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MODELLING CASCADE FAILURES IN POWER DISTRABUTION NETWORKS

Final Degree Project

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Abstract

Cascade Failure behaviour within power distribution networks was modelled and implemented in the programming language R. The power transported by each line in a network was calculated using DC flow analysis and compared to the capacity of the line. Lines were removed from the network and the flows were recalculated to simulate the behaviour of line protection devices. The model was used to demonstrate the varying degrees of failure which will occur depending on the spare capacity within a network at the time a failure occurs and depending on which line experiences the initial fault.

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1. Introduction

1.1 History and Motivation

This project was completed in conjunction with the broadband communications and distributed systems (BCDS) lab at the Universitat de Girona as part of a larger project which undertook Robustness Analysis of different networks under different “attack” scenarios. The aim of this project was to create a model which represents the behaviour of a power grid during a cascading failure. Initially the algorithm was developed using MATLAB and then was implemented within the larger simulator in R code.

A cascade is defined as “A succession of devices or stages in a process, each of which triggers or initiates the next” [1] and so, a cascade failure within a power grid is a succession of failed elements within the network causing further failures. In a power grid these line failures are typically caused by protection components triggering in response to over current.

Over current occurs when a larger current is flowing through a conductor than it is designed for, it can cause overheating which has several adverse effects. For example, a heated cable will lengthen and sag closer to the ground, buildings and vegetation than it is designed to. This was a major contributor to the 2003 North America blackout as well as the Italian one the same year, both of which will be discussed in the next section. Over current can also damage the electrical components and can be a fire risk [2].

For these reasons power network components are protected by devices which remove them from a network when too large a current is detected. When a line is suddenly disconnected the current that was being transported through it finds alternative paths from the generator to the consumer, increasing current flow through other power lines. If one of these affected lines now surpasses its own current ratings, it is also disconnected from the network. Again, their share of the current is redistributed across the network. The relation of this process is a cascade failure.

Cascade failures can have very large social and economic ramifications and pose a significant risk to the power network. Of the 15 largest black outs in the USA, 5 were cascades resulting from faults, all others were the result of severe weather including 7 hurricanes [5].

Cascade failures in power grids are rare as grids are generally designed to meet the N-1 criteria, that is a failure in one individual element should not cause the failure of another [3]. This means that it takes more than one near simultaneous fault to cause a cascade [4].

Cascade failures still occur more often in power grids than would be expected by a normal distribution, in fact according to figures from the North American Electric Reliability Cooperation (NERC) the probability of a Cascade failure follows a Power Law distribution [5]. This means that larges cascades happen at predictable intervals and there have not been any improvements in the number of occurrences of them from the 80s to early 2000s despite large investment [6].

The pressure on power grids will increase over the coming years as energy generation becomes more renewable and therefore distributed. The consumption is expected to increase, eroding the redundancy within the grid capacity. Cascade failures have also been found to be significantly more likely when the grids usage is close to maximum operational capacity [4].

Many factors inhibit grid improvement, there are environmental concerns limiting the types of generation which can be built easily. Where renewables are used the generation in the grid becomes more distributed which will likely increase congestion in certain areas. Distributed generation should also change the type of topology needed. Politically, large new power lines are opposed for aesthetic reasons where they will be crossing rural areas and economic barriers are also present. If the grid is not improved then the likelihood of cascade failures will increase further. Therefore, their mechanisms, causes and effects must be understood to identify the most vulnerable components and those in which failure would have the biggest impact. This research can be carried using models such as the one developed in this project.

1.2 Literature Review

The power grid is a complex interconnection of different components which allow power to be transferred from the point of generation to the point of consumption. The first power distribution systems were developed in the late 1800's to power the newly invented incandescent lightbulb, these systems were short range and Direct Current (DC). Since then the power distribution system has become an integral piece of global infrastructure. Modern designs span thousands of square miles connecting billions of consumers to thousands of generators.

The power grid primarily consists of vast quantities of cables interspaced between power stations, sub stations and consumers. The length of cable in Europe alone is 26 times the distance from the earth to the moon. The generators are mostly within the power stations and traditionally consist of a rotating turbine which turns a generator to produce the electricity. The turbine is normally turned by steam which is produced from heating water with coal, gas, oil or nuclear fission. The power produced is ordinarily in 3 phase form and held at the grid frequency, which is 50 Hz in Europe [7].

Power is transmitted long distances at a very high voltage to reduce infrastructure costs. The amount of current any cable can carry is proportional to its surface area therefore it is desirable to reduce the current carried by long cables to reduce the size of cables needed. The power transferred by a cable is described by equation (1) where P = power, V = voltage and I = current.

$$P = VI \tag{1}$$

This shows that if the voltage is increased then the current can be decreased for any given power. This approach to power distribution has the added benefit of reducing energy loss through heat during transmission. The energy lost is proportional to I^2R, where R is the resistance of the conductor and I is the current it carries.

After generation power is stepped up using a transformer, that is its current is decreased and voltage increased. A typical grid was designed to have several voltage, the closer to the final consumer the power gets the lower its voltage [5].

At generation, the power can be distributed above 100KV by the power transport network it will then be stepped down when it reaches a distribution network and stepped down again in the neighbourhood of the consumer. At each point where the power is stepped down normally it is also split into several lines. These points are called substations and are an intrinsic part of the power grid. Substations do not produce or consume power, they are bound by Kirchhoff's laws: all the energy which flows into them must flow out.

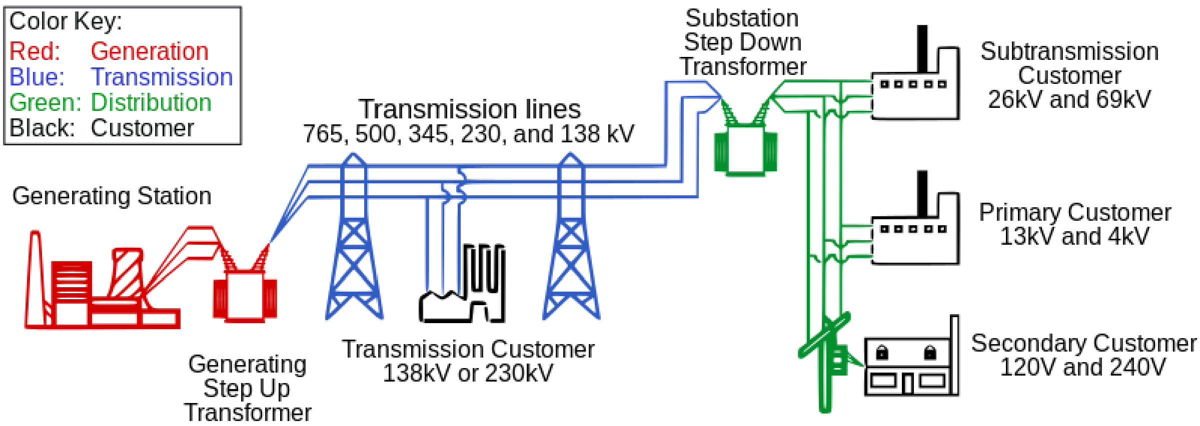

Figure 1 Components of power distribution system[1]

Figure 1 shows the three main stages of power distribution, the generation point in red, the transport network in blue and the distribution network in green. The transport network normally spans hundreds of square kilometres and carries very high voltage power. The distribution is normally a lot smaller than the transport network covering only a city or number of smaller towns. Both networks are vulnerable to Cascade Failures [5].

Within Europe 99% of the consumers are residential or small businesses who require their power to be delivered at 230 V. The other 1% of consumers are large industrial locations who consume substantial amounts of power and require it to be delivered at a much higher voltage.

[1] Figure available at: https://en.wikipedia.org/wiki/Electric_power_distribution

In recent years, the grid has evolved and can be expected to continue to do so. As traditional power plants reach the end of their life span and are increasingly being replaced by renewable energy sources. This has several effects on the power grid. Firstly, the tree like or radial structure which was the standard network structure is becoming obsolete. This structure consisted of very large generation points at power plants which then expanded outwards to all the consumers [5].

With the development of renewable energy, the location of energy production is decided with many more considerations than in a traditional power plant. The energy is often produced much further from the consumer than ever before. To transport power generated offshore for example, there has to be an increase in high voltage long distance power lines which are then connected to an existing infrastructure. Providing an intermittent sometimes very large power injection into a part of the grid that was not developed for it. This can cause the surrounding lines to begin operating much closer to their maximum capacity more often. When networks are operating with a large spare capacity failures are much less likely to cause catastrophic cascades [8].

As well as these large installations, there has been a rise in small scale generations, such as residences with solar panels or turbines on their houses and small on-shore wind farms. This kind of distributed generation has large implications for the grid as it requires it to be even more adaptive to changes in power levels. This kind of generation introduces more variability and intermittency into the system as power produced varies depending on the weather. Nodes are now able to be both consumers and generators of electricity at different points throughout the day [9].

