Employment effects of on-the-job human capital acquisition*†

Joaquín Naval[1], José I. Silva[2], and Javier Vázquez-Grenno[3]

[1]Universitat de Girona
[2]Universitat de Girona and University of Kent
[3]Universitat de Barcelona and Institut d'Economia de Barcelona

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Abstract

This paper quantifies the combined effect on-the-job training and workers' on-the-job learning decisions have on aggregate employment. Initially, we present an index of on-the-job human capital acquisition (OJHCA) based on data from the OECD program for international assessment of adult competencies. The data shows a positive relationship between the on-the-job training and on-the-job learning indexes and a strong positive correlation between the human capital index and employment rates across OECD economies. Next, we build a search and matching model that hinges on the training provided by firms as well as workers' learning. The model also includes education along with taxes that increase and decrease human capital investments. We calibrate the model to the United Kingdom and analyze the role of taxes and education to explain the variation in human capital acquired on-the-job, employment and productivity across countries. We also obtain the marginal costs in every country that reproduce the levels of on-the-job human capital as observed in the data. The model is almost able to reproduce the observed differences in employment rates between the groups of countries in the highest and lowest tertiles of the OJHCA distribution, and OJHCA accounts for forty percent of the simulated employment differences. Finally, we analyze subsidies to training costs and conclude that a 10% reduction in marginal training cost increases long-term employment by almost 0.5 percentage points.

JEL Classifications: E24, J24, J64.

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†Joaquín Naval, Campus de Montilivi, 17071 Girona, email: joaquin.naval@udg.edu, Tel.: (+34) 972418814; José I. Silva, Campus de Montilivi, 17071 Girona, email:jose.silva@udg.edu, Tel.: (+34) 972418779; Javier Vázquez-Grenno, Av. Diagonal 690, 08034 Barcelona, email:jvazquezgrenno@ub.edu, Tel.: (+34) 93 403 9934.

1 Introduction

In recent decades, the skills the workforce requires have become increasingly more demanding, with business and employees alike having to adapt to more increasingly complex technologies (see, OECD, 2006, 2010a, 2011). In this regard, participating in job training activities and learning on-the-job have gained importance, since these activities enable workers to update their competencies and acquire new skills which are, more often than not, compensated in the labor market (see, OECD, 2010b, 2013). Beyond the positive effects these activities have on productivity, they also make the workplace more attractive for workers, increasing their motivation, participation and, consequently, boost the employment rate (OECD, 2004; Grip, 2008). Even though the relevance for policy-makers to have empirical evidence concerning such issues, there are very few papers studying the economic impact that informal learning process has in the workplace. Most papers, following seminal studies in human capital theory (e.g., Arrow, 1962; Mincer, 1962) focus on experience as a proxy of learning in the workplace. These studies show that work experience (as a proxy of learning in the workplace and learning-by-doing) has been a key determinant of workers' economic outcomes.

In this paper we explore the role human capital and, in particular, on-the-job human capital acquisition (formally and informally) have in explaining differences in employment rates across OECD economies. We use data from the OECD Program for the International Assessment of Adult Competencies (PIAAC) that, through a number of questions, allows us to identify formal and informal learning in the workplace in OECD countries. Then, we build a dynamic model of the labor market that replicates the patterns we find and some non-targeted moments. Finally, we use the model to evaluate a public policy consisting in subsidize formal training activities.

First, using the PIAAC data, we construct an index of on-the-job human capital acquisition (OJHCA). This index combines two other indexes, on-the-job training index and on-the-job learning index. The former comprises formal on-the-job training sessions or training activities, while the latter includes the learning of new work-related competencies gained through the interaction between co-workers and supervisors (i.e. informal learning) and from the tasks workers perform on their own (i.e. learning-by-doing).

Second, we build an augmented search and matching model with on-the-job human capital acquisition that depends on both on-the-job training and learning. As search and matching models are workhorses for analyzing the labor markets, these models are suitable for our objective because, as shown by Acemoglu and Pischke (1999a,b), companies find it optimal to offer general training to workers in the presence of frictions in the labor market, contrasting with Becker (1964) results in a perfect labor market set-up. The idea is that firms make higher profits from trained workers because their wage increases are less than productivity increases. In our model, workers acquire skills in an initial stage and either become fully-trained employees or are dismissed. Acquiring skills is costly for both, the firm and the worker, the firm spends resources on training workers and the worker makes an effort to learn. Moreover, training activities encourage workers to increase their learning in the workplace, since the wage increases will offset the costs of learning and the part of the training costs transferred from their respective firms. Only fully-trained workers can transfer their acquired skills on-the-job to another firm. However, if they separate from the job, they may lose their human capital either due to the unemployment duration (as in Ortego-Marti, 2016) or because the unemployed workers find a job in a different occupation (as in Kambourov and Manovskii, 2009).

The model also incorporates payroll taxes and education as factors that affect, among others, on-the-job human capital investment. On the one hand, payroll taxes are negatively correlated with employment rates and the human capital acquisition index. In our model, not only do taxes increase employment opportunity costs (e.g. Prescott, 2004) but they also increase the implicit marginal cost of learning. Consequently, the worker's incentive to learn on-the-job drops and, due to its complementarity, so too does the level of training provided by the firm. On the other hand, formal education increases labor productivity, which increases the net returns of firm's training investments and reduces the workers effort to learn. An increase in formal education can also be interpreted as reduction in the implicit marginal costs of training and learning, which is in line with the observed complementarity between on-the-job training and formal education found in previous studies (see, e.g. Cairó and Cajner, 2018).

Third, to quantify the effect on-the-job human capital acquisition has on employment, we calibrate the model using the United Kingdom (UK) as our benchmark economy. Then, we decompose the effects of education and payroll taxes to account for the differences in on-the-job human capital acquisition, productivity and employment. The model with exogenous variation in education and payroll taxes accounts for 39.2% of the observed gap between countries in the highest and the lowest tertiles of the OJHCA, and 62.3% of the employment gap between these two groups of countries. In an additional exercise, we allow countries to differ in terms of their marginal costs of training and learning; specifically, we simulate the observed cross-country differences in the OJHCA index by adjusting the marginal costs of training and learning and compare the model's predictions for the employment rate with actual data. That is, on top of education and payroll taxes, we assume country-specific policies that determine the levels of training and learning as given. The model with differences in marginal costs is able to reproduce 87.7% of the differences in employment rates between the groups of countries in the highest and lowest tertiles of the OJHCA distribution, and OJHCA accounts for 40.2% of these differences. Furthermore, the difference between the model's predictions and the actual values of employment is less than 5 p.p. in 16 out of 26 OECD countries. The model also correctly predicts the cross-country patterns between education and employment (and labor productivity), and does a very good job at replicating the relations between taxes and employment (and labor productivity). We take these predictions as validation of our model, therefore, we think that our model is useful tool to analyze the relationships between human capital, taxes and employment.

Finally, we evaluate a public policy consisting in a subsidy of the marginal costs of training with the aim to foster human capital investment and employment. The subsidy not only encourages training, but also boosts workers to learn in the workplace, thus further increasing their on-the-job acquired human capital. We find that a 10% cut in marginal costs of training increases employment by almost 0.5 p.p. We also find that a training subsidy has a lower impact in a sclerotic labor market than in flexible one, the main reason is that workers spend more time in unemployment in an economy with low labor market flows.

This paper has obvious parallels with studies documenting a positive association between on-the-job training and employment. In a broader ranging report, the OECD (OECD, 2004) presents evidence of a positive relationship between job training participation and aggregate employment rates after controlling for formal education, GDP growth and labor market institutions.

Beyond formal training, some scholars have focused on the role of workplace learning. Barron et al. (1997) show the importance of these learning processes in the U.S., specifically, they document that, during the first quarter following the hiring of a new worker, more than one-third of training (54.5 hours) is provided through a so-called 'learning by watching co-workers' process, while the other two-thirds correspond mainly to formal sessions of on-the-job training or training activities provided by supervisors and co-workers. In the same vein, Bishop (1996) finds that learning-by-doing plays an important role in the increase in employee productivity during the first two years of job tenure in a firm. Thus, learning through experience and learning from co-workers and supervisors would appear to capture the essence of on-the-job learning.

More recently, some studies stressed that workers are continuously learning in the workplace (i.e., learning from co-workers and supervisors and learning-by-doing) and that has a positive effect on productivity and, subsequently, on economic performance (e.g. Grip, 2008; Destré et al., 2008; De Grip, 2015; Ferreira et al., 2017, 2018). In particular, Ferreira et al. (2017) document empirical evidence of complementarity between on-the-job training and on-the-job learning in an European context. Despite differences in the intensity in informal learning between temporary and permanent employees, using PIAAC data, Ferreira et al. (2018) corroborate complementarity between on-the-job training and on-the-job learning regardless the type of contract.

Our paper contributes to this literature since, to the best of our knowledge, this is the first paper that use a search and matching model to study the dual effects of on-the-job training and workers' on-the-job learning decisions on employment rates across OECD economies. Thus, we believe that our model is a good instrument to improve the understanding of how the different components (training and learning) of on-the-job human capital acquisition affect the labor market.

The remainder of this paper is organized as follows. In section 2 we describe our on-the-job training, learning and human capital acquisition indexes. Section 3 presents the model and Section 4 contains the calibration and main quantitative exercises. Finally, section 5 draws the conclusions.

2 On-the-job human capital acquisition index

The PIAAC developed and conducted the Survey of Adult Skills. This survey assesses adult (16-65-year-olds) proficiency in three key information-processing skills: literacy, numeracy and problem solving in technology-rich environments. The survey has been performed in 33 OECD countries (in two rounds: the first from August 2011 to March 2012 in 24 countries and the second from April 2014 to March 2015 in 9 countries). Among others, the PIAAC survey measures skills in the workplace, specifically, the relevance of on-the-job training and learning in the workplace (from co-workers/supervisors and from the worker's own experience).[1]

Thus, and based on data drawn from the PIAAC, we construct an On-the-Job Human Capital Acquisition index. The aim of this index is to capture both formal and informal learning in the workplace. As a measure of formal learning, the On-the-job Training index (OJT) includes information about worker participation in formal training programs provided by employers. In the case of informal learning, the On-the-job Learning index (OJL) incorporates both, worker interaction with co-workers and supervisors and, the acquisition of skills through learning-by-doing. In the Appendix A we present the construction of the index in greater detail.

Table 1 presents the indexes by country. As can be seen, the three indexes show a sizable variation across countries. More specifically, the OJT index ranges from countries in which less than 10 percent of workers reported having participated in formal on-the-job training sessions in the preceding year to countries in which formal training sessions involve more than 40 percent of employees. As with the training index, the OJL index ranges from almost 20 to nearly 70 percent. Finally the OJHCA index ranges from 14.1 in Turkey to 50.9 in New Zealand.

Figure 1(a) presents the raw correlation between on-the-job training and the employment rate for OECD countries.[2] Specifically, it shows that on-the-job training has a strong positive correlation with employment ($R^2 = 0.57$ in the OLS regression). This is in line with the pos-

[1] For details see http://www.oecd.org/skills/piaac/.

[2] Drawing its data from PIAAC, the training variable measures the proportion of workers receiving on-the-job training over the last 12 months.

Table 1: On-the-job human capital, training and learning indexes

Country	**OJT**	**OJL**	**OJHCA**
New Zealand	42.6	60.8	50.9
United States	37.9	57.8	46.8
Norway	30.6	68.4	45.8
Finland	40.4	51.7	45.7
Netherlands	38.7	49.8	43.9
Canada	33.3	56.9	43.6
Denmark	34.4	52.9	42.6
Chile	30.3	57.0	41.6
Sweden	30.2	57.1	41.6
Germany	32.3	51.1	40.6
United Kindom	34.7	47.3	40.5
Estonia	33.3	47.9	39.9
Israel	26.8	51.5	37.2
Czech Republic	34.7	37.7	36.1
Belgium	26.6	45.0	34.6
Spain	24.5	46.2	33.6
Ireland	27.7	40.5	33.5
Japan	25.3	43.8	33.3
Austria	20.6	49.7	32.0
Slovenia	23.8	39.6	30.7
Slovak Republic	20.0	43.5	29.5
France	15.7	45.5	26.8
Korea	25.4	25.4	25.4
Poland	19.5	31.7	24.9
Lithuania	22.9	26.0	24.4
Italy	12.2	36.2	21.0
Greece	8.1	28.1	15.1
Turkey	10.7	18.5	14.1

Data source: Based on PIAAC data. See Appendix A.

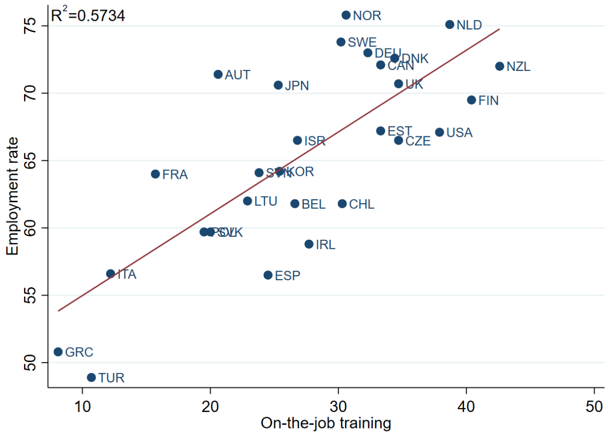
itive relationship between job training participation and aggregate employment documented by the OECD (OECD, 2004).

