez faire with private credit markets *cannot* be taken to be optimal, it follows, as a corollary, that the intergenerational state in Boldrin and Montes (2005), which allows to replicate the above-mentioned laissez faire, cannot be given any normative significance either.[16]

In contrast, to achieve the optimal Golden Rule, the arbitrage condition (7) is plainly irrelevant. The point is that the roles played by investing in education and in physical capital are completely

[16] With the subscript "$_{LF}$" denoting the laissez-faire resource allocation, the precise relationship between $\tilde{k}_{LF}$ and $\tilde{d}_{LF}$ on the one hand, and their optimal counterparts on the other depends, of course, on the welfare criterion adopted. Representing with "$**$" the case where the objetive funcion is an infinite discounted sum of utililies, as in Docquier et al. (2007), $\tilde{d}_{LF}$ can be greater or less than $\tilde{d}_{**}$, but $\tilde{k}_{LF} < \tilde{k}_{**}$ is not feasible. Using "$*$" to label the optimal values in terms of the Golden Rule discussed in Del Rey and Lopez-Garcia (2013), any relationship between $\tilde{k}_{LF}$ and $\tilde{k}_*$ is possible, but $\tilde{d}_{LF} \leqslant \tilde{d}_*$ is not feasible.

different: $\tilde{d}$ is addressed to generate the growth rate of productivity $e(\tilde{d})$, whereas $\tilde{k}$ is directly related to the generation of output through the production function $f(\tilde{k})$. Either the optimal public policy $(\tilde{m}_*, \tilde{z}_*^o)$, or, alternatively $(\tilde{x}_*, \tilde{z}_*^o)$ (in both cases with the required $\tilde{z}_*^m$), allows to decentralize $\tilde{k}_*$ and $\tilde{d}_*$, so that the optimal growth rate $(1+n)e(\tilde{d}_*)$, and the optimal amount of output, $f(\tilde{k}_*)$, will be attained. Individuals can now allocate their life-cycle savings by investing in the market for *physical capital*, and they can also intertemporally transfer resources with the *intergenerational transfers* associated with the optimal pay-as-you go social security. At the Golden Rule, the relevant arbitrage condition $f'(\tilde{k}_*) = (1+n)e(\tilde{d}_*)$ is fulfilled, i.e., the market interest factor of investing in physical capital at the optimal ratio $\tilde{k}_*$ is equal to the implicit return of investing in future generations with an education investment $\tilde{d}_*$ that is also optimal. This is the true intergenerational state that is consistent with, and allows to decentralize, the optimal Golden Rule in the present endogenous growth model.

6 Concluding comments

There is an increasing interest in public loan programs to finance higher education around the world. In this paper we have explored the welfare properties of a government-created credit market for education in which any difference between the issue of new credits by younger individuals and the repayment of previous ones by the middle-aged is covered by lump-sum taxes or subsidies on both the middle-aged and the old-aged. We have shown that the mere existence of a government-created credit market is, in itself, second-best in nature, and cannot decentralize the optimum. In this situation, an increase in the lump-sum tax paid by the old-aged (coupled with the required adjustment in the tax paid by the middle-aged) can either increase or decrease welfare, this being the case regardless of whether the interest factor is greater or less than the economy's growth factor. Clearly, this result is in sharp contrast to its counterpart in exogenous growth models, where the above-mentioned welfare effects depend in a systematic way on the relationship between the marginal product of physical capital and the economy's growth rate.

To achieve the optimal "Golden Rule" in the present endogenous growth model, we need that investments in education be optimal, as well as the adequate intergenerational transfer system. To this end, we have considered (i) a self-financing government created credit market where young individuals borrow the optimal amounts to fund their education (and new loans are just matched by the credits returned with interest), and (ii) an optimal system of intergenerational transfers from the middle-aged to the old-aged, that turns out to be a pure pay-as-you-go social security. Taken together, (i) and (ii) support the balanced growth path that fulfils the optimal no-arbitrage condition equating the marginal product of physical capital and the economy's growth factor. Under this welfare-maximizing scheme, individuals of each generation can transfer resources from the present to the future with the same return by saving through the physical capital market or investing in future generations by means of the optimal pay-as-you-go scheme. Thus, a truly optimal "intergenerational state" arises.

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Appendix

Consider Equation (24).

First, by (19), $\frac{\partial V}{\partial \tilde{\omega}} = \frac{\partial U}{\partial \tilde{c}^m}$; also, (20) and $\frac{dR}{d\tilde{k}} = f''(\tilde{k})$ imply $\frac{\partial V}{\partial R}\frac{dR}{d\tilde{k}} = \frac{\partial U}{\partial \tilde{c}^m}\frac{\tilde{c}^o}{R^2}f''(\tilde{k})$. From individual optimising behaviour we obtain:

$$\tilde{c}^o = R(\tilde{\omega} - \tilde{c}^m) \tag{A.1}$$

and using (15) and (17) we find:

$$\tilde{c}^m = w - (1+n)\tilde{d} + \frac{\tilde{z}^o}{(1+g)(1+n)} - (1+g)(1+n)\tilde{k}$$

which substituted back in (A.1) gives

$$\tilde{c}^o = R\left(\frac{-\tilde{z}^o}{(1+r)} + (1+g)(1+n)\tilde{k}\right)$$

Then

$$\frac{\partial V}{\partial R}\frac{dR}{d\tilde{k}} = \frac{\partial U}{\partial \tilde{c}^m}\left(\frac{-\tilde{z}^o}{R^2} + \frac{(1+g)(1+n)\tilde{k}}{R}\right) f''(\tilde{k})$$

Now, differentiating the right hand side of (15):

$$\begin{aligned}
\frac{\partial \tilde{\omega}}{\partial \tilde{k}} &= -\tilde{k}f''(\tilde{k}) + \tilde{z}^o\frac{f''(\tilde{k})}{R^2} \\
\frac{\partial \tilde{\omega}}{\partial \tilde{d}}\frac{d\tilde{d}}{d\tilde{k}} &= -(1+n)\phi' - \frac{e'(.)\phi'}{(1+g)^2(1+n)} \\
\frac{\partial \tilde{\omega}}{\partial \tilde{z}^o} &= \frac{(1+r)-(1+g)(1+n)}{(1+r)(1+g)(1+n)}
\end{aligned}$$

We can now substitute in (24) to obtain

$$\begin{aligned}
\frac{d\tilde{V}(\tilde{z}^o)}{d\tilde{z}^o} &= \frac{\partial U}{\partial \tilde{c}^m}\left(\left(-\tilde{k}f''(\tilde{k}) + \tilde{z}^o\frac{f''(\tilde{k})}{R^2} - (1+n)\phi' - \frac{e'(.)\phi'}{(1+g)^2(1+n)}\right)\frac{d\tilde{k}}{d\tilde{z}^o} + \frac{(1+r)-(1+g)(1+n)}{(1+r)(1+g)(1+n)}\right) \\
&\quad + \frac{\partial U}{\partial \tilde{c}^m}\left(\frac{-\tilde{z}^o}{R^2} + \frac{(1+g)(1+n)\tilde{k}}{R}\right) f''(\tilde{k})\frac{d\tilde{k}}{d\tilde{z}^o}
\end{aligned}$$

Reorganizing terms and simplifying, Equation (25) results.