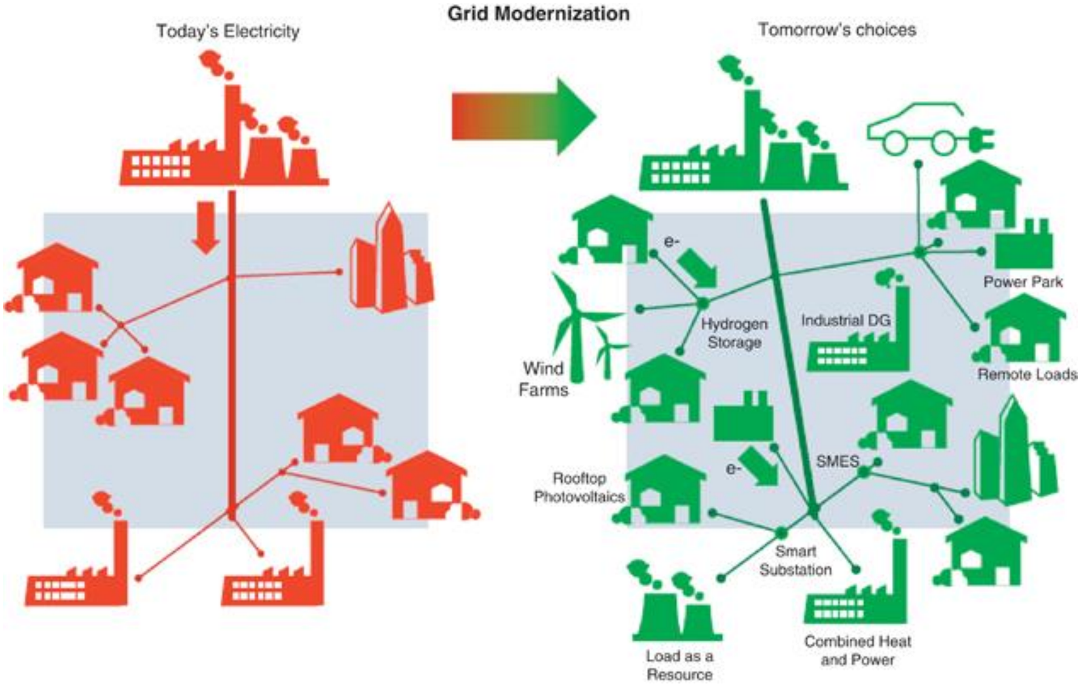

Figure 2 Grid Modernisation [2]

Figure 2 shows a simplified representation of these changes in the grid, the left side representing a typical grid structure which is much simpler containing few very similar generation points in comparison to the right side which shows several diverse types of generation which are distributed throughout the network. The diagram on the right also implies higher electricity consumption in the future as the number of electric vehicles in use is expected to continue to increase.

Some forms of renewable energy such as wind and hydro power can also introduce harmonics into the network increasing the risk of failures due to frequency instability. Increased wind power into the network will cause a significant effect on the quality of the power produced having causing voltage fluctuations and an increase in the presence of harmonics introduced from generation when compared to traditional generation [10]. Both are important to consider in terms of cascading failures because the protection relays that can be the mechanism of these failures are sensitive to abnormal changes in frequency and voltage.

As consumption of electricity continues to increase, the redundancy margins within the grid has been eroded. To help meet these challenges many countries are beginning to interconnect networks that in the past were separate national or regional networks. Figure 3 shows the lines which are interconnected by European Network of Transmission System Operators for Electricity.

[2] Figure available at: https://www.slideshare.net/CapeLightEnergy/51315-board-meeting-grid-modernization

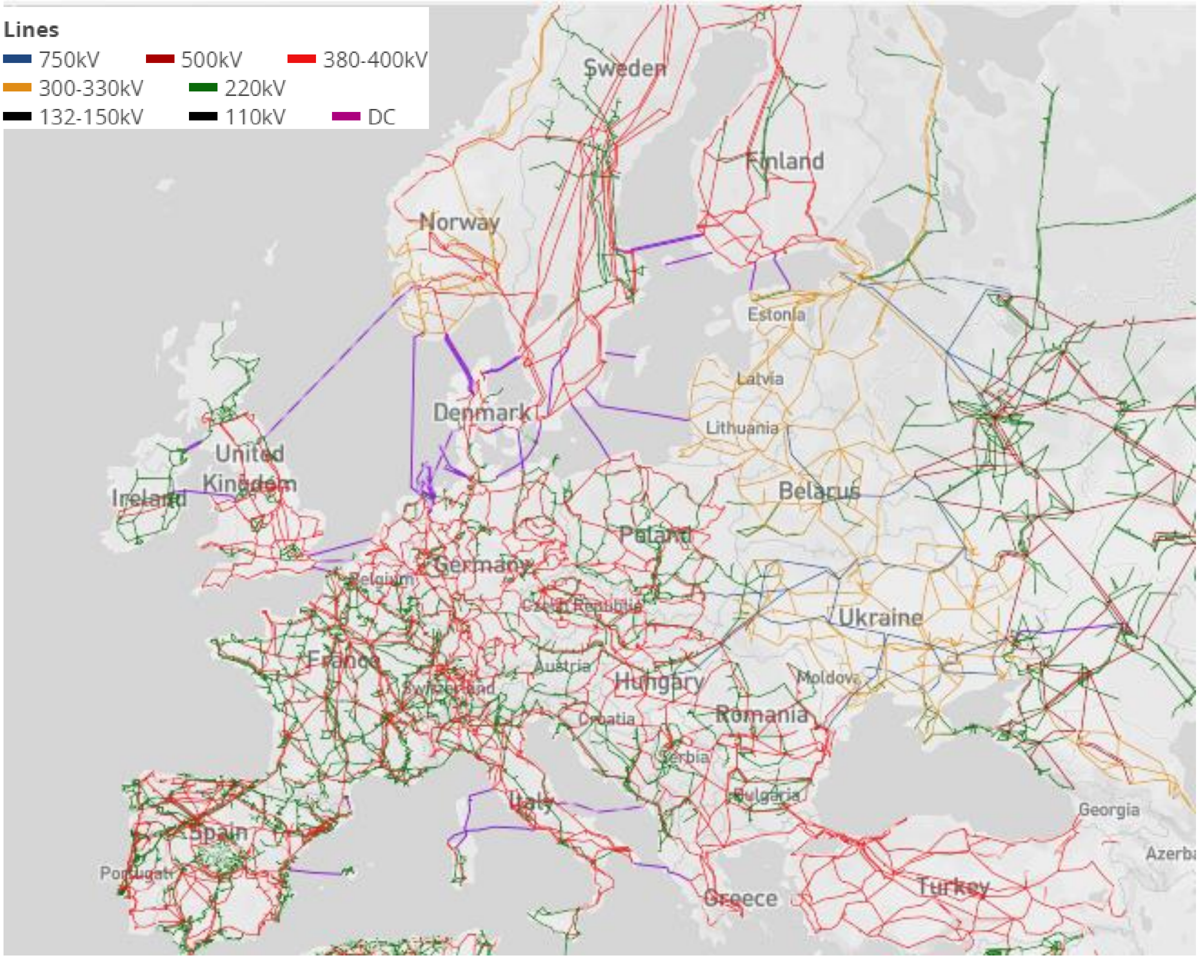

Figure 3 European Network of Transmission System Operators for Electricity [3]

European national networks all connected these connotations are represented in Figure 3. Each colour in the image represent different transmission voltages the purple being DC lines. This has many advantages including load and generation sharing as each country has different peak consumption and generating [11]. However, an increased grid size has been shown to increase the risk of Cascade Failures as well as increasing the abruptness at which they occur and so reducing the time available for mitigating actions to be taken by grid operators [12].

[3] Figure available at: https://www.entsoe.eu/map/Pages/default.aspx

Historical Cascade failures

There have been several notable cascade failures which have caused billions of euros in damage. These are triggered by a variety of different scenarios. The year 2003 is notable in terms of cascade failure history as there were massive blackouts in Northern America, Europe and Asia all within weeks of one another.

North American Blackout 2003

The first of these occurred in the North Eastern American interconnect affecting several American states and Canadian south east Canada the affected areas are shown in red in figure 4.

Figure 4 Area affected by 2003 Blackout [4]

The initial fault in this case was found to be over current through some cables which caused the lines to sag close to some trees. These trees had not been maintained diligently enough and were over grown and too close to the line. As consumption that day increased and the line dropped closer to the trees a flash over occurred as the trees created a short circuit. This short circuit caused the protective relays to correctly activate, the alarm system however malfunctioned so system operators were unable to find this initial fault. Three more lines sagged into overgrown trees and were in turn removed from the network triggering a massive cascade throughout the interconnect [13]. This black out lasted up to 2 days in some areas cost and estimated $6 billion and contributed to the deaths of at least 11 people [4].

[4] Figure available at: http://blogs.ei.columbia.edu/2017/04/18/microgrids-taking-steps-toward-the-21st-century-smart-grid/

Italian Blackout 2003

In September of 2003 most of Italy lost power due to a cascade failure resulting from the loss of 2 high voltage lines in Switzerland. The causes of this failure were later determined to be initial underestimation of the scale of the problem as well as overgrown vegetation causing a flashover between the line and the trees. These failures caused a cascade which effected many of the lines importing into Italy from Switzerland. This sudden change caused the Italian network to lose synchronicity with the Union for the Coordination of the Transmission of Electricity (UCTE) which is the network throughout Europe. The loss of synchronicity caused the Italian network to be completely removed from the rest of the network. As the Italian system had been importing at the time there was a huge imbalance in the system which could not be rectified and this caused the black out. [15]

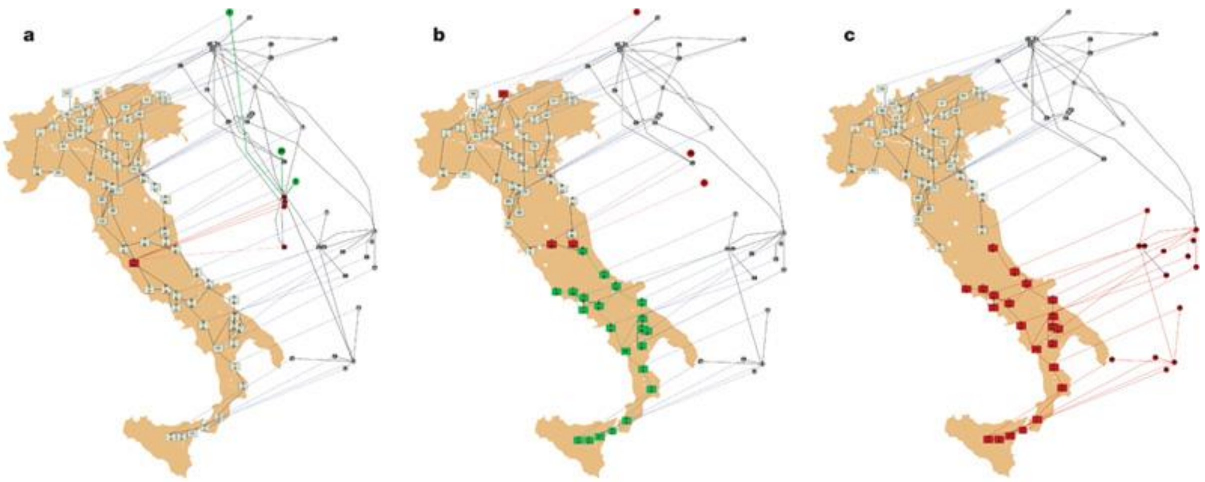

Figure 5 Italian blackout cascade [16]

The above diagram shows the mechanisms for how the cascade propagated through the Italian power network.