Our index also shows a positive association between on-the-job training and learning, as it has been empirically documented by Ferreira et al. (2017). We show the raw correlation in Figure 1(b). Taking this one step further, Figure 1(c) shows that the relation between human capital acquired on-the-job and the employment rate is strengthened when we combine training and learning activities as a measure of on-the-job human capital acquisition. Notice that the R^2 from the linear regression between on-the-job human capital and employment is 0.66, which is higher than the 0.57 obtained when we regress on-the-job training and employment in Figure 1(a).[3]

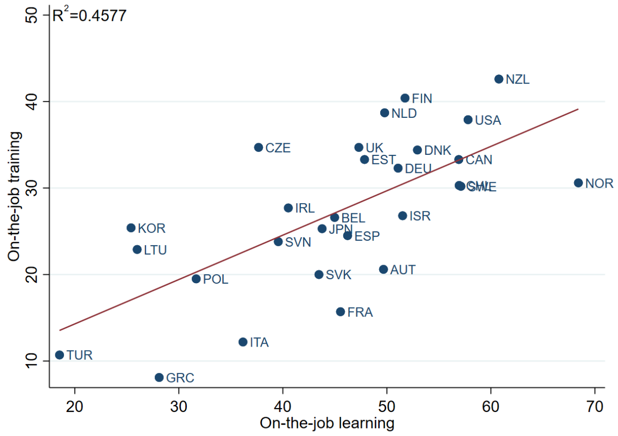
[3]The positive relationship between employment and the OJHCA remains significant even after controlling

Figure 1: Stylized facts: raw correlations between on-the-job human capital and employment

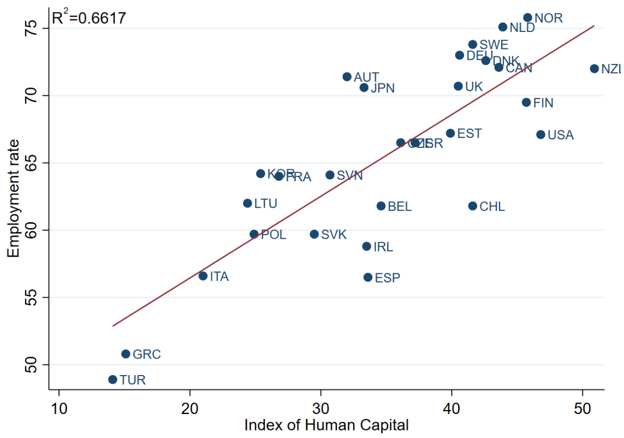
(a) Employment and On-the-job training



(b) On-the-job training and learning



(c) Employment and OJHCA



Data source: OECD-PIAAC and own calculations.

Moreover, a look at the labor market flows suggests that the strong relationship between the OJHCA index and employment mainly occurs throughout the job creation margin. Using annual data of job separation and job finding rates from Garda (2016), we observe a significant relationship between on-the-job human capital and job finding rates. Figure 2(a) shows that the R^2 from the linear regression between on-the-job human capital and job finding rates is 0.29, while the R^2 is equal to zero when we regress job separation rates on OJHCA (Figure 2(b)).[4] This is the main reason why we consider only endogenous job finding rates in our model.

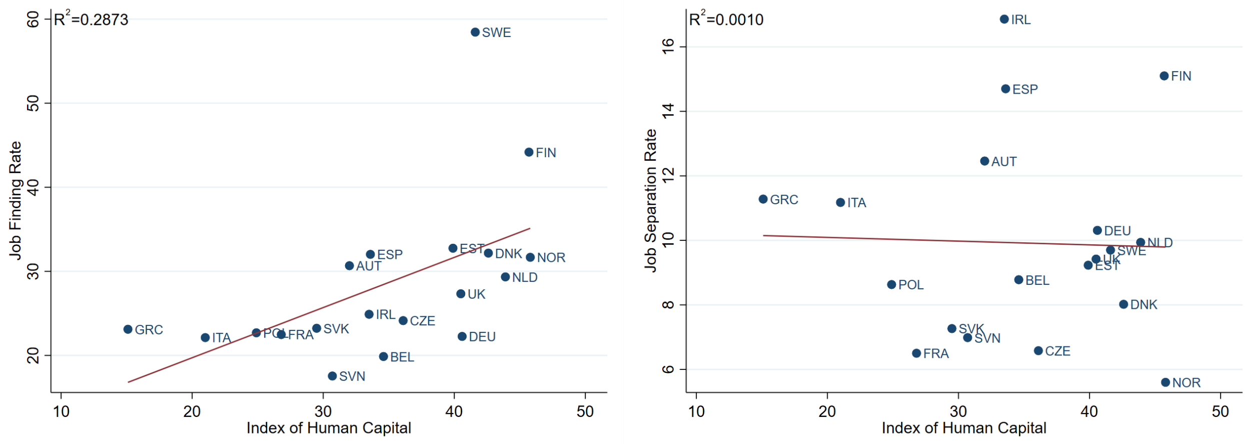
The on-the-job human capital index can also be correlated with other variables such as

for the share of workers with tertiary education as a proxy of the level of formal education.

[4] Garda (2016) calculates the annual job separation (finding) rate as the transition of workers from employment (joblessness) to joblessness (employment) in year t divided by the stock of employees (joblessness) in year $t-1$. Joblessness includes both unemployment and inactivity.

Figure 2: Stylized facts: Raw Correlations between job flows rates and on-the-job Human Capital Acquisition (OJHCA)

(a) Job finding rates and OJHCA (b) Job separation rates and OJHCA



Data source: OECD-PIAAC, Garda (2016) and own calculations.

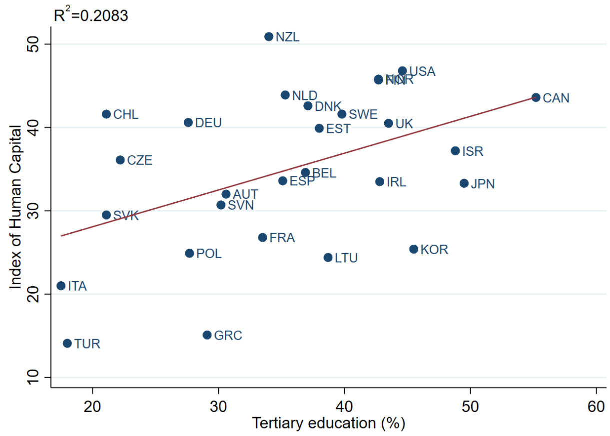
formal education, taxes and productivity. Figure 3(a) shows a positive relationship between tertiary education and the index of on-the-job human capital (the R^2 in the OLS regression is equal to 0.21). Along this line, existing empirical studies of on-the-job training show the presence of strong complementarities between education and training (e.g., Cairó and Cajner, 2018). In turn, Figure 3(b) shows a negative correlation between payroll taxes and OJHCA with a R^2 of 0.23. Comparing to tertiary education, labor productivity displays a weaker positive correlation with respect to OJHCA as shown in Figure 3(c) (with $R^2 = 0.13$).

Finally, Table 2 presents the average values observed in the data for the first (T1), the second (T2) and the third (T3) tertiles of the OJHCA distribution. The values are reported for the OJHCA, the employment rate, labor productivity, tertiary education and employer payroll taxes taken from the OECD database. Table 2 also includes the gaps between the third and second tertiles (T3-T2) and the third and first tertiles (T3-T1) for the OJHCA index, the employment rate, share of tertiary education and taxes, as well as, labor productivity ratios between the third and second tertiles (T3/T2) and the third and first tertiles (T3/T1). Specifically, we compare the results for an average economy in each tertile of the OJHCA index distribution.[5] On average, countries with higher levels of OJHCA, employment and labor productivity have a higher proportion of tertiary educated and lower taxes than countries with lower levels of OJHCA, employment and productivity. Countries in T3 have the highest

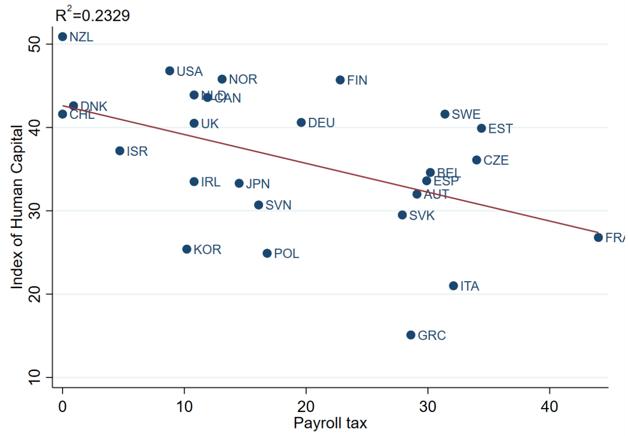
[5] T1 includes 9 countries with the lowest values of OJHCA, T3 includes the highest 9, and T2 includes the 10 countries with the middle values. Ordered from the minimum to the maximum level of on-the-job human capital index, we find Turkey, Greece, Italy, Lithuania, Poland, Korea, France, Slovak Republic, and Slovenia in T1, Austria, Japan, Ireland, Spain, Belgium, Czech Republic, Israel, Estonia, United Kindom, and Germany in T2, and Sweden, Chile, Denmark, Canada, Netherlands, Finland, Norway, United States, and New Zealand in T3. See Table 1 for the values of different countries.

Figure 3: Stylized facts: raw correlations between on-the-job human capital and other variables

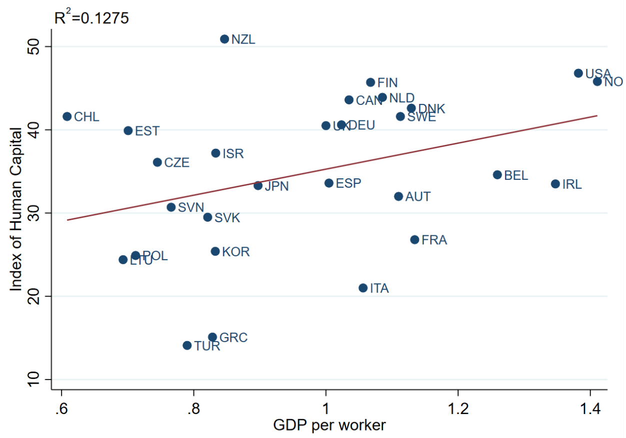
(a) OJHCA and tertiary education



(b) OJHCA and payroll tax



(c) OJHCA and labor productivity



Data source: OECD-PIAAC for OJHCA index, OECD database for labor productivity and tertiary education. Labor productivity values are reported relative to the UK, which we normalized to 1.

proportion of tertiary educated and lowest level of taxes, while countries in T1 have the lowest proportion of tertiary educated and highest taxes. Given these stylised facts, our theoretical model will incorporate employer payroll taxes and education as determinants of the on-the-job human capital investment.

3 The model

The economy consists of a continuum of measure one of risk-neutral, infinitely-lived workers and risk-neutral, infinitely-lived firms. Workers and firms discount future payoffs at a common rate r and capital markets are perfect. Time is continuous. There are employed and non-

Table 2: On-the-job human capital acquisition, productivity, education and taxes: data

	T1	T2	T3	T3-T2	T3-T1
OJHCA Index	24.0	36.4	44.9	8.5	20.9
Employment rate	58.9	66.3	71.1	4.8	12.2
Labor productivity	0.85	0.99	1.08	1.08	1.27
Tertiary educated (%)	29.0	37.5	39.2	1.7	10.2
Payroll taxes	0.251	0.218	0.111	-0.107	-0.140

Note: The on-the-job human capital acquisition index has been calculated using the PIAAC as explained in the Appendix A. Employment rate corresponds to the employment-to-working-age population rate in 2012, aged 15-64 years old, and taken from the OECD database. Labor productivity is also taken from the OECD database, it corresponds to the GDP per worker in 2012 based on purchasing power parity (PPP) in constant 2015 international dollars, and it is reported relative to the UK, which we normalized to 1. Population aged 25-64 years old with tertiary education is taken from the OECD database and is defined as those having completed the highest level of education in 2012. Payroll taxes correspond to the Employer Social Security Contribution in 2012 as percentage of average wage taken from the OECD tax database.

employed (unemployed) workers making up the working age population.[6] All workers have an exogenous level ϵ of formal education, which is complementary to the level of human capital acquired on-the-job (see Cairó and Cajner, 2018). Moreover, some employed workers are fully trained workers (denoted with subscript i) who have already acquired their endogenous level of on-the-job human capital h, while others are newly-employed workers (denoted with subscript e) involved in the training and learning process decided by the firm and the worker, respectively. These newly-hired workers become fully trained at the constant rate ι, where the inverse of ι captures the average duration of training/learning process. On the other hand, the variable h contains both the on-the-job training ξ per matched worker and the level of on-the-job learning l. The former corresponds to organized sessions for on-the-job training or training activities by supervisors and/or co-workers and is decided by the firm, while the latter is decided by the worker and includes their learning of new work-related things through the interaction with co-workers and supervisors and from the tasks they perform on their own. Only fully-trained workers keep their level of human capital when losing their jobs. However, during unemployment spells, these workers lose their on-the-job skills at a constant rate δ. This assumption is similar to Pissarides (1992) and Ljungqvist and Sargent (1998) when they assume that workers lose of human capital when unemployed.

[6] For simplicity, we put together both unemployed and inactive workers. This assumption is not unrealistic since many OECD countries show high flows between employment and inactivity. For example, according to the Eurostat labor market flow statistics, 52% of ins to employment and 60% of outs from employment are from/to inactivity. In turn, and according to the Bureau of Labor Statistics, the flows between employment and inactivity represent more than 70% of the total flows to/from employment in the U.S.