Diagram A

Initially the first power station was removed from the network this is shown in red.

The loss of this power station removed the internet nodes that depended on them they are the red nodes above the map.

The green nodes above the map are the nodes which will be disconnected from the cluster in the next step as a result.

Diagram B

The red nodes on the map show the power stations which were disconnected next

The red nodes above the map are additional nodes which were then disconnected

The green nodes above showing the nodes which were disconnected next.

Diagram C

Red Nodes on the map are additional nodes that were disconnected as the network degraded

Red nodes above the map are nodes in the Internet network that depend ed on those which disconnected [16].

At the time of the outage the major lines from Switzerland were very heavily loaded but the lines from France were not operating near capacity. After the flash over the Italian authorities were asked to reduce the unscheduled power consumption which was achieved after 10 mins. Before the pumped storage generation in Switzerland could be taken off line to allow the first failed line to be restored a second North-South High voltage line in Switzerland experienced a fault and was taken automatically off line. Italy had last all power within 30 minutes of the first fault occurring.

This power failure had significant cost associated with it. It occurred on a night in which a festival was taking place in Rome causing abnormal power consumption. Although it was late at night the city was full of people and the metro was still operating. This means that hundreds of people were trapped on the metro until the systems were restored.

The original line failures which caused this, were in a different country than the one that eventually bore the consequences. It is an example of the dangers of cascade failures and how fragile the networks can be.

Italy's imported energy was too heavily reliant on Switzerland to reduce cost when the system would have been more stable had the power been imported from the French network as well as the Swiss one. This shows the danger in connecting many different national grids because variations in policy and markets cause decisions to be made by each country or distribution company with a very narrow view of the network [10].

These two cases highlight the huge consequences a cascading failure can have and why all possible steps should be taken to mitigate against them.

Probability of a black out

The Probability of a blackout plotted against the size of the blackout follows a Power Law distribution in the United states. This means they occur more frequently than could be expected. Figure 6 Shows this distribution of large failures in the US over recent years.

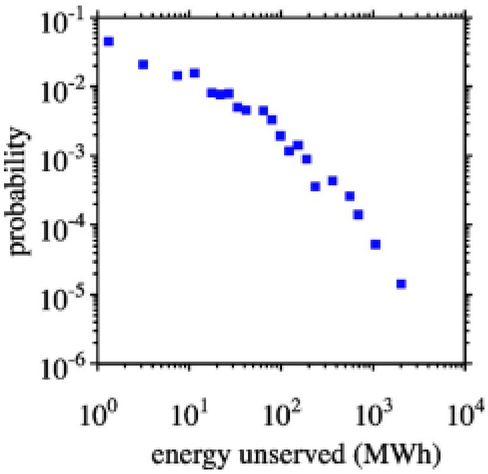

Figure 6 Probability of cascade failures [18, Fig. 1]

There are several theories as to why this occurs including the theory that blackouts are necessary in the evolution of a power grid. As the grid becomes more stressed through increased consumption or distributed generation or any other factor the grid becomes weaker until eventually a large failure occurs. Each time this happens and the reasons for the failure are investigated and the investment is provided to improve that section of the grid. In this theory, the grid is not considered to be static but a constantly evolving entity which requires a massive failure as a catalyst for real upgrade and improvement [18].

There is also a trend towards larger networks as national ones begin to be combined to increase efficiency as resources are shared as well as to create more competitive markets. This adds to the risk of blackouts though as the larger a power grid the more prone it is to cascades [12].

It would be much cheaper to identify and mitigate against the networks weaknesses without a failure occurring. The probability of a failure is linked to network structure so if as the structure is designed it can is modelled in such a way as to reduce the probability of a cascading failure billions could be saved. This means modelling the structure and not just creating changes based on local needs and a narrow view of one part of the network.

The number of failures in the Union for the Coordination of Transmission of Electricity suggests that the system is operating very near to its capacity and that further expansion if the grid will be necessary as consumption increased [4].

Power Grid Models

Power grids can be studied using Complex Network Analysis when the power grid is represented as a graph G (N, L), N is a set of nodes which represent generators, consumers and substations and L is the set of links which connect them. An N by N adjacency matrix is used to store connection information. Link L_{jk} is the link between nodes j and k and would be represented by a 1 in position (j,k) within the matrix. Where no link exists position (j,k) should contain a 0 [13].

Pagani and Aiello [9] Surveyed several existing models of power grids most modelled the American, European or Chinese networks. Their conclusions included, a recommendation that a more diverse range of power distribution networks should be studied, networks can be characterised by their average degree perimeter as they generally follow an exponential distribution. All studies agreed power grids are very robust against random attack but are vulnerable to targeted attack. The study of these models also showed that the results are like those obtained from traditional electrical engineering approaches to network analysis which goes to verify the validity of Complex Network Analysis for power grids.

In the past, power grids have been analysed by simulating contagious failures but power grids behave very differently from other types of graph modelled networks [19]. In epidemic processes and network synchronisation, the critical breakdown threshold is related to the spectral radius of the network. Analysing the network using pure graph theory can be a useful supplement to traditional network monitoring [20]. However, this is not the most effective method to study the behaviour of a network during a cascade failure, a much more accurate measurement of system robustness is the spectral radius of the graph [3].

Modelling of grids as complex networks has several applications which can be used to improve grid design or develop knowledge of grid behaviour as a network. For example, it can be used to demonstrate the presence of Braess' Paradox in the power network. Which is a phenomenon that occurs in some networks where increasing capacity decreases overall efficiency. Further analysis of the model showed that the presence of certain substructures created this effect using modelled grids these substructures can be avoided [17].

Source [4] analyses separately the three major 2003 blackouts which effected large populations and assess and characterises the tools which have been used to analyse and model cascading failure in the past. It argues that the Robustness of power grid against cascade failures is typically studied one of two ways: an artificial system dynamics approach or a conventional reliability approach. Some common conventional reliability approaches are fault tree analysis and Markov analysis.

A fault tree analysis is a way to visualise and determine the causes for a potential critical fault. It is a diagram which uses a top down method to find underlying connections between network conditions and topology with the potential for specific failure modes.

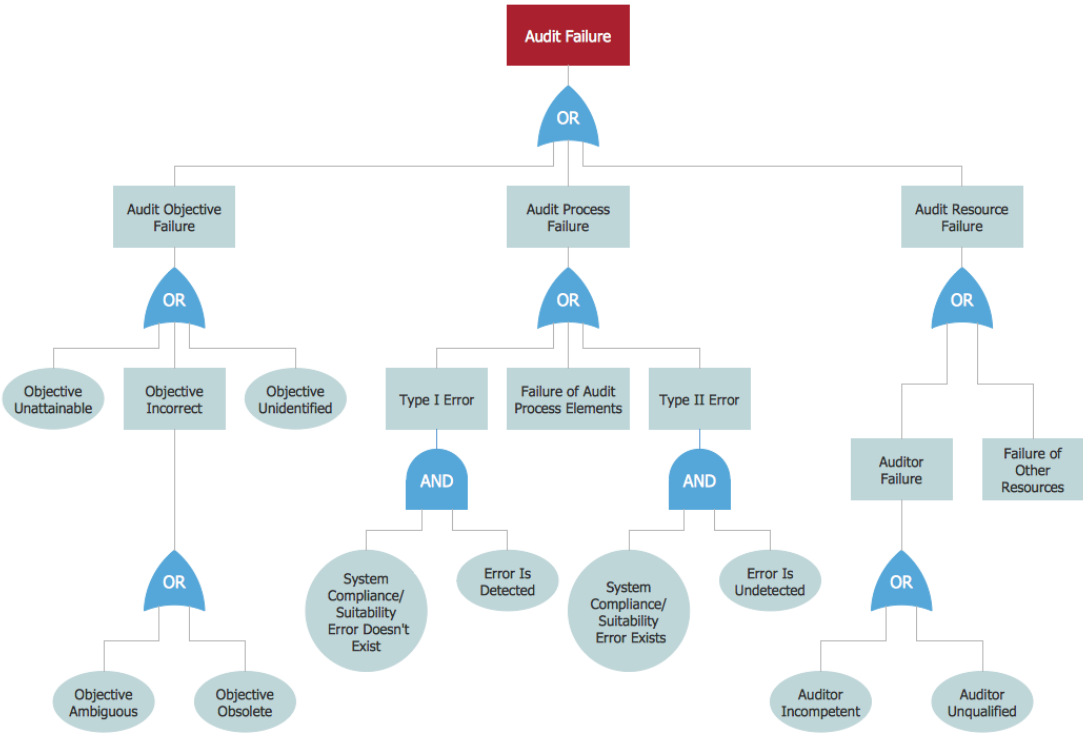

Figure 7 Example Fault Tree Analysis [5]

The diagram above shows a finished fault tree analysis for a generic problem, logic gates are used to create the diagram. Initially boundary conditions such as physical state of the system, internal and external stresses and depth resolution are defined. The depth resolution level of the potential failure is defined. Each level is connected to a cause using logic gates and the diagram is created level by level. The diagram is complete when all the possible faults have been linked to a catastrophic failure. The probability of such an event can then be calculated as a function of the occurrence of these basic events. This approach is desirable in its simplicity and is easy to understand for non-specialists. [4]

Markov analysis is a method of failure probability calculation that describes the system in terms of specific states and uses the probability of each state transition to evaluate the overall probability of a certain failure occurring. The system is in one state at a time and the state transitions occur based on a probability distribution. The system does not contain memory, the state transitions are made based solely on the current state. This provides a major

[5] Figure available at: http://www.conceptdraw.com/examples/what-is-a-fault-tree-analysis

descriptor of the system that is the probability of each state transitions is constant with respect to time [21].

The artificial system dynamics approaches are more appropriate for the Robustness Analysis needed for the UdG simulation. They include ORNLPSerc-Alaska (OPA) modelling, this is a method of modelling a cascade failure based on line overload values. Linear programming is used to create a DC flow estimation of the currents through the line and therefore identify those which have been overloaded and should be removed from the system. This process can be iterative terminating when there are no further line outages or when there are no longer any lines still carrying current. This kind of model however includes a function which allows gradual grid upgrades and their effects to be considered. As well as containing a faster time scale which shows the immediate effect and mechanism of a cascade propagating through the network being investigated. This method has importantly shown that efforts to mitigate cascade risk such as increasing the generator capacity margin or improving the network tend to move the system closer to criticality [4].

The CASCADE model can be used to see the way a cascade can propagate through a network. The network is made from identical components which are assigned random loads. To start the cascade after each component is loaded, a disturbance is caused somewhere in the system causing some components to fail. When a component is overloaded it is removed and its load is subsequently redistributed. This type of model is too simple to accurately represent the type of failures which occur in a power grid as it does not consider system structure, the time between adjacent failures and generation adaptation during the failure [4].

A hidden failure is a common cause of cascade failure. A hidden failure is a permanent fault within the network. This fault may be a result of many different things such as human error or system degradation. It will cause a line to be inappropriately disconnected when over current has not been reached. After a line is tripped the probability of incorrect tripping occurring in all the lines connected to the initial line is evaluated. Linear programming load shedding is adopted to keep the system stable. The Importance Sampling method is used to increase the frequency of these failures, by attaching a likelihood index to hidden failures. It is achieved by using altered probabilities when simulating unlikely events to cause them to happen more frequently [22].

When Hidden failures are modelled they are split into two types, line failure is the simpler type and occurs when a line is inappropriately removed from the network. Typically, this occurs in one of two ways either as a result a faulty protection device or because of human error. A faulty protection device could cause a line to be disconnected before reaching its maximum capacity or the same thing could happen due to human error if the capacity of the protection device has been set incorrectly.

A voltage based failure is when the generator incorrectly trips because of low voltage conditions. Hidden failure analysis is very important as a study of the American power system revealed that 75% of blackouts were related to a hidden failure in a protection device [23].

A complete analysis of a network for cascade failure behaviour before the event of a failure rather than an analysis of what went wrong is very difficult to achieve due to the vastness of the network and therefore the number of possible combinations of very unlikely events.

A commonly used cascade failure simulation is MACASC [6] this is a program developed in MATLAB which can be used to analyse a network for to see how it reacts to cascade failure. The program implements a DC power flow system and checks for line overload failures. In [6] its effectiveness as a model is demonstrated by its use to find the Robustness of IEEE Networks and it is used to evaluate the effectiveness of different attack strategies on certain networks.

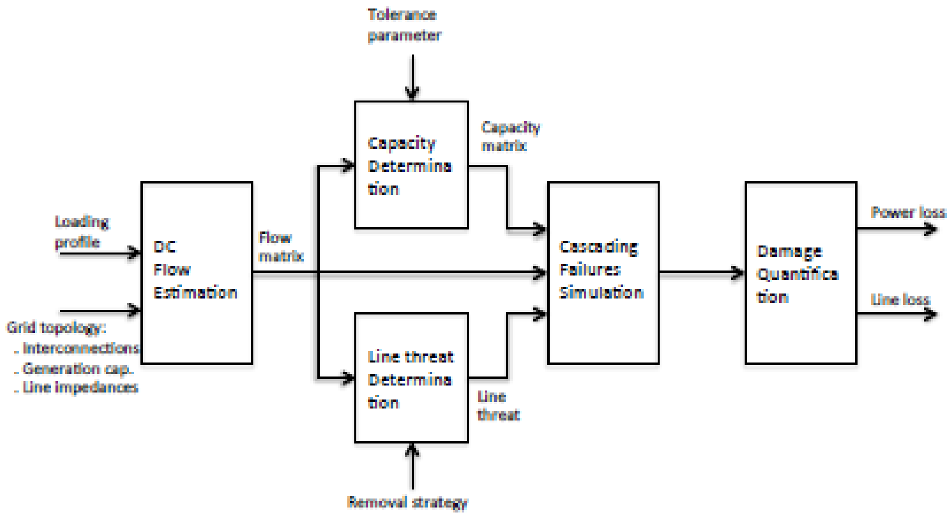

Figure 8 MATCASC Structure [6, Fig 1]

Source [23] describes the development of a model which combines line overload cascade modelling. Hidden failure, Node load and betweenness is also integrated. This means that as the failure propagates through the system, nodes fail due to overload but the hidden failures calculated to be present also cause incorrect node removal so that the cascade is more representative of a real-life system. The two types of failure interact as a hidden failure could trigger a node failure without which the cascade would have stopped.

This model also considers the electrical behaviour of a power grid meaning some standard complex network modelling techniques cannot be used. The two key features that must be considered are the fact that all nodes are not equal and can be classified as either generator consumer or transmission nodes. Secondly all paths between generator and consumer must be considered in the flow of power not just those with the shortest path or lowest impedance.

All of these models have their shortcomings in that they do not and cannot comprehensively model the entire system behaviour. All have a focus and can be used only in a narrow area of research. A comprehensive model would need accurate representation of power relays as well as a probability component as the majority of large blackouts have at least a component of device malfunction or human error contributing to black out. The generator and load should be properly modelled as well.

The purpose of this project was narrow enough that these parameters were not all necessary. The model does not need to be show evolution over time in response to failure nor does the probability of failure need to be considered as this model was required to show the propagation path of a failure given the starting parameters of the system

2. Proposed Model

2.1 Per unit

The pre-unit (PU) system is used throughout this model to simplify calculations and make it more easily scalable. Per unit is a method of representing power, voltage, current, impedance or admittance used where there will be large variations in these quantities within a system. It's especially useful when modelling the transport or distribution networks because the presence of transformers within the network results in large variations in voltage levels which are eliminated when the system is represented in this way.

For example, the impedance in per unit is represented as:

$$Z\ PU = \frac{Z}{Zbase} \qquad (2)$$

The base values of other parameters can be calculated by first principles and other base parameters for example Ohms Law can be used as follows:

$$V\ base = \frac{I\ base}{Z\ base} \qquad (3)$$

The Network used in this project uses a base of 100MVA and operates at 60Hz.

2.2 DC Power flow

Equations (4) and (5) describe the real and reactive power transferred by a power line between nodes j and k respectively.

$$P_k = \sum_{j=1}^{N} |V_k||V_j|\left(G_{jl}\cos(\theta_k - \theta_j) + B_{jk}\sin(\theta_k - \theta_j)\right) \quad (4)$$

$$Q_k = \sum_{j=1}^{N} |V_k||V_j|\left(G_{jl}\sin(\theta_k - \theta_j) - B_{jk}\cos(\theta_k - \theta_j)\right) \quad (5)$$

Where, P_k = net real power injected at bus k

Q_k = the net reactive power injected at bus k

θ_k and θ_j = The voltage phase difference at busses k and j respectively when measured against a common reference

G_{jk} and B_{jk} as the real and imaginary parts of the admittance of the link jk

To solve these equations for every line within a power grid requires non-linear programming methods. Most commonly the Newton-Raphson technique is employed. This is computationally expensive as it is an iterative technique that must be carried out for every link within the network. While the network being analysed is small this is an acceptable expense for accuracy. As the grid gets bigger this begins to be too much for each simulation. As explained in the next section the analysis of the network is an iterative process and it is for this reason that most cascade failure analysis is carried out using DC flow analysis.

DC flow analysis describes several different methods of representing flow through a power network and is based on several assumptions that can be used to convert equations (4) and (5) to linear equations which can be easily programmed and calculated using matrices.

There are several advantages to using a DC power model. It is a linear programming method in comparison to AC power flow models it has the advantage of being non-iterative and much more simply programmed. The results are also consistent and consider link properties. The data needed to carry out the analysis is also very basic and accessible. The approximation of the MW flows is accurate especially for the heavily loaded branches which are the most common locations of the failures. [24]

The DC Power flow model used can be developed from equations (4) and (5) when 3 assumptions are made. The method is explained below.

$$P_k = \sum_{j=1}^{N} |V_k||V_j|\left(G_{jl}\cos(\theta_k - \theta_j) + B_{jk}\sin(\theta_k - \theta_j)\right) \tag{4}$$

$$Q_k = \sum_{j=1}^{N} |V_k||V_j|\left(G_{jl}\sin(\theta_k - \theta_j) - B_{jk}\cos(\theta_k - \theta_j)\right) \tag{5}$$

Assumption one

The resistance of the transmission lines is significantly less than the reactance and so can be considered as a negligible component to the impedance of each line.