3.1 Job and worker value functions

There is a time-consuming and costly process of matching unemployed workers and job vacancies, which is captured by a standard constant-return-to-scale matching function

$$m(u,v) = m_o u^{\alpha} v^{1-\alpha}, \tag{1}$$

where m_o and α are the matching function parameters, u is the number of unemployed workers, and v the number of vacancies. Among the unemployed, there are u_i workers whose skills acquired in previous jobs are portable to other jobs and u_e workers who do not have acquired portable skills or the skills became obsolete, thus $u = u_i + u_e$. However, all unemployed workers compete for the same jobs. Hence, the aggregate rates at which unemployed workers find jobs, $f(\theta) = m(u,v)/u$, and vacancies are filled, $q(\theta) = m(u,v)/v$, both depend on the vacancy-unemployment ratio θ (labor market tightness). Note as well that $f(\theta) = \theta q(\theta)$, $f'(\theta) > 0$, and $q'(\theta) < 0$.

Vacancies may either be filled or not. If the position is not filled, the firm incurs a flow cost c. A vacancy is filled at the endogenous rate $q(\theta)$, and with probability $\lambda_k = u_k/u$, the position is filled with a worker of type $k = e, i$, yielding a positive value $J_k - V$ during job creation. The value functions J_k and V stand for the value that the firm attributes to a filled and vacant position, respectively.

Each firm has constant returns to scale production technology with labor as the sole production factor. Filled positions can be destroyed at a constant hazard rate s. The firm's output per worker depends on the level of education ϵ, the human capital acquired on-the-job h, and a scale parameter A, that captures the determinants of labor productivity other than those related to the total level of human capital. In this context, h can be understood as the level of human capital required to work in the economy. For entrants who do not have this level of human capital, the firm provides additional resources in the form of training to maximize the value of a filled position, and the worker makes an additional effort in terms of learning to maximize the worker's value of being employed. When the worker becomes an incumbent, training and learning processes are no longer needed in order to be fully productive.

Thus, we assume that a filled job produces $A\epsilon^{\psi}h^{\phi}$ with $\phi, \psi \in (0,1)$. Hence, we also assume decreasing returns to education and to the level of on-the-job human capital $h = (\xi l)^{1/2}$, which is the geometric mean of training ξ and learning l. In turn, firms pay wages w_k, payroll taxes τ, and incur linear training costs $\mu\xi$ during the training process of newly-hired workers, where $\mu \geq 0$ is the marginal cost of training. Hence, the cost for the firm increases with the level of human capital required to work in the economy. In contrast, job positions with fully-trained workers do not incur training costs. The values V and J_k are given by the following expressions:

$$rV = -c + q(\theta)\left[\lambda_i(J_i - V) + \lambda_e(J_e - V)\right], \tag{2}$$

$$rJ_e = A\epsilon^{\psi}h^{\phi} - (1+\tau)w_e - \mu\xi + \iota(J_i - J_e) - s(J_e - V), \tag{3}$$

and

$$rJ_i = A\epsilon^{\psi}h^{\phi} - (1+\tau)w_i - s(J_i - V). \tag{4}$$

Unemployed individuals receive an unemployment benefit b and, at rate $f(\theta)$, find a job that yields net value $W_k - U_k$, where W_k and U_k stand for the value that the worker attributes to employment and unemployment, respectively. Employed workers earn the endogenous wage w_k and newly-hired workers receiving on-the-job training also incur on-the-job learning with flow costs σl. These learning costs can be related, for example, to the leisure forgone when the worker allocates part of their daily rest or breaktime at work to improve their job related skills. The values associated with different worker status – unemployed and employed – are given by the following expressions:

$$rU_e = b + f(\theta)(W_e - U_e), \tag{5}$$

$$rU_i = b + f(\theta)(W_i - U_i) - \delta(U_i - U_e), \tag{6}$$

$$rW_e = w_e - \sigma l + \iota(W_i - W_e) - s(W_e - U_e), \tag{7}$$

$$rW_i = w_i - s(W_i - U_i). \tag{8}$$

According to equations (3)-(4) and (7)-(8), there are two productivity states: incumbents i and entrants e, with flow productivity (net of on-the-job human capital accumulation costs) equal to $y_i = A\epsilon^{\psi}h^{\phi}$ and $y_e = A\epsilon^{\psi}h^{\phi} - \mu\xi - \sigma l$. Notice that on-the-job human capital investment has an immediate effect on labor productivity in line with Bishop (1996). Using cross-sectional firm-level survey for the U.S, Bishop (1996) shows that simultaneous with the training process, the reported average productivity of a newly hired employee increases significantly, by roughly a third during the first quarter and by an additional 32% between the second quarter and the end of the second year of job tenure in the firm.

We also include a free entry condition for vacancies. Hence, we assume that firms open up vacancies until the expected value of doing so becomes zero, that is

$$V = 0. \tag{9}$$

3.2 Wage determination

We assume wages to be the result of bilateral Nash bargaining between workers and employers. The solution is the wage that maximizes the weighted product of the worker's and the firm's net return from the job match. The first-order conditions yield the following equations:

$$(1-\beta)(1+\tau)(W_k - U_k) = \beta J_k, \quad \text{for} \quad k = e, i, \tag{10}$$

where β and $1-\beta$ represent the bargaining power of the worker and the firm, respectively. We define the surplus S_k resulting from the match to be $J_k + (1+\tau)(W_k - U_k)$. To divide this surplus between the firm and the worker, Nash bargaining implies that workers obtain a fraction $\beta/(1+\tau)$ of the total surplus generated from the match (the tax reduces the value of a job for the worker) and firms receive a fraction $1-\beta$, i.e. $(1+\tau)(W_k - U_k) = \beta S_k$ and $J_k - V_k = (1-\beta)S_k$.

3.3 Training and learning investments

To close the model we assume that firms choose the training level ξ, taking as given the learning effort l of workers, that maximizes the firms' value of job position J_e,

$$\max_{\xi} J_e,$$

and workers choose the learning effort l, taking as given the training level ξ, that maximizes the worker's value of being employed W_e,

$$\max_{l} W_e.$$

These decisions take as given the job market tightness θ, and assume that firms and workers internalize the effect of training and learning decisions on wages.[7]

Hence, the timing of the model is as follows: firms decide whether to open a vacancy or not. Unemployed workers are matched randomly to vacancies, and firms learn whether workers need to acquire human capital on-the-job to be fully productive or not. In case workers need to be trained, the surplus of the match is reduced because firms and workers incur in costs to acquire human capital. At a rate ι, the worker becomes fully productive and the costs do not need to be incurred any more. Job separation takes places at a rate s. While unemployed, those who were fully productive in their job can lose their human capital acquired on-the-job at a rate δ.[8] Thus, skills accumulated on-the-job are not necessarily general because of two reasons. Firstly, workers can be separated while they are in a training position and their skills

[7]Notice that $\phi \in (0,1)$ ensures a concave relation between training (learning) and its returns in terms of productivity. When training (learning) is close to zero the returns are infinity and they decrease because of the concavity, while costs to training are linear. Hence, there exist a positive amount of training (learning) chosen by the firm (worker).

[8]The parameter δ can capture both, skill losses due to the unemployment duration (as in Ortego-Marti (2016)) and skill losses because the unemployed worker finds a job in different occupation and the acquired skills were specific to the previous one (as suggested by Kambourov and Manovskii (2009)).

are not portable to another job, and, secondly, fully productive workers can lose their skills while being unemployed.

3.4 Stationary equilibrium

Dynamics of employment

We normalize the working age population to one and consider the fact that individuals are either employed (n) or unemployed (u). There are unemployed workers with portable skills (u_i) and without (u_e), and employees who are fully trained (n_i) and new entrants (n_e),

$$n + u = 1, \tag{11}$$

$$u_i + u_e = u, \tag{12}$$

$$n_i + n_e = n. \tag{13}$$

Then, using (11)-(13) and given the state-contingent ratio of vacancies to unemployment θ, employment n_k and unemployment u_k evolve according to the following backward-looking differential equations:

$$\dot{n_e} = f(\theta)u_e - (\iota + s)n_e, \tag{14}$$

$$\dot{n_i} = f(\theta)u_i + \iota n_e - sn_i, \tag{15}$$

$$\dot{u_i} = sn_i - (\delta + f(\theta))u_i, \tag{16}$$

$$\dot{u_e} = sn_e + \delta u_i - f(\theta)u_e. \tag{17}$$

At equilibrium, $\dot{n_k} = \dot{u_k} = 0$ for $k = e, i$. Thus, we obtain the equilibrium employment rate

$$n = \frac{f(\theta)}{s + f(\theta)}. \tag{18}$$

Surplus

Using equations (4), (5), (6), (8), and (10) we obtain the surplus resulting from the match of an incumbent worker to a job position,[9]

$$S_i = \frac{A\epsilon^{\psi}h^{\phi} - (1+\tau)b - \frac{\delta}{r+\delta}\beta f(\theta)S_e}{r + s + \beta f(\theta)\frac{r}{r+\delta}}. \tag{19}$$

Similarly, using equations (3), (5), (6), (7), and (10), we obtain the surplus resulting from the match of an entrant worker to a job position,

[9] A more detailed exposition of the derivation of stationary equilibrium surpluses, wages, and on-the-job training and learning decisions can be found in Appendix B.

$$S_e = \frac{A\epsilon^{\psi}h^{\phi} - \mu\xi - (1+\tau)(\sigma l + b) + \iota\left(1 + \frac{\beta f(\theta)}{r+\delta}\right)S_i}{r + s + \iota + \left(1 + \frac{\iota}{r+\delta}\right)\beta f(\theta)}. \tag{20}$$

Notice that, payroll taxes not only increase employment opportunity costs by τb (as in Prescott, 2004) but they also increase the implicit cost of learning by $\tau\sigma l$ (equation 20).

Job creation by firms

Using equations (2), (19) and (20), we obtain the job creation condition, which implies that the expected value to the firm of filling a position must, in equilibrium, be equal to the cost of opening the vacancy,

$$\frac{c}{q(\theta)} = \lambda_i(1-\beta)S_i + (1-\lambda_i)(1-\beta)S_e. \tag{21}$$

Since ξ and l increase the surpluses S_i and S_e, a higher level of on-the-job human capital increases the firms expected value, which induces them to post more vacancies. As a result, the labor market tightness θ rises, which in turn increases both, the job finding rate $f(\theta)$ and, according to equation (18), the equilibrium employment rate n.

Equilibrium wage

To find the equilibrium wages of fully-trained workers, we first calculate $W_i - U_i$ using (6) and (8), and then we plug it in (10). After some algebra, we obtain

$$(1+\tau)w_i = \beta A\epsilon^{\psi}h^{\phi} + (1-\beta)(1+\tau)b + (1-\beta)\beta f(\theta)\frac{\delta S_e + rS_i}{r+\delta}. \tag{22}$$

Next, we calculate $W_e - U_e$ using (5) and (7) and then, we plug it in (10). After some manipulation, we obtain the wage of newly-hired workers involved in on-the-job training and learning process

$$(1+\tau)w_e = \beta\left(A\epsilon^{\psi}h^{\phi} - \mu\xi\right) + (1-\beta)(1+\tau)(b+\sigma l) + (1-\beta)\beta f(\theta)\frac{(r+\delta+\iota)S_e - \iota S_i}{r+\delta}. \tag{23}$$

Equations (22) and (23) show that on-the-job learning and training increase workers' wages because they increase workers' productivity. In turn, equation (23) shows that training costs $\mu\xi$ have a direct negative effect on the wages of newly-hired workers involved in on-the-job human capital acquisition because firms transfer a fraction β of these costs to the workers in the form of lower wages. Similarly, learning costs σl have a direct positive effect on w_e since workers transfer a fraction $(1-\beta)$ of the learning costs to the firm.

On-the-job training and on-the-job learning

Workers and firms choose the level of learning and training to maximize the value of a worker and an occupied position given the job market tightness θ. Using (3)- (8), the optimal level of on-the-job training and on-the-job learning are given by the following expressions:

$$\xi = \left(\Omega \frac{\phi}{2} \frac{A\epsilon^{\psi}}{\mu} l^{\frac{\phi}{2}} \right)^{\frac{1}{1-\frac{\phi}{2}}}, \tag{24}$$

$$l = \left(\Omega \frac{\phi}{2} \frac{A\epsilon^{\psi}}{(1+\tau)\sigma} \xi^{\frac{\phi}{2}} \right)^{\frac{1}{1-\frac{\phi}{2}}}, \tag{25}$$

where

$$\Omega \equiv 1 + \frac{\iota}{r+s} + \frac{\iota}{r+s} \frac{s\beta f(\theta)}{(r+\delta)(r+s) + r\beta f(\theta)}.$$

Equations (24) and (25) show that training decisions ξ taken by the firm complement the efforts made by the workers in their on-the-job learning process l. This complementarity implies that training induces workers to increase their learning activities in the workplace to raise their wages, and then, to offset part of the training costs transferred from firms to workers.[10] In turn, the Ω term, in equations (24) and (25), shows that a higher rate ι and a lower rate δ increase the return of both training and learning investments. Finally, note that Ω captures the discounted value of training and learning while the worker is being trained, while the worker is fully trained, and the discounted value of the portability of on-the-job human capital from job to job.

It is straightforward to see that, since formal education ϵ has a direct positive effect on the worker's productivity, then, according to equations (24) and (25) training and learning increase if education raises. ϵ can also be interpreted as a reduction in the implicit marginal costs of training from μ to μ/ϵ^{ψ} and that of learning from σ to σ/ϵ^{ψ}, which leads to higher training and learning investments, and to higher labor productivity. In contrast, payroll taxes increase the marginal cost of learning from σ to $(1+\tau)\sigma$ (equation 25), which reduces the level of learning l and, by complementarity, brings down the level of training provided by the firm ξ (equation 24).

Average labor productivity

Finally, as we mentioned before, there are two productivity states: incumbents i and entrants e, with flow productivity net of on-the-job human capital accumulation costs equal to $y_i = A\epsilon^{\psi}h^{\phi}$ and $y_e = A\epsilon^{\psi}h^{\phi} - \mu\xi - \sigma l$. Thus, the average labor productivity in this economy is equal to

[10] In line with our theoretical model, Ferreira et al. (2018), using the PIAAC data, test whether there is complementarity between training and informal learning. They find that, on average, participation in training activities increases informal learning.

$$\bar{y} = \frac{y_i n_i + y_e n_e}{n} = \frac{A\epsilon^{\psi} h^{\phi} n_i + (A\epsilon^{\psi} h^{\phi} - \mu\xi - \sigma l) n_e}{n}. \tag{26}$$

4 Calibration and simulated results

This section undertakes a quantitative assessment of the role of training and learning investments have on patterns observed in the on-the-job human capital acquisition, employment and productivity. First, we calibrate the model's parameters using the UK as our benchmark economy. Second, we analyze the role of taxes and education to explain the variation in human capital acquired on-the-job, employment and productivity across the OECD countries. Third, we obtain the marginal costs of training and learning in every country that reproduce the levels of on-the-job human capital as observed in the data. Finally, we analyze the impact of a policy that subsidizes on-the-job training activities.

4.1 Calibration

We calibrate the model at a quarterly frequency in order to match it with several empirical facts of the UK economy. Some of the targets and calibrated parameters correspond to the main year of the PIAAC (2012). Thus, our calibration is in line with the on-the-job human capital acquisition index presented in section 2. Table 3 summarizes all the calibrated parameters and presents the steady-state values of the endogenous variables.

The interest rate is set at $r = 0.012$, similar to Shimer (2005). Following Garda (2016) we set quarterly transition rate from employment to joblessness of 2.6% ($s = 0.026$).[11] The matching function's elasticity parameter with respect to unemployment is set at $\alpha = 0.5$, which is in the range of plausible values according to Petrongolo and Pissarides (2001). Payroll taxes are set at $\tau = 0.108$ from OECD data for the UK.

We target an average employment-to-working-age population rate $n = 0.707$ (OECD database) and, to be consistent with equation (18), we obtain a job finding rate of $f(\theta) = 0.063$. Similar to Shimer (2005), we normalize job market tightness to $\theta = 1$ and use the matching function (equation 1) to obtain $m_0 = f(\theta)/\theta^{1-\alpha} = 0.063$.

We normalize the productivity of a fully trained worker ($y_i = A\epsilon^{\psi} h^{\phi} = A\epsilon^{\psi}(\xi l)^{\phi/2} = 1$), and calibrate its components using OECD data for the proportion of tertiary educated ($\epsilon = 43.5\%$), and PIAAC data for the level of on-the-job training ($\xi = 34.7$), learning ($l = 47.3$) and human capital acquisition ($h = 40.5$) in the UK. Konings and Vanormelingen (2015) find that increasing the proportion of workers receiving training by 10 percentage points can increase

[11] The job separation rate is equal to the transition of workers from employment to joblessness in quarter t divided by the stock of employees in quarter $t - 1$. Joblessness includes both unemployment and inactivity. This separation rate is close to $s = 0.032$ that we calculate using data from Figure 1 in Gomes (2012).

productivity by 3.2% in Belgium, similar to the 3% value found by Barron et al. (1989) for the U.S.

Then, we calibrate the residual productivity $A = 0.142$ and $\phi/2 = 0.116$ by solving, simultaneously, the expression $A = y_i / \left(A\epsilon^{\psi}(\xi l)^{\phi/2}\right)$ and the target of a 3% semielasticity of productivity with respect to a 10 percentage points increase in ξ (*target* 1). Thus, the parameter ϕ is equal to 0.232, and the contribution of on-the-job human capital acquisition to labor productivity is $h^{\phi} = 2.36$. In addition, we estimate the elasticity of output per worker with respect to education running the following OLS regression: $\ln(y_c) = a_0 + \psi \ln(\epsilon_c) + \varepsilon_c$, where y_c is the GDP per worker, ϵ_c is tertiary education, ε_c is the error term and sub-index c indicates the OECD country. The estimated parameter of tertiary education ψ is significant at 5% level and it is equal to 0.290.[12] Hence, the contribution of education to the labor productivity is $\epsilon^{\psi} = 2.980$.[13]

Using equations (12), (14), (15), and (16) in equilibrium to obtain the probability to fill a vacancy with a fully-trained worker

$$\lambda_i = \frac{u_i}{u} = \frac{\iota f(\theta)}{\iota f(\theta) + \delta(\iota + s)}. \tag{27}$$

We set a rate $\iota = 0.25$ at which the workers finish the training and learning process implying a 1-year period of training and learning consistent with the evidence presented in Silva and Toledo (2009) using the 1982 Employer Opportunity Pilot Project survey in the U.S. In addition, using the PIACC database we calculate an average wage ratio of incumbent to entrant (with less one year of tenure in the firm) workers equals to $w_i/w_e = 1.34$, and set it as target (*target* 2).

Using wages and unemployment history from the U.S. Panel Study of Income Dynamics, Ortego-Marti (2016) shows that (log) wages are reduced around 1.22% per month of unemployment. Thus, we target the quarterly skill loss at 3.66% (*target* 3). To calibrate the skill loss process in our model, we combine the wage ratio w_i/w_e with the proportion of time among unemployed workers incurring in a wage loss when they find a new job. Hence, the expected quarterly (log) wage loss due to unemployment is

$$\textit{skill loss} = \log\left(\frac{w_i}{w_e}\right) \frac{\left(\frac{1}{f(\theta)} - \frac{1}{\delta}\right)}{\frac{1}{f(\theta)}} \frac{u_i}{u} = \left(1 - \frac{f(\theta)}{\delta}\right) \frac{u_i}{u} \log\left(\frac{w_i}{w_e}\right). \tag{28}$$

Notice that $\log(w_i/w_e)$ is the size of the (log) wage loss, u_i/u is the proportion of unemployed

[12]See the data sources in Table 2. To check the robustness of our estimated results, we run an additional regression replacing tertiary education by the measure of human capital from the Penn World Table 9.1 and we obtain almost the same coefficient (0.299), which is similar to the value of one third suggested by Mankiw et al. (1992) for the contribution of human capital to output.

[13]This elasticity is irrelevant for the benchmark calibration because it simply changes the residual A. However, it plays an important role in the cross-country comparison (see section 4.4).

Table 3: Calibrated parameter values for UK

Parameters	Value	Source/Target
Interest rate, r	0.012	Shimer (2005)
Separation rate, s	0.026	Own calculation, similar to Garda (2016)
Rate of skill loss, δ	0.091	Solves (19)-(25), (27) and *targets* 2-4
Acquisition rate of the OJHCA, ι	0.250	Silva and Toledo (2009)
Matching function scale, m_o	0.063	Solves (1)
Matching function elasticity, α	0.500	Petrongolo and Pissarides (2001)
Workers' bargaining power, β	0.491	Solves (19)-(25), (27) and *targets* 2-4
Productivity residual, A	0.142	Solves $y = Ah^{\phi}\epsilon^{\psi}$ and *target* 1
Output elasticity w.r.t. OJHCA, ϕ	0.232	Solves $y = Ah^{\phi}\epsilon^{\psi}$ and *target* 1
Output elasticity w.r.t. education, ψ	0.290	Own estimation using the OECD database
Marginal training costs, μ	0.030	Solves (19)-(25), (27) and *targets* 2-4
Marginal learning costs, σ	0.020	Solves (19)-(25), (27) and *targets* 2-4
Cost of vacancy, c	0.261	Solves (19)-(25), (27) and *targets* 2-4
Employment opportunity cost, b	0.319	Solves (19)-(25), (27) and *targets* 2-4
Tertiary education, ϵ	0.435	OECD database
Payroll taxes, τ	0.108	OECD database
Targets		
Semielasticty of y with respect to ξ,	0.030	Konings and Vanormelingen (2015) (*target* 1)
Wage ratio, w_i/w_e	1.360	PIAAC (*target* 2)
Skill loss, equation 28	0.037	Ortego-Marti (2016) (*target* 3)
Participation tax rate, $b/\bar{w}$	0.462	Jara et al. (2017) (*target* 4)
Variables		
Employment rate, n	0.707	OECD database
Incumbent labor productivity, y_i	1.000	Normalization
Average labor productivity, $\overline{y}$	0.887	Solves (26)
Labor market tightness, θ	1.000	Normalization
Job finding rate, $f(\theta)$	0.063	Solves (18)
Incumbents unemployed share, λ_i	0.385	Solves (19)-(25), (27) and *targets* 2-4
Entrants joblessness, u_e	0.180	Solves $u_e = 1 - n - u_i$
Incumbents joblessness, u_i	0.113	Solves $u_i = \lambda_i(1 - n)$
Entrants employment, n_e	0.041	Solves $n_e = \delta u_i/\iota$
Incumbents employment, n_i	0.666	Solves $n_i = n - n_e$
Entrants surplus, S_e	6.031	Solves (19)-(25), (27) and *targets* 2-4
Incumbents surplus, S_i	11.61	Solves (19)-(25), (27) and *targets* 2-4
Entrants wage, w_e	0.515	Solves (19)-(25), (27) and *targets* 2-4
Incumbents wage, w_i	0.700	Solves (19)-(25), (27) and *targets* 2-4
On-the-job training index, ξ	34.70	PIAAC
On-the-job learning index, l	47.30	PIAAC
On-the-job human capital index, h	40.50	PIAAC

workers who can suffer a wage loss, $1/f(\theta)$ is the average time a worker spends unemployed, $1/\delta$ is the average time the incumbent (trained) unemployed worker stays unemployed without losing skills. Therefore, $1/f(\theta) - 1/\delta$ is the average time incumbent workers stays unemployed without the skills acquired in the previous job.

Moreover, Jara et al. (2017, in Table 1) calculate a one year participation tax rate (short-run PTR) of 46.2% for all individuals in the UK.[14] Since PTR measures the proportion of earnings kept when workers transition from work into joblessness, we set $b = 0.462\bar{w}$ (*target* 4), where $\bar{w} = (w_i n_i + w_e n_e)/n$.[15]

To jointly calibrate the six remaining parameters of the model (β, δ, b, c, μ, σ), and find the equilibrium values of the five variables ($S_e, S_i, w_e, w_i, \lambda_i$), we use $\xi = 34.7$, $l = 47.3$, $\theta = 1$, the three targets (*target* 2)-(*target* 4), and the eight equilibrium equations (19)-(25) and (27). We obtain the workers' bargaining power $\beta = 0.491$, the skill loss rate $\delta = 0.091$, the unemployment benefits $b = 0.319$, the vacancy costs $c = 0.261$, the marginal costs of training $\mu = 0.030$ and learning $\sigma = 0.020$, the equilibrium wages $w_e = 0.515$ and $w_i = 0.700$ of entrant and incumbent workers, the surpluses $S_e = 6.031$ and $S_i = 11.608$, and the probability to fill a vacancy with a fully-trained worker $\lambda_i = 0.385$.

Finally, the average productivity $\overline{y} = 0.887$ is obtained using equation (26), while the implied shares of unemployed and employed workers are equal to $u_i = (1-n)\lambda_i = 0.113$, $u_e = 1 - n - u_i = 0.180$, $n_e = u_i\delta/\iota = 0.041$, $n_i = n - n_e = 0.666$.

4.2 Quantitative assessment: education and taxes

Firstly, countries are treated as being identical to the benchmark economy except in their proportion of tertiary educated individuals (ϵ) and payroll taxes (τ). To explore the quantitative implications education and payroll taxes have on on-the-job human capital acquisition, employment and productivity, we use the parameters summarized Table 3 for every country and let the variables of interest be determined by the model. Considering the exogenous variation in ϵ and τ, we address the following question: how much of the observed differences in the OJHCA, employment rates, and productivity can be accounted for by the model?

Table 4 presents the simulation results and the values observed in the data and predicted by the model for the first (T1), the second (T2) and the third (T3) tertiles of the OJHCA distribution. The first two rows report the values of taxes (0.251, 0.218, 0.111) and education

[14] The PTR includes unemployment benefits, social assistance benefits and other benefits and pensions such as family benefits and public pensions. A short-run PTR of 100 means that the worker income will remain the same if she is separated from her job and remains jobless for one year, thus, a low work incentive, on contrary, a PTR of 0 indicates a high work incentive. This rate (PTR) it is also known as the effective tax rate of entering employment (for details on the calculation of the PTR see Jara et al. (2017) and OECD database https://stats.oecd.org/viewhtml.aspx?datasetcode=PTR&lang=en).

[15] Notice that the policy parameters τ and b are independent of each other, implying that the government budget constraint is absent in our model.