The longer a transmission line is the larger the proportion of total impedance the reactance represents. The reactance of a line is caused by the magnetic interaction between current carrying conductor. The difference can be seen in the IEEE systems used in this project with the X/R ration typically being between 2 and 10. This allows us to simplify the impedance of the line as follows:

$$g + jb = \frac{r}{r^2 + x^2} + j\frac{-x}{r^2 + x^2} \tag{6}$$

$$g = \frac{r}{r^2 + x^2}, \quad b = \frac{-x}{r^2 + x^2} \tag{7}$$

If R >> X then it can be approximated that:

$$g = 0, \qquad b = \frac{-1}{x}$$

If g = 0 so does G having the following effect:

$$P_k = \sum_{j=1}^{N} |V_k||V_j|\left(B_{jk}\sin(\theta_k - \theta_j)\right) \tag{8}$$

$$Q_k = \sum_{j=1}^{N} |V_k||V_j|\left(-B_{jk}\cos(\theta_k - \theta_j)\right) \quad (9)$$

Assumption two

For most operating conditions the phase change between any 2 adjacent busses is less than 15 degrees.

The difference in voltage phase (ϴ) for each link are only used within the equations as the arguments for the sine and cosine functions. When it is assumed that all angles will be small, typically between 10 and 15 degrees, we can also assume that the cosine function will approximate 1 and the sine function will approximate the argument angle when given in radians. The changes the function as shown in equations (10) and (11).

$$P_k = \sum_{j=1}^{N} |V_k||V_j|\left(B_{jk}(\theta_k - \theta_j)\right) \quad (10)$$

$$Q_k = \sum_{j=1}^{N} |V_k||V_j|(-B_{jk}) \quad (11)$$

Assumption three

When analysis is carried out in per unit the magnitude of the voltage at each bus is very close to one typically between 0.95 and 1.05.

These three observations allow us to convert the AC power flow equations (4) and (5) to the DC ones shown as (12) and (13).

$$P_k = \sum_{j=1, j\neq k}^{N} \left(B_{jl}(\theta_k - \theta_j)\right) \quad (12)$$

$$Q_k = -b_k + \sum_{j=1, j\neq k}^{N} |b_{jk}| \left(|V_k| - |V_j|\right) \tag{13}$$

In real power systems Q_k is much smaller than P_x meaning it too is considered negligible in this simplified model which leaves one linear equation to be used in the model. This equation can be adapted for matrix manipulation and so the algorithm was initially designed in MATLAB.

Equation (13) was rewritten to for matrix multiplication and linear programming to give equation (14):

$$\mathrm{P} = \mathrm{B}'\,\Theta \tag{14}$$

Where B is the Laplacian version of the adjacency matrix of the network and ϴ is, a vector containing the difference in voltage angle across each link. The algorithm for creating and calculating this equation was first developed in MATLAB as explained in the following section which contains first the pseudo code and followed by an explanation of the development of the matrices.

2.3 Pseudo Code

```
If Input ≠ New Input
            Create A matrix
            Remove reference node
            Generate Graph G (E,N)
            D → Diagonal matrix containing admittance values on the diagonal
            B' → Laplacian matrix of graph G
            Θ → B' P
            PL → D*A*Θ
            For n = 0 : size (PL)
                  If PL(n) > Capacity(n)
                        Remove edge in Input matrix
                  End
            End
            If graph is connected
            Input → New Input
            Else
                  Find number of islands
                  create and store adjacency matrix for each island
                  New input → adjacency matrix for next unprocessed matrix
            End
End
```

2.4 MATLAB Development

The algorithm was first developed in MATLAB as it is easy to use and debug it was developed using equation (14) as a basis.

The Power through each line was approximated as described in the previous section by equation (14).

$$P = B' \Theta \tag{14}$$

Where B' is, a square matrix derived from the adjacency matrix called the Laplacian matrix of the network and Θ a vector containing the voltage phase difference across each link when measured against a reference node. This equation is used to find a vector which represents the power through each link of the network PL.

Each value in the vector PL is compared to the corresponding value in the capacity vector to determine whether the line is carrying more than its rated power. If the power through the line is larger than the capacity of the line, the line was removed from the network, a new network was formed from the remaining lines and the process is repeated. The final output is a list of lines which have failed in the order in which they fail.

Initially the simple network shown in figure 1 was used to develop the method and will be used in the following examples of how the code was developed.

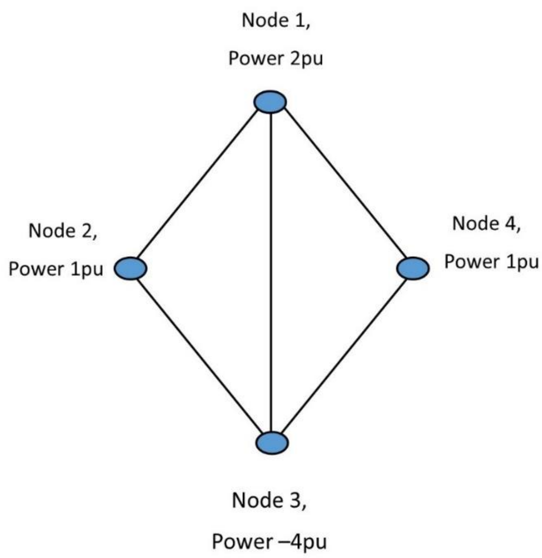

Figure 9 Example network

Each edge on this graph represents a power line in the network. The links were weighted by their susceptance (B) which is the imaginary component of admittance. In this basic model, all links were given a susceptance of 10 Siemens PU.

The input to the algorithm was a matrix which was similar to an adjacency matrix in that it contains the weights of the links in the correct position to indicate a link between two nodes. This matrix however contains negative values of the admittance in the corresponding position to suggest a bi-directional matrix however these admittance values have a negative polarity indicating the link is in the same direction as the positive entry suggests. For example, a link between nodes one and two with an admittance of 10 WPU would cause a 10 to be placed in position (1,2) and a -10 in position (2,1).

To create the square matrix B' this is the Laplacian version of the matrix the input matrix was used. The first step was to find the absolute value of all elements in the matrix. Then the diagonal element within each matrix was calculated by summing all the other elements in the row their respective rows. The non-diagonal elements of the matrix are then multiplied by -1 and the first row and column are removed this as shown in figure 2.

```
Input =

     0    10    10    10
   -10     0    10     0
   -10    10     0   -10
   -10     0   -10     0
```

→

```
B =

    20   -10     0
   -10    30   -10
     0   -10    20
```

Figure 10 Input conversion to Laplacian matrix

After this matrix was developed it was used to calculate the values of Θ. This is calculated by multiplying the inverse matrix of B' with the vector of power values. The direction of power flow in the matrix is incorporated into the equations using another matrix named here matrix A.

Matrix A is an n-1 by m matrix where n is the number of nodes in the matrix and m is the number of links connecting them. The matrix is constructed line by line. For example, if link 1 carried current from node 1 to 3 the first row in the matrix would contain a 1 in column 1 and a -1 in column 3. Figure 3 shows the matrix A that would accompany the network shown in Figure 11.

```
A =

     1     0     0    -1
     1    -1     0     0
     0     1    -1     0
     0     0    -1     1
     1     0    -1     0
```

Figure 11 Complete A matrix

After the matrix is formed all information about the reference node must be removed so that the matrix will have a unique inverse. In this case, the reference node is node 1 as this is the same node that was removed from the adjacency matrix. This involves removing column 1.

```
A =

     0     0    -1
    -1     0     0
     1    -1     0
     0    -1     1
     0    -1     0
```

Figure 12 Matrix A without reference node information

The final matrix needed for the calculation is a square m by m matrix in which the diagonal elements for each row are equal to the reactance on the line it represents. This was created in MATLAB by first producing a weighted graph of the network. From this a vector of line weights were derived and used to create this matrix.

```
D =

    10     0     0     0     0
     0    10     0     0     0
     0     0    10     0     0
     0     0     0    10     0
     0     0     0     0    10
```

Figure 13 Matrix D

The final calculation is made by multiplying A, D and Θ to return a vector containing the power through each line.

After this vector was produced a further matrix was created for comparison of the power through each line and the capacity associated with that line. It was made by concatenating matrix A with the vector of the power through each line and the vector of the capacity through the line.

```
compare =

    1.0000         0         0   -1.0000    1.3500    0.2500
    1.0000   -1.0000         0         0    1.3500    0.2500
         0    1.0000   -1.0000         0    1.3500    1.2500
         0         0   -1.0000    1.0000    1.3500    1.2500
    1.0000         0   -1.0000         0    1.3500    1.5000
```

Figure 14 Comparison matrix

This matrix shows the nodes that each line connects as well as the power carried by it as calculated in the most recent iteration and the capacity of the link. Each row of the matrix represents a single link, each row contains a positive and negative 1. The column with the positive one indicates the starting point of the link and the negative 1 shows where the link terminates. The first row shown in Figure 14 represents the link from node 1 to node 4 which has a capacity of 1.35. In this example, the capacity is uniform in all links, power carried by the line is 0.25Wpu. The last line represents link 5 and is the only line to fail in this iteration it has a capacity of 1.35 and a power of 1.5.

The line of the matrix which represents the failed line was then used to delete that line from the network before the next iteration. The non- zero elements of the line were found and the columns in which they appear represent the location of the line within the input matrix. For example, if the link between nodes 1 and 3 were removed in the comparison matrix shown above in figure 14 there would be a 1 in column 1 and a negative 1 in column 3. This means the in the input matrix positions (1,3) and (3,1) must be set to zero to remove the line for the next iteration.