(29.0, 37.5, 39.2) used to compute the simulations in T1, T2 and T3, respectively. These values are the average proportion of tertiary educated and payroll taxes of the countries in every tertile. The remainder rows report values for the OJHCA, the employment rate, and labor productivity. Table 4 also includes information about the OJHCA and employment gaps between the third and second tertiles (T3-T2) and the third and first tertiles (T3-T1) of the OJHCA, and the productivity ratios between the third and second tertiles (T3/T2) and the third and first tertiles (T3/T1). Specifically, we compare the results for an average economy in each tertile of the OJHCA index distribution.[16]

Table 4: On-the-job human capital acquisition, employment and productivity: data and model

	T1	T2	T3	T3-T2	T3-T1
Tertiary educated (%)	29.0	37.5	39.2	1.7	10.2
Payroll taxes	0.251	0.218	0.111	-0.107	-0.140
Panel A					
OJHCA	T1	T2	T3	T3-T2	T3-T1
Data	24.0	36.4	44.9	8.5	20.9
Model (ϵ, τ)	30.3	35.0	38.5	3.5	8.2
Education	33.5	37.8	38.6	0.8	5.1
Taxes	36.8	37.6	40.4	2.8	3.6
Panel B					
Employment	T1	T2	T3	T3-T2	T3-T1
Data	58.9	66.3	71.1	4.8	12.2
Model (ϵ, τ)	61.9	67.0	69.4	2.5	7.6
Education	65.5	68.9	69.5	0.5	4.0
Taxes	68.4	69.0	70.7	1.7	2.2
Panel C					
Productivity	T1	T2	T3	T3/T2	T3/T1
Data	0.85	0.99	1.08	1.08	1.27
Model (ϵ, τ)	0.83	0.93	0.96	1.03	1.16
Education	0.84	0.94	0.96	1.02	1.13
Taxes	0.98	0.99	1.00	1.01	1.02

Note: Productivity values are reported with respect to the UK, which we normalized to 1.

On average, taxes and education explain almost 40% of the differences observed in OJHCA among tertiles. In particular, in Panel A of Table 4, we observe that the model accounts for 41.2% (3.5 out of 8.5 p.p.) and 39.2% (8.2 out of 20.9 p.p.) of the differences of OJHCA observed in the data between T3 and T2, and between T3 and T1, respectively. Then, we decompose the role of taxes and education separately. Between the first and third tertile, education differences are more pronounced than tax differences: education alone accounts

[16] A robustness check analysis, available from the authors on request, shows that the simulated results remain practically unchanged if we assume, like in Cairó and Cajner (2018), that untrained workers are less productive than trained workers by a fraction $\varphi = 0.80$. The reason behind this result is that the entrants' productivity net of training and investment costs $\tilde{y}_e = \varphi A \epsilon^{\psi} h^{\phi} - \sigma l - \mu \xi$ remains almost unchanged when we recalibrate the marginal costs μ and σ to match the observed values of training and learning in the UK.

for 62.2% (5.1 out of 8.2 p.p.), while taxes account for 43.9% (3.6 out of 8.2 p.p.) of the simulated differences in OJHCA. In contrast, taxes account for 80.0% (2.8 out of 3.5 p.p.) of the differences generated by the model between the third and second tertiles, while education for 22.9% (0.8 out of 3.5 p.p.).[17]

In terms of employment (Panel B of Table 4) taxes and education account for a greater share of the differences. The model accounts for 62.3% (7.6 out of 12.2 p.p.) of the differences between T3 and T1, and 52.1% (2.5 out of 4.8 p.p.) between T3 and T2. Education explains 52.6% (4.0 out of 7.6 p.p.), while taxes account for 28.9% (2.2 out of 7.6 p.p.) of the simulated differences in employment between T3 and T1. In contrast, taxes account for 68.0% (1.7 out of 2.5 p.p.) of the differences generated by the model between the third and second tertiles, while education accounts for 20.0% (0.5 out of 2.5 p.p.).

Productivity results are presented in Panel C of Table 4. The model accounts for 52.3% (16 out of 27 p.p.) of the productivity gap between T3 and T1, with education explaining the 81.3% (13 out of 16 p.p.) of the simulated productivity gap. Besides, the model only accounts for the 37.5% (3 out of 8 p.p.) of the productivity gap between T3 and T2.[18]

4.3 Quantitative assessment: training and learning marginal costs

In the exercise that we perform in this section, we allow countries to differ not only on taxes and education but also in the marginal costs of training and learning to quantify differences in employment and productivity across countries. In this case, countries are treated as being identical to the benchmark economy except in their proportion of tertiary educated individuals ϵ, payroll taxes τ, and marginal costs of learning σ and training μ. We use the parameters summarized in Table 3 and equations (24) and (25) to compute the marginal costs (σ and μ) that generate the levels of learning and training observed in every country.

Table 5 reproduces data values (first row), the simulated values with variation in education and taxes (second row), the simulated values with additional variation in marginal costs of training and learning to reproduce the variation of OJHCA in the data (third row), and the simulated values with variation in education and taxes but without variation in OJHCA (fourth row). The simulated model with variation in marginal costs accounts much better for differences in employment and productivity across tertiles.[19] Specifically, the model accounts for 66.7% of the differences between T3 and T2 (3.2 out of 4.8 p.p.) and 83.4% between T3 and T1 (11.4 out of 12.2 p.p.) of employment differences observed in the data (Panel B).

[17] We can compute the elasticity $(OJHCA_{T3} - OJHCA_{T1})/(X_{T3} - X_{T1}) \times (X_{T1})/(OJHCA_{T1})$, where X is either the proportion of tertiary educated and taxes. We obtain that the elasticity of OJHCA with respect to taxes and education is -0.49 and 0.78, respectively.

[18] The results remain practically unchanged if we consider the labor productivity of the full productive worker instead of the average labor productivity in equation (26).

[19] Table 5 Panel A presents the observed values of the OJHCA for the clarity of the exposition.

Table 5: On-the-job human capital acquisition, employment and productivity: data and model (with and without variation of training and learning marginal costs)

Panel A					
OJHCA	T1	T2	T3	T3-T2	T3-T1
Data	24.0	36.4	44.9	8.5	20.9
Model (ϵ, τ)	30.3	35.0	38.5	3.5	8.2
Model $(\epsilon, \tau, \sigma, \mu)$	24.0	36.4	44.9	8.5	20.9
Model $(\epsilon, \tau)_{h=h_{UK}}$	40.5	40.5	40.5	0.0	0.0
Panel B					
Employment	T1	T2	T3	T3-T2	T3-T1
Data	58.9	66.3	71.1	4.8	12.2
Model (ϵ, τ)	61.9	67.0	69.4	2.5	7.6
Model $(\epsilon, \tau, \sigma, \mu)$	59.1	67.3	70.5	3.2	11.4
Model $(\epsilon, \tau)_{h=h_{UK}}$	63.3	67.7	69.7	2.0	6.3
Panel C					
Productivity	T1	T2	T3	T3/T2	T3/T1
Data	0.85	0.99	1.08	1.08	1.27
Model (ϵ, τ)	0.83	0.93	0.96	1.03	1.16
Model $(\epsilon, \tau, \sigma, \mu)$	0.78	0.93	0.99	1.06	1.27
Model $(\epsilon, \tau)_{h=h_{UK}}$	0.86	0.95	0.96	1.02	1.12

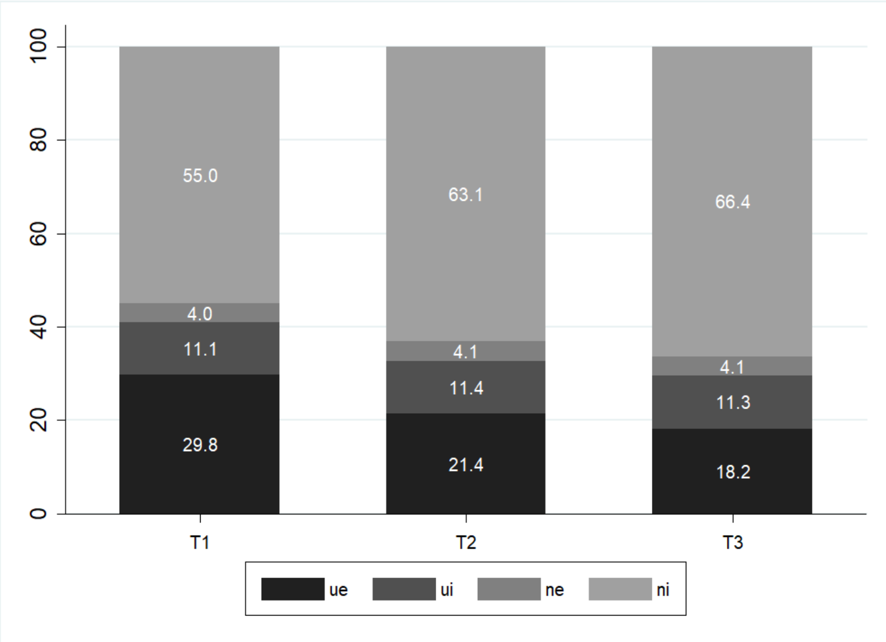
Note: Productivity values are reported with respect to the UK, which we normalized to 1.

Regarding productivity (Panel C), the model explains 75.0% (6 out of 8 p.p.) between T3 and T2 and 100.0% (27 out of 27 p.p.) between T3 and T1 of productivity gaps.

To know whether the channel of OJHCA is important to quantitatively explain employment and productivity differences, in last row of Panels B and C of Table 5, we consider the case with constant OJHCA in the model simulations. We obtain that the direct effect of education and taxes on employment are 2.0 out of the 3.2 points between T3 and T2, and 6.3 out of 11.4 between T3 and T1, which implies that OJHCA explains 37.5% and 44.7% of the simulated employment differences between T3 and T2 and between T3 and T1, respectively. Hence, this quantitative exercise reveals that around forty percent of the employment differences between countries, are consequence of human capital acquisition at the workplace. Regarding productivity (Panel C), the direct effect of education and taxes are 2 out of 6 p.p. (33.3%) between T3 and T2, and 12 out of 27 p.p. (44.4%) between T1 and T3, implying that OJHCA accounts for the 66.7% and 55.6% of the simulated productivity gaps.

Finally, Figure 4 shows the model distribution of workers skills across tertiles. Countries in T1 with a low level of OJHCA and low job finding rates show a higher proportion of jobless workers without skills ($u_e = 0.298$) than countries in T3 with a high level of OJHCA and high job finding rates ($u_e = 0.182$). In contrast, T3 countries have a higher proportion of fully trained employees n_i than T1 countries (0.664 and 0.550, respectively). According to our model, countries in the third tertile, with a high proportion of highly educated workers and low taxes, have more incentives to invest in on-the-job human capital than countries in the first tertile, with low proportion of highly educated workers and high taxes. Thus, unemployed workers find jobs more quickly as OJHCA increases, which reduces the time in jobless spells (and skill losses) and increases the proportion of fully trained employees in equilibrium.

Figure 4: Distribution of workers by skills across tertiles



4.4 Quantitative assessment: cross-country differences

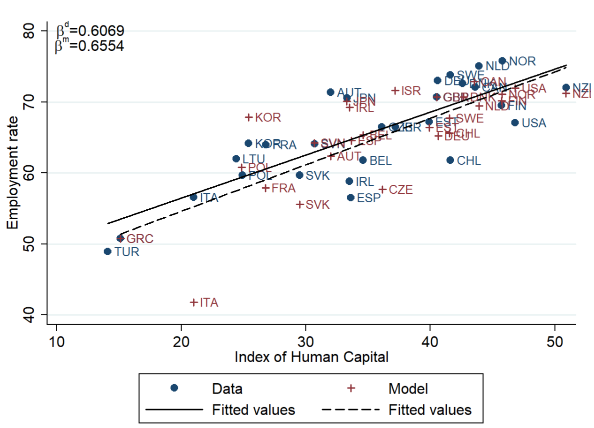
Following the approach of the previous section with variation in education, taxes and marginal costs of training and learning, we now turn to look at country-level predictions. Hence, we take the values of tertiary educated and taxes for every country, and find marginal costs of training and learning that reproduce the values of training and learning in every country.[20] Figure 5(a) shows the data versus the simulated values of employment rates, while Figures 5(b) and 5(c) show the values of productivity and job finding rates, respectively. In general, the model performs well in explaining the positive relationship observed in the data between on-the-job human capital and employment, productivity and job finding rates. Moreover, the model is able to match the average differences in job finding, employment rates and productivity between different levels of OJHCA. In particular, the slopes of the regression line of the data and the simulated values are 0.607 and 0.655 for employment and 0.0014 and 0.0012 for job finding rates. Similarly, the simulated slope for labor productivity (0.001) is similar to the one obtained from the data (.0082). Moreover, 16 out of the 26 countries, show a difference

[20] Table C.1 in the Appendix shows the simulated values and estimated marginal costs . The number of model simulations is lower than observations because there is no tax data for Lithuania and Turkey.

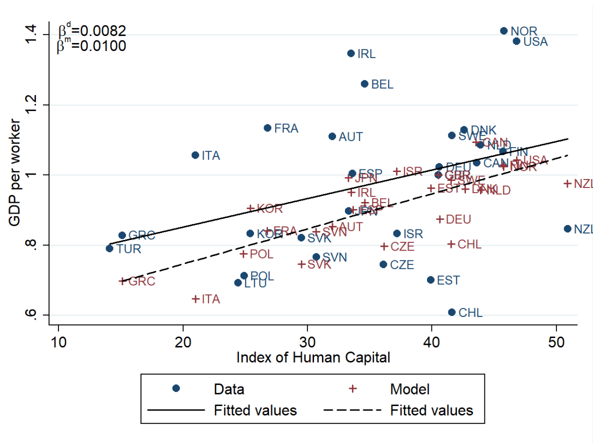
between the simulated and the actual values of employment lower than 5 p.p.

Figure 5: Employment and productivity against OJHCA: data and model

(a) Employment and OJHCA



(b) GDP per worker and OJHCA



(c) Job finding rate and OJHCA

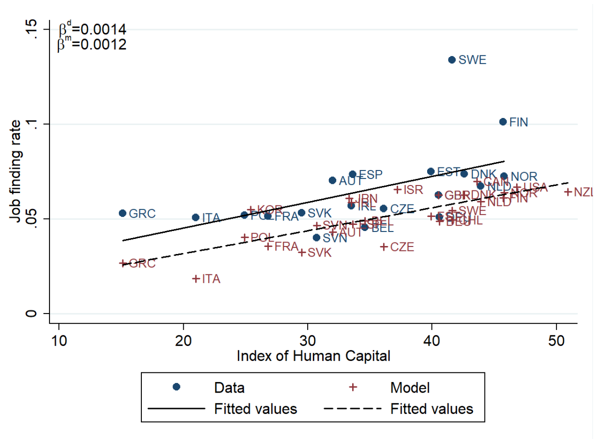


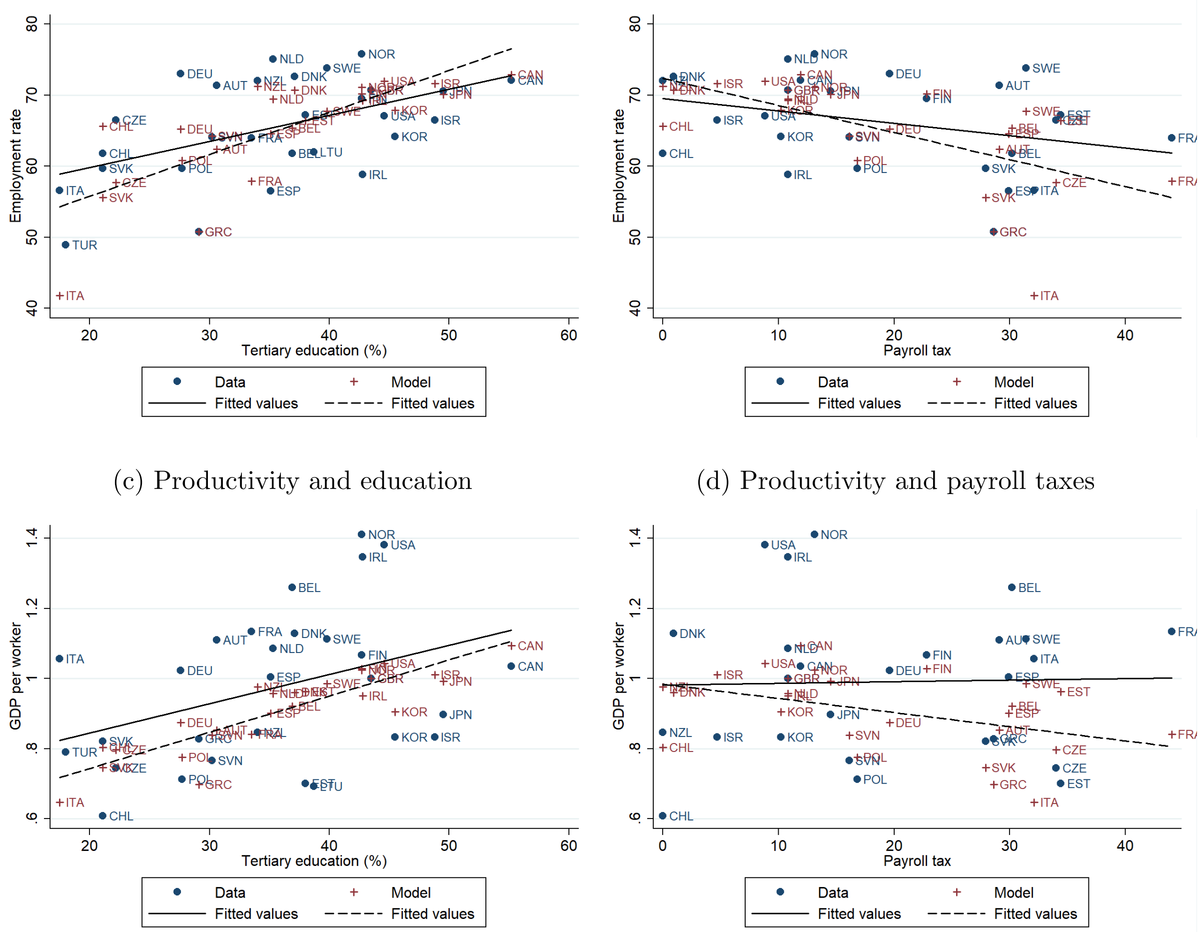
Figure 6 shows the relationships between education and taxes with employment and productivity. The model simulations perform quite well to reproduce the relationships between the share of tertiary educated and productivity in Figure 6(c), although model variation is lower than data. In terms of taxes, the model introduces a small negative relationship between taxes and productivity (Figure 6(d)) that is not present in the data. With regard to the correlation with employment, the model performs well to reproduce the signs of the relationships in both cases of education and taxes (Figure 6(a) and (b)).

In the Appendix, Figure C.1 shows the simulated model distribution of workers skills across countries. It reflects a high degree of dispersion in the skills distribution. For example, on one side, we find Anglo-Saxon and Scandinavian countries with more than 65% of fully trained employees n_i and, on the other side, this group of workers represents 47% in Greece and 38% in Italy. Hence, Anglo-Saxon and Scandinavian countries with a high proportion of highly educated workers and low taxes, have more incentives to invest in on-the-job human capital than countries such as Italy and Greece, with a low proportion of highly educated workers and high taxes. Hence, in countries with high OJHCA, unemployed workers find jobs more quickly, which reduces the skill losses while unemployed and increases the proportion of fully trained employees.

Figure 6: Employment and productivity against education and payroll taxes: data and model

(a) Employment and education (b) Employment and payroll taxes

(c) Productivity and education (d) Productivity and payroll taxes



4.5 Quantitative assessment: subsidizing training costs

Finally, we perform two different exercises to analize the effects of a policy that subsidizes training activities.[21] First, we analyze the effects of adjusting the costs of training in our benchmark economy (UK). Second, since the effect of the policies are likely to be different in countries with flexible and sclerotic labor markets, we recalibrate the model and simulate the impact of reducing the training costs in both economies, with high (flexible) and low (sclerotic) worker flow rates.

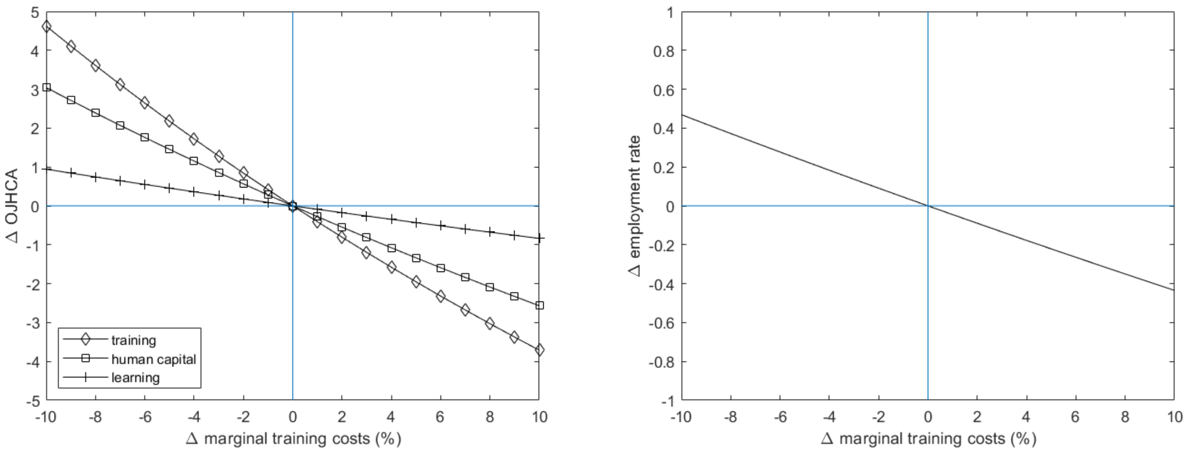
[21] We abstract from the government sector and implicitly assume that these policies are financed with lump sum taxes and the associated effects are of second order.

The first exercise considers the UK as benchmark economy and we modify the marginal cost of training μ to shed light on the effects of subsidizing training costs. In contrast to the previous exercises, we do not alter marginal learning costs. Figure 7(a) shows the changes in training, learning and human capital predicted by the model, when the marginal costs of training are modified exogenously. Increasing marginal cost of training reduces human capital, not only because training falls but also because learning decreases, whereas the opposite is true when reducing marginal cost of training. The complementarity between training and learning observed in the data and captured in equations (24) and (25) of the model, explains the relationships of our simulations. Hence, our model is able to rationalize the fact that policies targeted at reducing marginal training costs μ can help accumulate human capital, raise labor productivity and increase the employment rate. More precisely, our simulations show that a 10% reduction in marginal costs of training generates an increase of 3.0 points in the OJHCA index and, as a result, a 0.46 p.p. increase in the employment rate (see Figure 7(b)).

Figure 7: OJHCA and employment against marginal training costs

(a) OJHCA and marginal costs

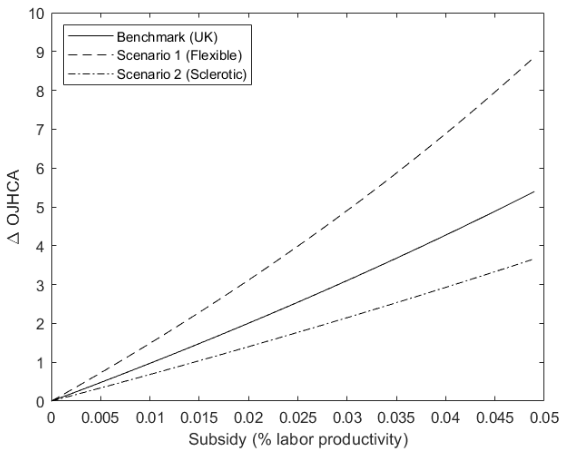
(b) Employment and marginal costs



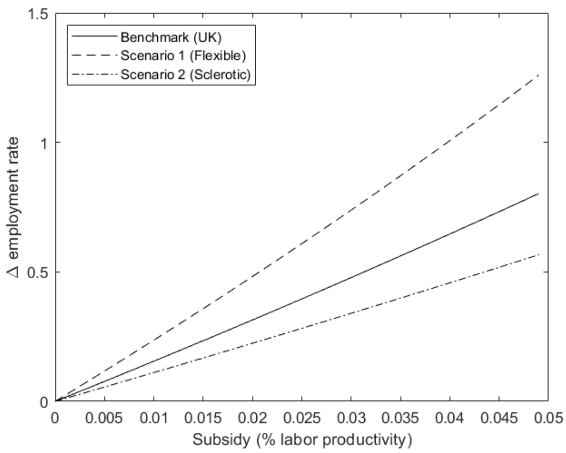
Similar to Elsby et al. (2013), Garda (2016) finds that the UK is an average OECD country in terms of job separation and job finding rates. More in detail, countries with highly flexible labor markets such as the US and Northern European countries have double job separation and job finding rates than the UK, while countries with sclerotic labor markets such as Continental European countries have half of the values, on average. Since the policy effects are likely to be different in countries with flexible and sclerotic labor markets, we next simulate the impact of reducing the marginal training costs in these two types of labor markets. Specifically, we consider a subsidy that reduces the cost of training up to 5% of labor productivity in two different scenarios: scenario 1 with job separation and finding rates which are two times higher than those of our benchmark economy (UK); and scenario 2 with job finding and separation rates which are half those of the benchmark. We recalibrate the parameters values to match the same targets of our benchmark economy, implying that the marginal costs of training and learning do change to match the data. As a result, training and learning marginal costs are around two times higher in scenario 2 than in scenario 1. Figures 8(a) and 8(b) show that OJHCA and employment increase relatively more in a flexible labor market (scenario 1) than in a more sclerotic labor market (scenario 2) due to the subsidy. In countries with low labor market flows (Scenario 2), jobless workers will experience greater skill losses than jobless workers in more flexible labor markets (Scenario 1) due to a longer time spent in unemployment.

Figure 8: The impact of a training subsidy in different labor markets

(a) OJHCA and training subsidy



(b) Employment and training subsidy



5 Conclusion

In a globalized world competitiveness is a key element for economic development and on-the-job human capital acquisition can be considered as an essential tool. This happens in a context in which workforce skills are increasingly gaining in importance, thus requiring firms and workers alike to adapt to the use of more complex technologies. Against this backdrop, it is crucial improve the understanding of the channels trough which the different components of human capital (including, formal on-the-job training and workers' on-the-job learning) affect labor market outcomes.

In this paper, we explore the role of on-the-job human capital acquisition in explaining differences in employment among OECD countries. We build an index of on-the-job Human Capital Acquisition for 28 OECD economies using PIAAC data. The index, which combines formal on-the-job training and informal learning in the workplace, reveals substantial variation across countries. We document a positive correlation between the index and the employment rate. On top of that, we show that the two components of the human capital index, on-the-job training and on-the-job learning, have a strong positive correlation.