After all the adjustments are made to the input matrix it was tested for connectivity, if the entire network is still connected together then the new input matrix was used to recalculate the power carried by each line and the process was repeated.

Each time a line is removed the capacity list must be updated to reflect the removed lines. In the event a node is completely disconnected this nodes entry must be removed from the power matrix.

If the network is no longer fully connected each new "island" formed must be analysed separately. For each island the laws of conservation of energy must be upheld this means that the power flowing in to the node must equal any power flowing out. The excess power was distributed amongst the remaining nodes proportionally to the power initially dissipated in them for example for an Island containing four nodes of powers [20, -5,- 5, -10] if the second node was removed from the network the new power injection vector would look like this [20,-6.67, 13.33]. This was calculated as shown below.

For power through each Node P_N, Removed Node P_R.

$$P_{N\,new} = \frac{P_R\, P_N}{Sum\ of\ all\ other\ negative\ nodes} \tag{15}$$

The new network was used to recalculate the power through each line and their powers were compared to the capacity in the same way as above.

The failures at each time step were noted along with the time step at which it failed. When a new island is introduced to the formula the time step should be reset back to the point at which the island separated so it is as if the algorithm iterates over each island simultaneously.

2.5 R- Implementation

The R implementation followed the algorithm developed in MATLAB but is more sophisticated, it used GML data. This meant much larger networks could be analysed more quickly as the matrices were no longer hard coded.

The R implementation was incorporated into the larger simulator being developed by the BCDS lab. The results discussed in the next section will be from the implementation of the R code before it is integrated into the larger system. Each time a network was analysed by the code a new JPEG image is created showing the network or island which was analysed and which of the original nodes are within it.

With this implementation, the IEEE 39 bus network was analysed. This network is represented in Figure 15. All busses are numbered and each link is named Link ij for the link between nodes i and j. The figure also shows the generators and the downward arrows on some of the busses indicate power consumers. The busses which do not have a generator and do not consume energy follow Kirchhoff's law, all current flowing into them must also flow out.

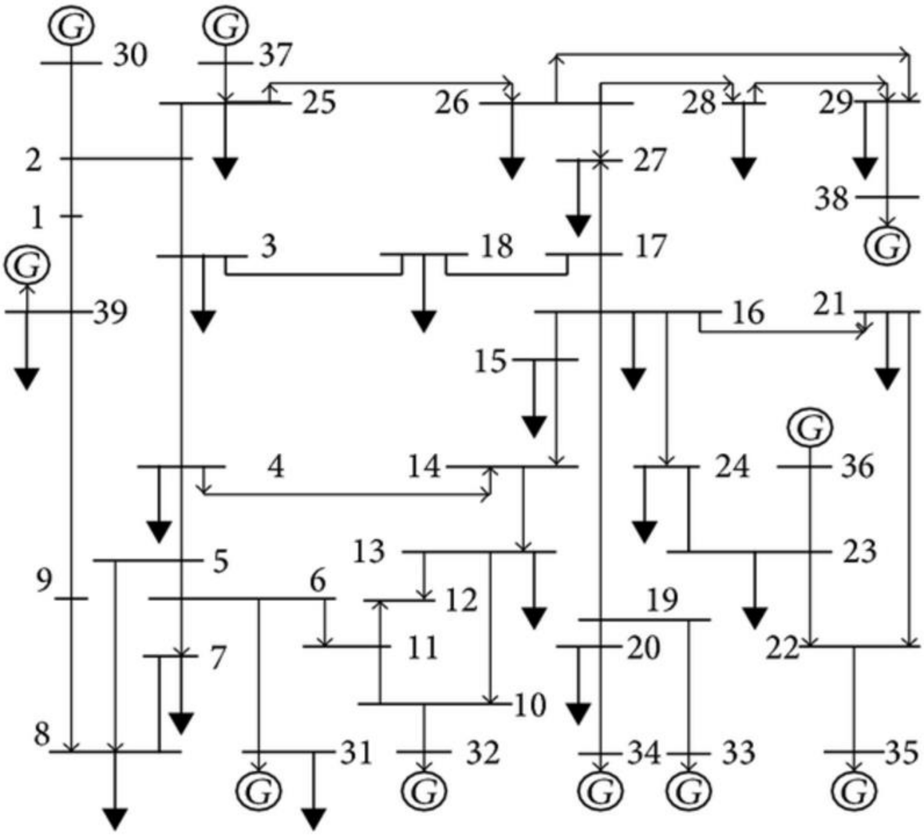

Figure 15 IEEE 39 bus network [23]

The graphical representation of the system shows only the nodes and the edges with no visual indicators of consuming or generating nodes. The information about power generation and consumption is stored in a pre-determined vector and was updated each iteration. The graph layout was created randomly and did not represent grid topology an example of a graphical representation of the IEEE 39 bus system is shown in Figure 17.

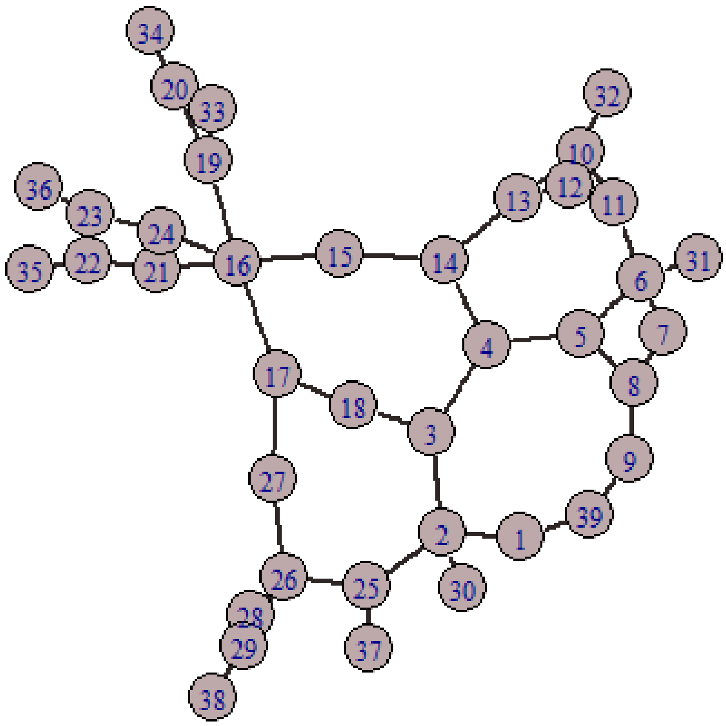

Figure 16 IEEE 39 Bus Network as a Graph

A significant difference between the MATLAB algorithm and the implementation is that the capacity of each line was calculated as a percentage above the power calculated to be flowing through each line before a fault is introduced. This value can be considered the spare capacity of the system and could be altered to study the effectiveness of increasing redundancy in the network as a means of mitigating the risk of cascading failure. This was because capacity data on networks are not available in most cases and so had to be estimated. Estimating the capacity based on balanced power flow represents the physical properties of a power system more accurately than a uniform capacity being applied.

To start the cascade one line must transport more power than its capacity. In the first instance capacities of all the lines are set very high so the algorithm can execute and create the PL vector from which the capacities are calculated. To start the cascade the capacity of a link or links is set very low by the user to cause the initial failure and analyse the failure which occurs as a result and the extent to which this failure cascades throughout the system. The power through each line in the stable case ranges from 5Wpu to 830 Wpu so the size of the cascade and the abruptness of the cascade depends on which line is first removed.

3. Results

When a link is removed from the system, the damage from removal varies wildly depending on the location of the link, the current it was carrying and the spare capacity in the system at the time of removal. A line removal results in one of four cases:

- The failure does not propagate
- The failure only removes a small number of lines but all power was still delivered
- The failure propagates through the network reducing the power delivered significantly
- The failure propagates through the entire network until no power is delivered

An example of each case is described in the following sections for each of these examples the capacity of the links in the system were set to 20% larger than the initial power flowing through the system when it is stable.

3.1 Variation in the link which is remove first

The following are examples of each of the four cases and which links need to be removed to create each result. The networks are referred to as Network 1-4 because the networks used in effect are all similar but have one line removed from the original.

Case 1: Small cascade network remains connected

If the link between nodes 3 and 4 we create Network 1 shown in the below figure 17 in red was removed the only other links to fail are the two shown in yellow that is link 3-18 and link 4-14. The output of this simulation is:

“3-4” “3-18” “4-14”

It is a list of links which failed in the order in which they failed. This is because the power transferred through this part of the network is relatively small and can be absorbed by the rest of the network. Without any one line taking a more than a 20% increase in current.

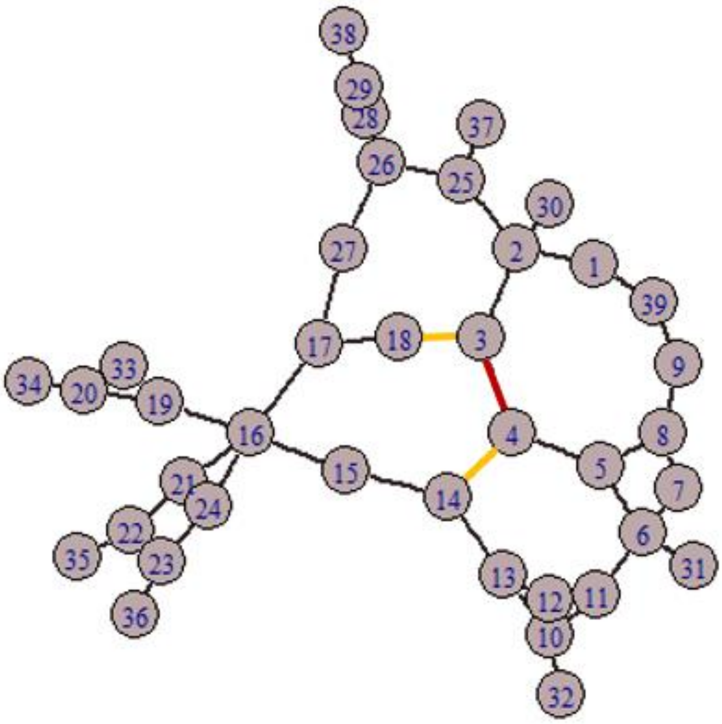

Figure 17 Input Network 1 indicating failure of line 3-4

Link 3-4 carried 250 Wpu whereas link 3-18 carried only 5 Wpu and 4-14 carried 22 Wpu, both nodes 3 and 4 consume power so without the link to 3 all of the power that node 4 would have consumed that ran through link 3-4 must be diverted through link 3-18 causing this link to fail.