To explain these raw stylized facts, we build a search and matching model incorporating on-the-job human capital acquisition that depends on both on-the-job training determined by firms and workers' decisions regarding on-the-job learning. We consider two important factors that may affect human capital acquisition, productivity and employment: formal education and payroll taxes. While formal education increases human capital investments, because it raises productivity, payroll taxes decrease human capital investments because the marginal cost of learning rises.

To quantify the employment effects skills acquired in the workplace have, we calibrate the model to the UK economy. We decompose the effect of education and payroll taxes to determine differences in on-the-job human capital acquisition, productivity, and employment. We find that taxes and education explain almost 40% of the variation in on-the-job human capital acquisition and around 60% of employment. We also find that payroll taxes contribute more than education to explain the differences between countries in the high part of the distribution of the human capital index, while education plays a greater role than payroll taxes in explaining the differences between countries in the bottom part of the distribution. Moreover, the quantitative exercise reveals that human capital acquired on the job accounts for around forty of the employment differences. Additionally, we adjust the learning and training marginal costs to match the observed cross-country levels in the human capital index, which enables the model to almost reproduce differences in productivity and employment rates across countries.

Finally, we evaluate a public policy consisting in a subsidy of the marginal costs of training with the aim to foster human capital investment and employment. The subsidy not only encourages training, but also boosts workers to learn in the workplace, thus further increasing their on-the-job acquired human capital. We find that a 10% cut in marginal costs of training would increase the share of trained workers by 3.0 p.p., workers learning in their workplace by 1 p.p., and the employment rate by almost 0.5 p.p. We also find that a training subsidy has a lower impact in a sclerotic labor market than in flexible one, the main reason is that workers spend more time in unemployment in an economy with low labor market flows.

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Appendix

A Construction of On-the-job human capital acquisition index

We use three variables from the PIAAC survey to build our **On-the-Job Human Capital Acquisition Index**. First, the on-the-job training (OJT) variable which measures whether the worker claims to have attended (or not) formal training sessions, either organized in the workplace or provided by their supervisors/colleagues over the preceding 12 months.[22] Second, using two qualitative variables, we build an on-the-job learning index (OJL). Specifically, we consider two variables from the survey: i) how often workers declare themselves as having learned new work-related competencies from co-workers or supervisors ("learning from co-workers") and, ii) how often their jobs involve learning-by-doing from the tasks that they perform ("learning-by-doing").[23] In all three cases, we normalize these indexes by considering the different scales of the raw data before integrating them into the OJHCA index.[24].

On-the-job training index

The OJT index measures just how widespread formal training activities are on a country level (extensive margin). Specifically, in building this index, we draw on responses to the following question in the PIAAC survey:

"**During the last 12 months, have you attended any organized sessions for on-the-job training or training by supervisors or co-workers**?"

Given that the answer to this question is either "yes" or "no", we can compute the OJT index as the percentage of individuals who have received on-the-job training in the last 12 months:

$$Index\ OJT = \frac{yes}{total} \times 100.$$

Figure A.1(a) presents the histogram for the OJT index.

[22]This question corresponds to the B_Q12c according to the PIAAC questionnaire (PIAAC, 2010).

[23]These two questions correspond to D_Q13a and D_Q13a according to the PIAAC questionaire. See PIAAC (2010) for further details about the questionnaire.

[24]Owing to problems of data availability, we had to exclude five countries from our sample. Thus, our final sample is made up of 28 of the 33 OECD countries surveyed. Austria, Belgium, Canada, Chile, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Ireland, Israel, Italy, Japan, Korea, Lithuania, Netherlands, New Zealand, Norway, Poland, Slovak Republic, Slovenia, Spain, Sweden, Turkey, United Kingdom and United States

On-the-job learning index

In building this index, we draw on responses to a further two PIAAC questions:

1. "**In your own job, how often do you learn new work-related things from co-workers or supervisors?**"
2. "**How often does your job involve learning-by-doing from the tasks you perform?**"

The possible answers to both questions are as follows: Never; Less than once a month; Less than once a week but at least once a month; At least once a week but not every day; Every day. Then, we compute the two indexes as the percentage of individuals participating in these activities at least once a week.

$$Index\ learning\ from\, coworkers\ =\ \frac{at\ least\ once\ a\ week}{total} \times 100,$$

$$Index\ learning\ by\, doing\ =\ \frac{at\ least\ once\ a\ week}{total} \times 100.$$

These partial indexes are then integrated to compute the OJL index as the geometric mean.

$$Index\ OJL\ =\ \sqrt{Index\ learning\ from\, coworkers\ \times\ Index\ learning\, by\ doing}.$$

Figure A.1(b) presents the histogram for the OJL index.

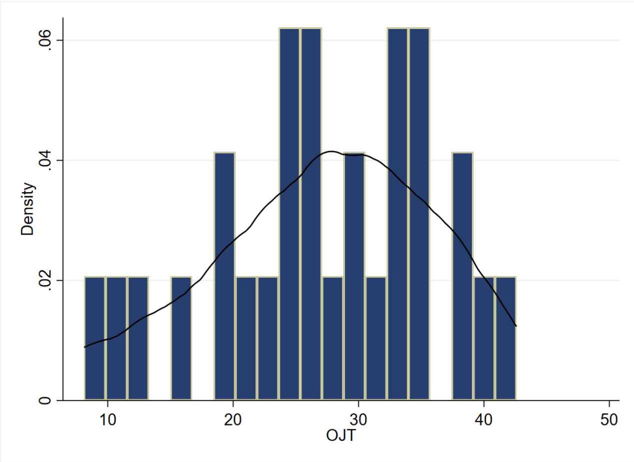
On-the-job human capital acquisition

Finally, we integrate the training and learning indexes (OJT and OJL) to compute the OJHCA index by taking the geometric mean of these two indexes.[25]

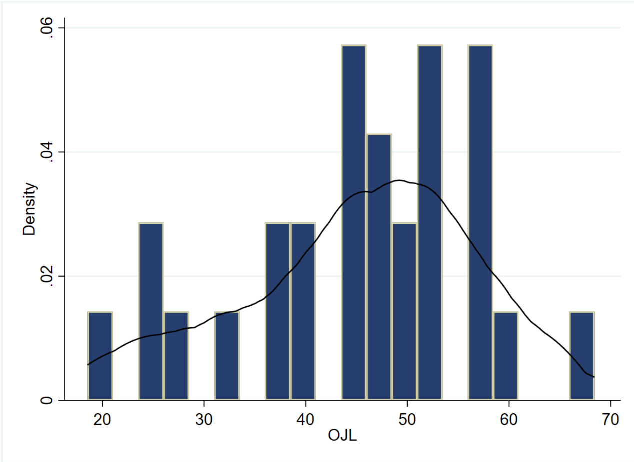
[25] Our OJHCA index aggregates the intermediate indexes using the geometric mean. It means that implicitly we are assuming specific weights (0.5, 0.25 and 0.25) for the different components. However, the results obtained were very similar when using the weights suggested by the principal component analyses (PCA). Specifically, we find a very strong correlation (98.6%) between the OJHCA index using PCA weights and our original OJHCA index using the geometric mean.

Figure A.1: OJT & OJL & OJHCA

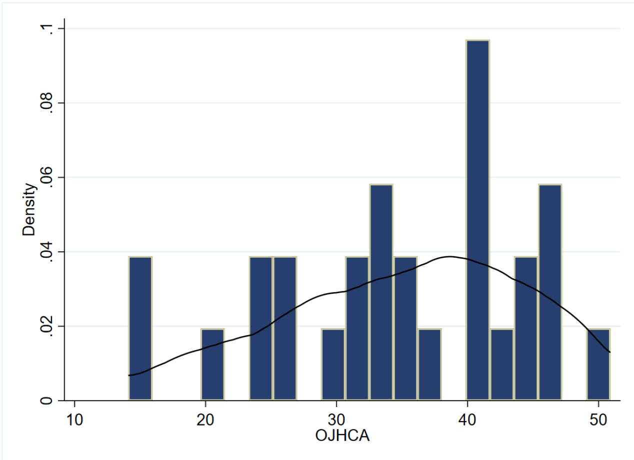
(a) On-the-job training index



(b) On-the-job learning index



(c) On-the job human capital acquisition index



Data source: OECD-PIAAC, own calculations.

$$Index\ OJHCA = \sqrt{Index\,OJT \times Index\ OJL}.$$

Figure A.1(c) shows that the OJHCA index varies considerably across the OECD countries, reflecting the high degree of dispersion in both of its components (the OJT and OJL indexes).

B Stationary equilibrium equations

This section describes the steps taken to obtain the equations in section 3.4.

B.1 Surplus

To obtain the surplus expressions (19) and (20), first notice that from (5), (6), and the surplus sharing rule $(1+\tau)(W_k - U_k) = \beta S_k$ for $k = e, i$, we obtain

$$(1+\tau)(U_i - U_e) = \frac{\beta f(\theta)(S_i - S_e)}{r+\delta}. \tag{B.1}$$

Then, plugging in expressions (4), (6) and (8) into $S_i = J_i + (1+\tau)(W_i - U_i)$, it follows that

$$\begin{aligned} rS_i &= r(J_i + (1+\tau)(W_i - rU_i)) \\ &= y - (1+\tau)b - s(J_i + (1+\tau)(W_i - rU_i)) - \beta f(\theta)S_i + \delta(1+\tau)(U_i - U_e), \end{aligned} \tag{B.2}$$

where $y = A\epsilon^{\psi}\left(\xi l\right)^{\frac{\phi}{2}}$. Then, plugging in (B.1) into (B.2) and rearranging terms we obtain

$$S_i = \frac{A\epsilon^{\psi}\left(\xi l\right)^{\frac{\phi}{2}} - (1+\tau)b - \frac{\delta}{r+\delta}\beta f(\theta)S_e}{r + s + \beta f(\theta)\frac{r}{r+\delta}}.$$

Similarly, we use (3), (5), (7), (B.1) and $S_k = J_k + (1+\tau)(W_k - U_k)$ for $k = e, i$ to obtain

$$\begin{aligned} rS_e &= y - \mu\xi - (1+\tau)\sigma l - (1+\tau)b - (s + \beta f(\theta))S_e + \iota(J_i - J_e + (1+\tau)(W_i - W_e)) \\ &= y - \mu\xi - (1+\tau)(\sigma l + b) - (s + \beta f(\theta))S_e + \iota\left(1 + \frac{\beta f(\theta)}{r+\delta}\right)(S_i - S_e), \end{aligned}$$

which can be rewritten as

$$S_e = \frac{A\epsilon^{\psi}\left(\xi l\right)^{\frac{\phi}{2}} - \mu\xi - (1+\tau)(\sigma l + b) + \iota\left(1 + \frac{\beta f(\theta)}{r+\delta}\right)S_i}{r + s + \iota + \beta f(\theta)\left(1 + \frac{\iota}{r+\delta}\right)}.$$

B.2 Wages

To obtain wage expressions (22) and (23), we plug (4) and (8) in (10) and obtain

$$\beta J_i = (1-\beta)(1+\tau)(W_i - U_i) \Leftrightarrow \beta\left(\frac{y - (1+\tau)w_i}{r+s}\right) = (1-\beta)(1+\tau)\left(\frac{w_i - rU_i}{r+s}\right).$$

Thus,

$$
\begin{aligned}
(1+\tau)w_i &= \beta y + (1-\beta)r(1+\tau)U_i \\
&= \beta y + (1-\beta)(1+\tau)(b + f(\theta)(W_i - U_i) - \delta(U_i - U_e)) \\
&= \beta y + (1-\beta)\left((1+\tau)b + \beta f(\theta)S_i - \frac{\delta}{r+\delta}\beta f(\theta)(S_i - S_e)\right) \\
&= \beta y + (1-\beta)(1+\tau)b + \beta(1-\beta)\frac{f(\theta)}{r+\delta}(\delta S_e + rS_i),
\end{aligned}
$$

where we use (6) and (B.1) to go from the first to the second and third lines, respectively.

Similarly, we use (3), (7), and (10) and obtain

$$
\begin{aligned}
&\beta J_e = (1-\beta)(1+\tau)(W_e - U_e) \Leftrightarrow \\
&\beta\left(\frac{y-(1+\tau)w_e - \mu\xi + \iota J_i}{r+s+\iota}\right) = (1-\beta)(1+\tau)\left(\frac{w_e - \sigma l + \iota W_i - (r+\iota)U_e}{r+s+\iota}\right).
\end{aligned}
$$

Next, we rewrite it and use (5) to obtain

$$
\begin{aligned}
(1+\tau)w_e =& \beta(y-\mu\xi) + (1-\beta)(1+\tau)(b+\sigma l + f(\theta)(W_e - U_e)) \\
& + \iota(\beta J_i - (1-\beta)(1+\tau)(W_i - U_e)) \\
=& \beta(y-\mu\xi) + (1-\beta)((1+\tau)(b+\sigma l) + \beta f(\theta)S_e) \\
& + \iota(\beta J_i - (1-\beta)(1+\tau)(W_i - U_e)) \\
=& \beta(y-\mu\xi) + (1-\beta)((1+\tau)(b+\sigma l) + \beta f(\theta)S_e) \\
& + \iota(1-\beta)(1+\tau)(U_e - U_i) \\
=& \beta(y-\mu\xi) + (1-\beta)(1+\tau)(b+\sigma l) \\
& + \beta(1-\beta)\frac{f(\theta)}{r+\delta}((r+\delta+\iota)S_e - \iota S_i)
\end{aligned}
\tag{B.3}
$$

where we use $(1+\tau)(W_e - U_e) = \beta S_e$, (10) and (B.1) and to go from the first to the second, third and fourth lines, respectively.

B.3 On-the-job training and learning

Firms choose training ξ to maximize the value of an occupied job position J_e,

$$
\arg\max_{\xi} J_e = \arg\max_{\xi} y - (1+\tau)w_e - \mu\xi + \iota J_i, \tag{B.4}
$$

where $y = A\epsilon^{\psi}(\xi l)^{\frac{\phi}{2}}$. Notice that since $(1-\beta)S_e = J_e$, then $\partial S_e/\partial\xi = 0$ because the first order condition (FOC) to the maximization problem implies $\partial J_e/\partial\xi = 0$. Similarly, taking into account that $\partial U_e/\partial\xi = 0$, then $\partial W_e/\partial\xi = 0$. Next, we obtain from expression (B.3) above that

$$(1+\tau)\frac{\partial w_e}{\partial\xi} = \beta\left(\frac{\partial y}{\partial\xi} - \mu + \iota\frac{\partial J_i}{\partial\xi}\right) - (1-\beta)(1+\tau)\iota\frac{\partial W_i}{\partial\xi}.$$

Then, the FOC to the maximization problem (B.4) simplifies to

$$\begin{aligned}\mu = \frac{\partial y}{\partial\xi} + \iota\left(\frac{\partial J_i}{\partial\xi} + (1+\tau)\frac{\partial W_i}{\partial\xi}\right) &= \left(1+\frac{\iota}{r+s}\right)\frac{\partial y}{\partial\xi} + \frac{\iota s(1+\tau)}{r+s}\frac{\partial U_i}{\partial\xi}\\ &= \left(1+\frac{\iota}{r+s} + \frac{\iota s}{r+s}\frac{f(\theta)}{r+\delta}\frac{\beta}{r+s+\frac{r\beta f(\theta)}{r+\delta}}\right)\frac{\partial y}{\partial\xi} = \Omega\frac{\partial y}{\partial\xi},\end{aligned}\tag{B.5}$$

where we use

$$\frac{\partial J_i}{\partial\xi} + (1+\tau)\frac{\partial W_i}{\partial\xi} = \frac{1}{r+s}\left(\frac{\partial y}{\partial\xi} - (1+\tau)\frac{\partial w_i}{\partial\xi}\right) + \frac{1+\tau}{r+s}\left(\frac{\partial w_i}{\partial\xi} + s\frac{\partial U_i}{\partial\xi}\right),$$

$$(1+\tau)\frac{\partial U_i}{\partial\xi} = \frac{\beta f(\theta)}{r+\delta}\frac{\partial S_i}{\partial\xi} = \frac{\beta f(\theta)}{r+\delta}\frac{1}{r+s+\beta f(\theta)\frac{r}{r+\delta}}\frac{\partial y}{\partial\xi},\tag{B.6}$$

and

$$\Omega \equiv 1 + \frac{\iota}{r+s} + \frac{\iota s}{r+s}\frac{f(\theta)}{r+\delta}\frac{\beta}{r+s+\frac{r\beta f(\theta)}{r+\delta}}.$$

Expression (B.6) follows from the derivative of (B.1) with respect to ξ and taking into account that $\partial S_e/\partial\xi = 0$ and $\partial U_e/\partial\xi = 0$; Ω captures the value of training over productivity while the worker is being trained, while the worker is fully trained, and the portability of skills to other jobs. We finally obtain (24) in the text plugging in the derivative of output y with respect to training ξ into equation (B.5) above

$$\mu = \Omega\frac{\partial y}{\partial\xi} \Leftrightarrow \mu = \Omega A\epsilon^{\psi}\frac{\phi}{2}\xi^{\frac{\phi}{2}-1}l^{\frac{\phi}{2}} \Leftrightarrow \xi = \left(\Omega\frac{\phi}{2}\frac{A\epsilon^{\psi}}{\mu}l^{\frac{\phi}{2}}\right)^{\frac{1}{1-\frac{\phi}{2}}}.$$

Equivalent to firms choosing training, workers choose learning l to maximize the value of a working position W_e,

$$\arg\max_{l} W_e = \arg\max_{\xi} w_e - \sigma l + \iota W_i + sU_e.\tag{B.7}$$

Notice that since $\beta S_e = W_e - U_e$, then $\partial S_e/\partial l = 0$, because the FOC to (B.7) implies $\partial W_e/\partial l = 0$, and $\partial U_e/\partial l = 0$. Hence, from expression (B.3) we obtain

$$(1+\tau)\frac{\partial w_e}{\partial l} = \beta\left(\frac{\partial y}{\partial l} + \iota\frac{\partial J_i}{\partial l}\right) + (1-\beta)(1+\tau)\left(\sigma - \iota\frac{\partial W_i}{\partial l}\right).$$

Next, the FOC to the maximization problem (B.7) simplifies to

$$(1+\tau)\sigma = \frac{\partial y}{\partial l} + \iota\left(\frac{\partial J_i}{\partial l} + (1+\tau)\frac{\partial W_i}{\partial l}\right) = \left(1+\frac{\iota}{r+s}\right)\frac{\partial y}{\partial l} + \frac{\iota s(1+\tau)}{r+s}\frac{\partial U_i}{\partial l} = \Omega\frac{\partial y}{\partial l},$$

where we use

$$\frac{\partial J_i}{\partial l} + (1+\tau)\frac{\partial W_i}{\partial l} = \frac{1}{r+s}\left(\frac{\partial y}{\partial l} - (1+\tau)\frac{\partial w_i}{\partial l}\right) + \frac{1+\tau}{r+s}\left(\frac{\partial w_i}{\partial l} + s\frac{\partial U_i}{\partial l}\right),$$

and

$$(1+\tau)\frac{\partial U_i}{\partial l} = \frac{\beta f(\theta)}{r+\delta}\frac{\partial S_i}{\partial l} = \frac{\beta f(\theta)}{r+\delta}\frac{1}{r+s+\beta f(\theta)\frac{r}{r+\delta}}\frac{\partial y}{\partial l}.$$

Finally, expression (25) in the text follows

$$(1+\tau)\sigma = \Omega\frac{\partial y}{\partial l} \Leftrightarrow l = \left(\Omega\frac{\phi}{2}\frac{A\epsilon^{\psi}}{(1+\tau)\sigma}\xi^{\frac{\phi}{2}}\right)^{\frac{1}{1-\frac{\phi}{2}}}.$$

C Figures and Tables

Figure C.1: Distribution of workers by skills across countries

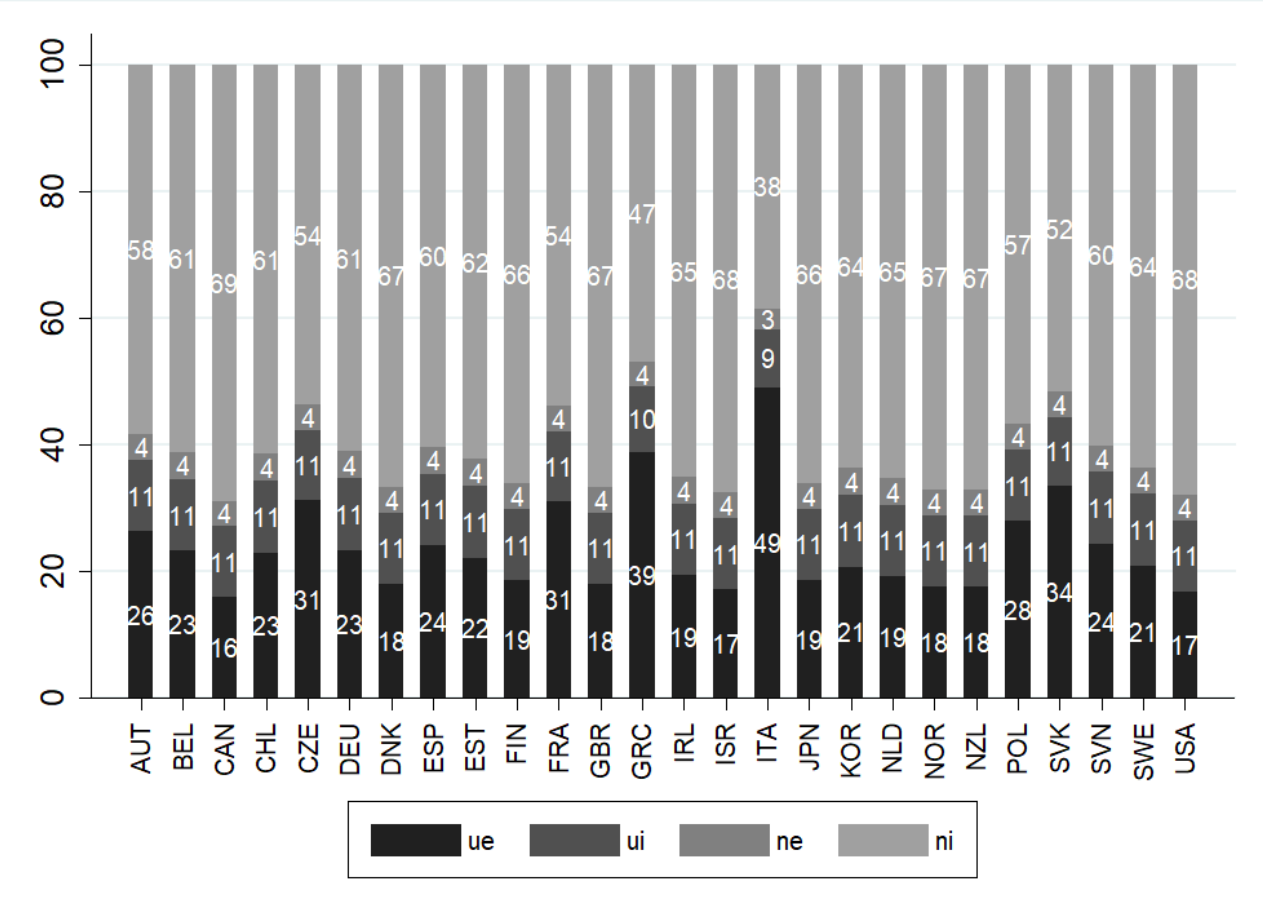


Table C.1: Country values: OECD data and simulations

Country	Tertiary ed. (%)	Payroll taxes (%)	OJHCA	Marginal Cost[1]		Employment		GDP pw[1]		Job finding rate	
				Training	Learning	Data	Model	Data	Model	Data	Model
Austria	30.6	29.1	32.0	1.40	0.67	71.4	62.4	1.11	0.85	7.04	4.31
Belgium	36.9	30.2	34.6	1.17	0.80	61.8	65.4	1.26	0.92	4.56	4.91
Canada	55.2	11.9	43.6	1.15	0.91	72.1	72.9	1.03	1.09		6.99
Chile	21.1	0.0	41.6	0.91	0.73	61.8	65.6	0.61	0.80		4.98
Czech Republic	22.2	34.0	36.1	0.76	0.78	66.5	57.7	0.75	0.80	5.54	3.54
Denmark	37.1	0.9	42.6	0.98	0.95	72.6	70.7	1.13	0.96	7.38	6.28
Estonia	38.0	34.4	39.9	0.98	0.76	67.2	66.4	0.70	0.96	7.51	5.15
Finland	42.7	22.8	45.7	0.88	0.84	69.5	70.2	1.07	1.03	10.14	6.11
France	33.5	44.0	26.8	1.75	0.63	64.0	57.9	1.13	0.84	5.16	3.58
Germany	27.6	19.6	40.6	0.91	0.73	73.0	65.2	1.02	0.87	5.11	4.87
Greece	29.1	28.6	15.1	2.80	0.95	50.8	50.8	0.83	0.70	5.30	2.68
Ireland	42.8	10.8	33.5	1.18	1.10	58.8	69.3	1.35	0.95	5.71	5.86
Israel	48.8	4.7	37.2	1.32	0.99	66.5	71.6	0.83	1.01		6.57
Italy	17.5	32.1	21.0	1.70	0.65	56.6	41.8	1.06	0.65	5.07	1.87
Japan	49.5	14.5	33.3	1.36	1.03	70.6	70.1	0.90	0.99		6.09
Korea	45.5	10.2	25.4	1.22	1.67	64.2	67.9	0.83	0.91		5.49
Lithuania	38.7		24.4			62.0		0.69			
Netherlands	35.3	10.8	43.9	0.85	0.90	75.1	69.5	1.09	0.96	6.73	5.92
New Zealand	34.0	0.0	50.9	0.80	0.85	72.0	71.2	0.85	0.98		6.44
Norway	42.7	13.1	45.8	1.16	0.69	75.8	71.1	1.41	1.03	7.27	6.40
Poland	27.7	16.8	24.9	1.33	1.06	59.7	60.8	0.71	0.78	5.21	4.04
Slovak Republic	21.1	27.9	29.5	1.22	0.66	59.7	55.6	0.82	0.75	5.33	3.25
Slovenia	30.2	16.1	30.7	1.19	0.93	64.1	64.2	0.77	0.84	4.03	4.67
Spain	35.1	29.9	33.6	1.23	0.76	56.5	64.6	1.00	0.90	7.35	4.74
Sweden	39.8	31.4	41.6	1.11	0.67	73.8	67.7	1.11	0.99	13.41	5.45
Turkey	18.0		14.1			48.9		0.79			
Unitec Kingdom	43.5	10.8	40.5	1.00	1.00	70.7	70.7	1.00	1.00	6.27	6.27
United States	44.6	8.8	46.8	0.96	0.88	67.1	72.0	1.38	1.04		6.68

[1] The values are relative to the UK.