After nodes 3 and 4 are no longer connected node 4 is a power consumer with no power flowing into it, this reverses the polarity between nodes 4 and 14 causing he line to be marked as failing within this model.

The disconnections here do not cause the algorithm to produce a new graph as the network remains connected with no further failure occurring.

Case 2: After several steps some nodes continue to consume some do not

Network 2 has 15-16 removed and is shown below in red 11 lines initially fail due to overload. This is because this line is near the middle of the network and had 650 Wpu flowing through it. A much higher proportion of the power than line 3-4. After the initial 11 lines fail the graph is split into several islands.

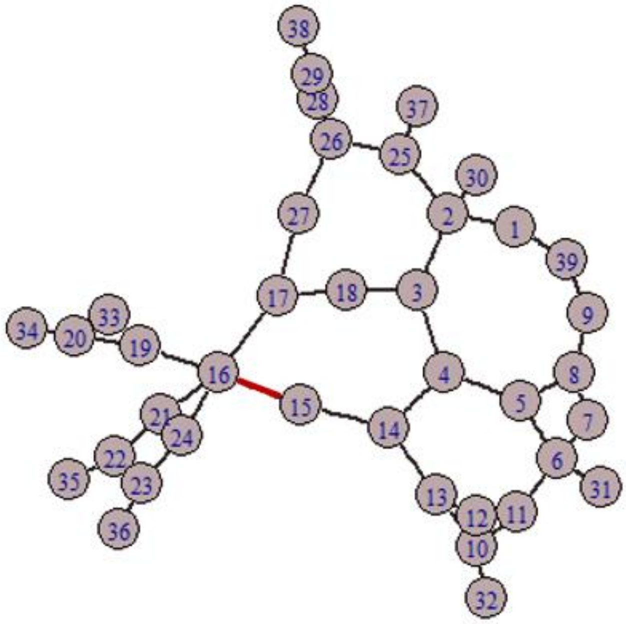

Figure 18 Input Network 2 Graph Indicating Failure of line 15-16

The algorithm asses which of the islands have at least one consumer node and at least one generator. For this failure, there are 3 such islands shown below in figure 19.

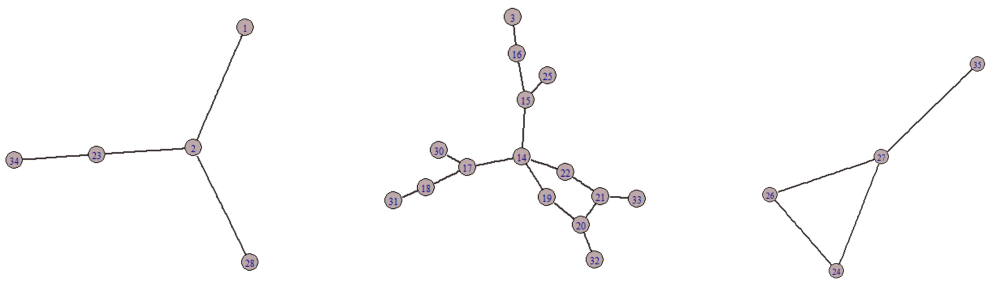

Figure 19 three Islands produced by the failure of line 15-16

Each of these three networks were considered separately and treated as new input matrices for the algorithm.

The first network contains a generator at node 34 and consumers at both nodes 23 and 28. After the new powers through each of these lines is calculated no further failures occur. This island can be self-sufficient and can continue to operate independently of the rest of the network.

The second island contains 4 generators which is a very large proportion of the power produced for the whole network. When the power is recalculated through this island a further 9 edges fail. The result is several completely disconnected nodes and no new connected networks which contain a generator and a consumer.

The final island contains one generator and 3 consumers this island also functions as a sub network. That means that for this network the calculations stop here. The larger the networks the more generations of islands could be formed from a single failure.

The following is the final output of this failure:

```
"15-16" "23-36" "20-34" "17-18" "16-24" "10-13" "10-11" "4-14" "3-18" "2-3" "1-39"
       "19-33" "12-13" "26-28" "26-27" "25-26" "23-24" "21-22" "17-27" "4-5"
```

Case 3: There is No Cascade

Network 3 represents the network with link 16-17 removed this link has the smallest amount of current flowing through it of all the links in this network it is shown in the figure below in red.

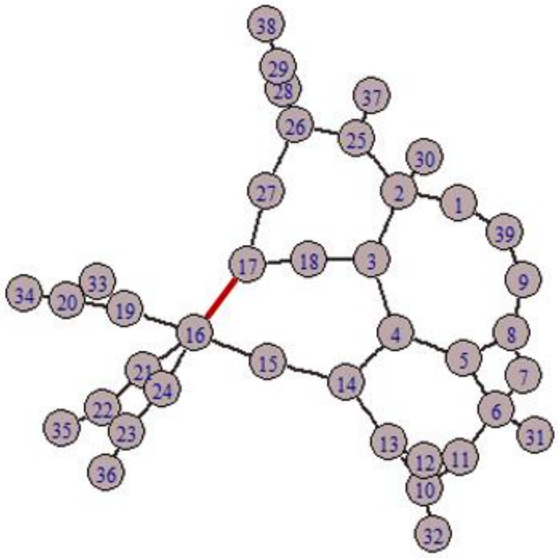

Figure 20 Input Network 3 Indicating Failure of Line 16-17

When this link is removed, it is the only line to fail and every node still receives the power it is supposed to. So in this care the output is simply "16-17"

Case 4: The system fails entirely

Network 4 is represented below it does not contain link 25-26, when this network is used the failure propagates through the entire network until there are no generating node connected to any consumer nodes and the entire network is without power.

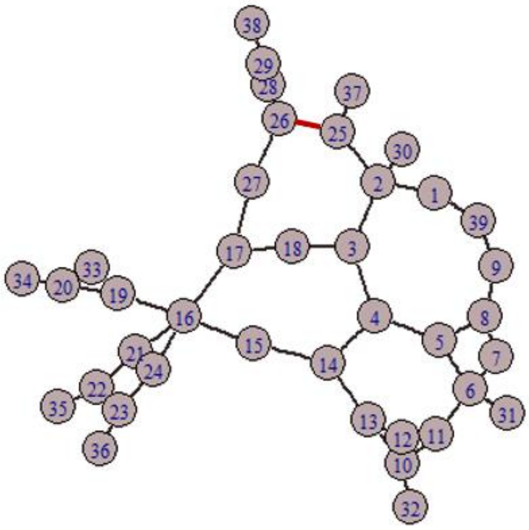

Figure 21 Input Network 4 Indicating failure of line 25-26

Failure in this line is investigated further in the following section.

3.2 Variation in spare capacity in the network

The capacity of the network was varied and link 15-16 was removed each time which is the same as in case 2 above.

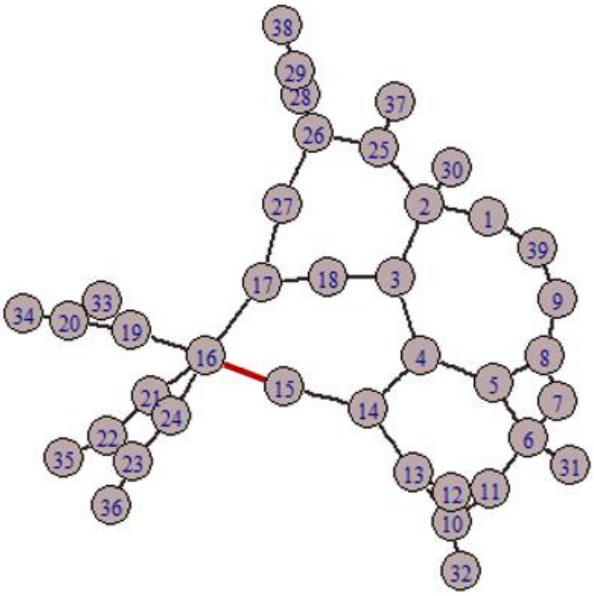

Figure 22 Replication of Figure 18: Input Network 2

Table 1 shows the resultant power delivered and the number of nodes connected in a network containing both power consumer's and power generators.

Table 1 Varying the capacity of Network 2

Spare Capacity	Connected nodes	Power Delivered	% of total power delivered
10%	9	1236.6	24.31
50%	13	2056.28	40.42
100%	14	2426.6	47.70

This was repeated with the removal of link 1-2 as shown below in figure 23 and the results are shown in table 2.

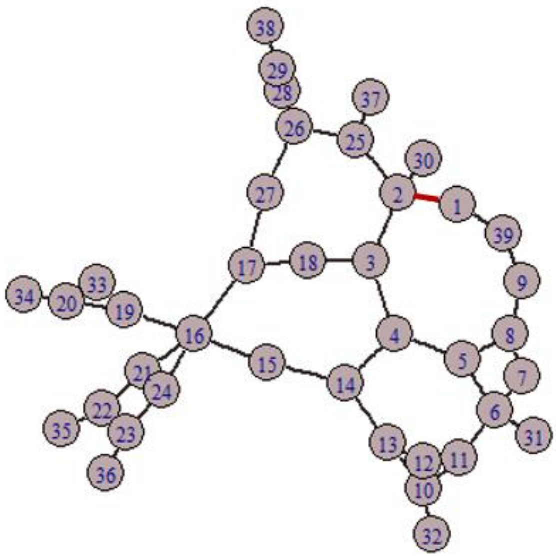

Figure 22 Input Network 5 indicating failure of line 1-2

Table 2 Varying the Capacity of Network 5

Spare Capacity	Connected Nodes	Power Delivered	% of total power delivered
10%	8	1236.6	24.31
50%	13	2056.28	40.42
100%	14	2596.28	51.03

Table 3 shows the results from the removal of line 25-26 under different levels of loading.

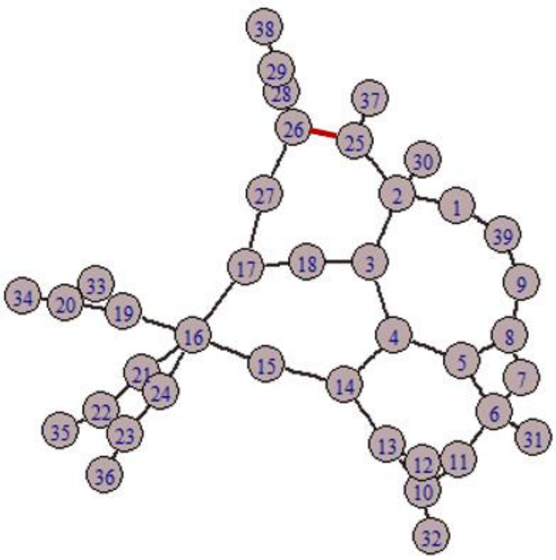

Figure 234 Replication of figure 21: Input Network 4

Table 3 Varying the Capacity of Network 4

Spare Capacity	Connected nodes	Power Delivered	% of total power delivered
10%	0	0	0
50%	20	2177.3	42.80
100%	20	2177.3	42.80

3.4 Discussion of the Results

The results show the size and propagation of the cascade changes significantly when the initial failure is moved throughout the network. This supports the theory that to protect a system from catastrophic failure some lines should be protected more than others. For line 16-17 in this system, less protection would be necessary than for line 25-26 because we can see that failure on line 16-17 is likely not to cause any power outage whereas on line 25-26 it is likely that the majority of the system will lose power. These kinds of results are important and can be used effectively when designing grid protection as it shows where money can be saved on redundancy and where it is most important.

The failures in this model are forced and so they do not show where the grid is most likely to fail but the second set of results does show clearly that the spare capacity of the grid at the time of the failure has an effect on the power which can be delivered. With all three line failures showing a difference in the power which is delivered after a failure occurs when the grid is operation near full load with a spare capacity of only 10% in comparison to when the grid is operating at half load.

Increased spare capacity at the time of the failure does having a significant effect on the size of the failure within a system. If the system is working at 100% spare capacity meaning the grid is operating at half its maximum load in all three locations the after the failure stabilised there was 40-50% of power being delivered to a consuming in comparison to when it is operating with a spare capacity of only 10% where the power delivered halved for 2 Networks or the network was completely destroyed as in network 4.

Links 1-2 and 25-26 were carrying 277.876 and 233.138 respectively when they system was stable this makes them lines that are just under the average line power of 289.926. Although both lines are similar in terms of the power they carry even with a very small capacity the system partially survives a failure in line 1-2 but does not in line 25-26.

Given a larger spare capacity the failure of either line results in a similar level of power delivery. This highlights the importance of simulation to power grid development because these two lines look very similar before the simulation is carried out but their importance to the stability of the grid is significantly different.

In network 2, the initial failure occurs on link 15-16 which had an initial power of 650Wpu making it significantly higher power than links 1-2 or 25-26 however it's loss from the grid had a similar result as that of line 1-2 at the various levels of capacity. This suggests that the current a specific line is carrying does not define the consequences of its loss very accurately alone and that there are several factors involved again supporting the case for power grid failure simulation.

4. The Broadband Communications and Distributed Systems Simulation

The R code discussed previously was integrated into the Simulator so that the robustness analysis could take place. Figure 24 shows the same network as was analysed in the previous section in the simulator.

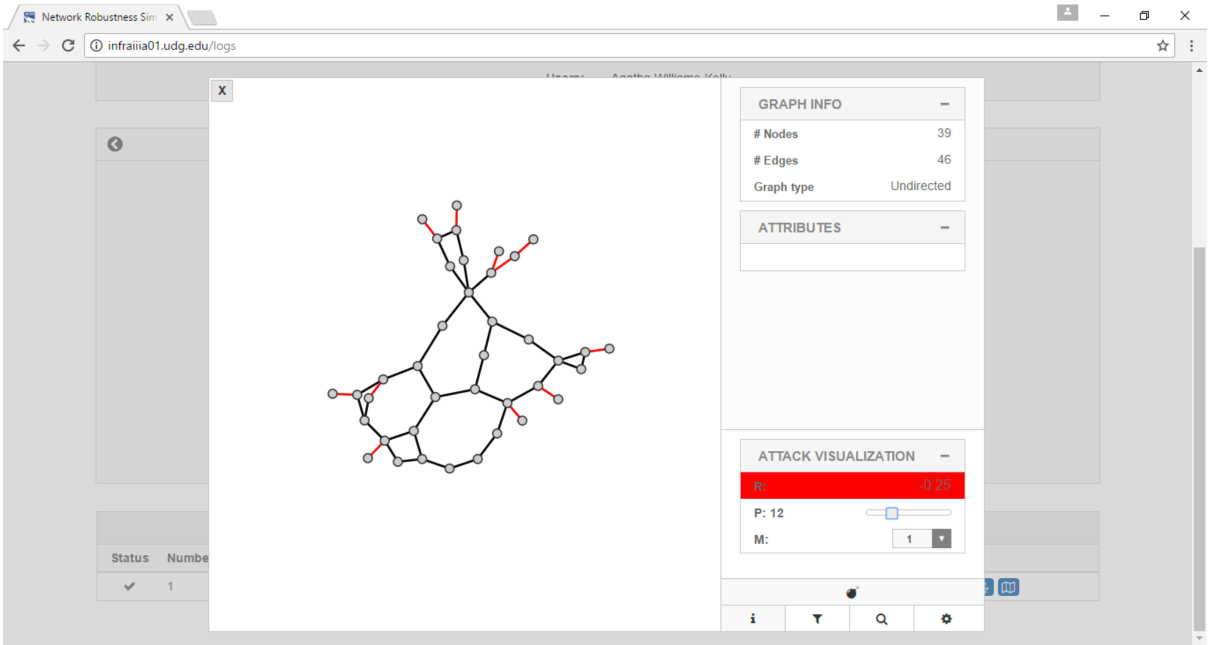

Figure 24 BCDS Simulator

On the right-hand side of the screen we can see the controls for the attack visualisation. As the slider P, is moved the propagation of the failure through the network can be observed. Each link turns red when it fails. At this point in the simulation the slider is at point P which corresponds to the 12th time slot and so we can see the first 11 lines that have failed the first slot being occupied by the system with no failure.

Any graph that has the correct parameters can be uploaded to this simulator and analysed to see the progression of a cascading failure throughout it.

5. Improvements/Further Work

For this model to accurately represent a cascade in a power system it would need to incorporate a method to calculate the probability of hidden failures and incorporate that into the cascade propagation. This however will never be an accurate model as it will rely on probabilities. Hidden failure includes device malfunction and human error the location of which occur randomly and cannot be predicted. It is not necessary to take hidden failures into account when viewing the propagation patterns on a failure.

The model could become more accurate if AC power flow was calculated instead of DC. This model uses a simplified version of the power equations to estimate the power through each line using an equation which can be linearly programmed. If the full versions of the equations were used the accuracy of the power being transported through each line would have increased accuracy, however there is a huge computational cost to this as well as it being far more complex. For this to be meaningful accurate capacities would also need to be included.

The model analysed within the project was a very small unrealistic power grid as they tend to have thousands of nodes and links. The complex nature of a cascade failure can be better viewed with a larger network and the islands formed are more likely to be self-sufficient.

This model also assumes the failures to be near instantaneous with no changes being made to power generation over the course of the failure. Cascade failures do happen fast but not instantaneously, as could be seen in the Italy blackout of 2003 there was enough time for operators to begin reducing generation before the cascade cause the networks to disconnect. This kind of human intervention could be incorporated into further models which would more accurately represent a cascade.

The model is relatively unstable when compared to the other types of failures being analysed in the simulation. That is a cascade model is very likely to have several links fail simultaneously. This leads to uncertainty in the output as the order of failed links which fail at the same time is random currently in future versions the output could include several versions of the ordered list to account for any variation this will cause in robustness analysis.

6. Conclusion

A computer model to represent a cascading failure was created in MATLAB the same algorithm was then used to implement the model in R. The R code was used to incorporate the model into a larger simulator which looks at different forms of network failure.

The largest difference between a cascading failure from other network failures such as epidemics is that each failure could be caused by a failure in a line which is not connected to a line which previously failed. This gives rise to the islanding behaviour we have seen which is a problem in simulating the cascading failure as it means the algorithm must be applied to several subnetworks before a final output is reached.

This kind of model can be used to effectively analyse the robustness of a network as well as identify the points which if attached will have the most catastrophic results. This is important both in grid design and in grid improvement.

The spare capacity in the system and the location of the original fault have a significant effect on the damage done by a cascading failure highlighting the importance of built in redundancy as well as showing why certain times of day or year mean the grid is more volatile.

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