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Analysing assets' performance inside a portfolio: From crossed beta to the net risk premium ratio[i]

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Abstract: This paper is focused on enlarging the performance inside a portfolio that provides the Treynor ratio by relating portfolio weights with performance indicators. Intuition suggests that the higher the weight of an asset, the higher should be its expected performance. These weights, and the information that we can obtain from their analysis, are not only relevant for investors but also for corporate managers. Nevertheless, the available performance indicators are not linked to portfolio weights. In order to fulfil this gap we answer three questions: which is the minimum risk premium that justifies holding an asset in long position? How can we analyse if the performance of an asset justifies the budget's weight invested in it? And, how can we apply ex-post optimisation to performance analysis? Methodologically, we centre the analysis on the definition of crossed beta and the net risk premium ratio that stems from it. The latter fulfils the axioms of risk/reward performance measures. The three answers to the questions are related to the net risk premium. The analysis in developed for the Mean-Variance and Mean-Gini models. The empirical illustration, based on DJIA assets, that completes the paper shows how the analysis of portfolio weights provides relevant information about the performance of assets.

ABOUT THE AUTHORS

Maria Teresa Bosch-Badia is a lecturer at the Universitat de Girona. Joan Montllor-Serrats is Professor of Finance, and Maria Antonia Tarrazon-Rodon is Associate Professor of Finance, both at the Universitat Autonoma de Barcelona. Their main research interests are asset valuation, including capital budgeting and real options, performance analysis, corporate financial policy, and Corporate Social Responsibility. Previous articles of the authors in the field of performance have been focused on the functional relationships among different performance measures (*Journal of Banking & Finance*, 2014, 42, 154–165). This paper is part of a wider research on the analysis of the financial meaning of optimal portfolio weights and the information they embed. About this subject, the authors have published a paper focused on the CAPM inverted matrix and the Bamberg and Dorfleitner model (*Review of Managerial Science*, 2016). A previous version of this paper was presented at the XXII Finance Forum of the Spanish Finance Association.

PUBLIC INTEREST STATEMENT

Does the performance of any asset justify its weight in the portfolio to which it belongs? This is the question that this paper aims to answer. Intuition suggests that the higher the weight the higher must be the expected performance of the asset, and, also the higher its ex-post performance. Each optimal weight shares with performance measures the fact of being the outcome of a combination between risk and return. Depurating risk measures and the required risk premium from the impact of the asset under analysis on the benchmark portfolio, we obtain the crossed risk indicator and the net risk premium ratio. This ratio fulfils the properties of coherent performance measures, and, together with the ratio between crossed and total risk, explains the weight of each asset. It has a straightforward relationship with the Treynor ratio. The analysis we develop combines ex-ante and ex-post perspectives, and CAPM and Mean-Gini methodologies.

Subjects: Corporate Finance; Investment & Securities; Risk Management

Keywords: common stocks performance; beta; Treynor ratio; Gini portfolios

JEL code: G11

1. Introduction

The aim of this paper is centred on studying the performance of an asset inside a portfolio. Among the classical performance measures, the Treynor ratio (Treynor, 1966) is the appropriate one to evaluate the return/risk relationship of an asset that is part of a portfolio (Amenc & Le Sourd, 2003, p. 109; Bodie, Kane, & Marcus, 2009, p. 830). In these cases, Jensen alpha (Jensen, 1969) also provides a valuable piece of information, but in order to make comparable the alphas of the assets that belong to a portfolio it is necessary to submit them to the beta equalising procedure depicted by Bodie et al. (2009, pp. 829–830). The theoretical framework of these performance measures is the CAPM. Although its explanatory capacity and applicability have been widely discussed in literature, the CAPM and their associated performance measures, continue being a useful tool for theoretical and practical analysis, if the analyst is aware of its limitations. For instance, Bilinski and Lyssimachou (2014) find that beta turns out to be a strong predictor of large positive and negative returns showing that the relation between beta and returns is U-shaped instead of linear. Loviscek (2015) shows that Mean-Variance portfolios outperformed the market during the period 2000–2009 when the S&P 500 rate of return became negative. Levy (2012, pp. 372–401) presents an updated analysis of the CAPM. The main reason why we focus our analysis on the CAPM, together with the Mean-Gini model, is because for our purpose of linking performance with optimal weights we need a theoretical framework that includes an optimisation model. Moreover, since, for our knowledge, the relationship between weights and performance has been scarcely studied, it is essential to deal with the basic models to explore this topic.

Later developments in performance measurement have been centrally focused on investment funds and often on the incorporation of new risk measures to performance indicators. These are, among others, the cases of Sortino ratio (Sortino & van der Meer, 1991), Omega ratio (Shadwick & Keating, 2002), Rachev ratio (Stoyanov, Rachev, & Fabozzi, 2007), Farinelli and Tibiletti (2008) and Homm-Pigorsch ratios (Homm & Pigorsch, 2012). Farnsworth, Ferson, Jackson, and Todd (2002) and Gusset and Zimmermann (2014), among others, explore the properties of the Stochastic Discount Factor as a substitute for beta. Gomez-Bezares and Gomez-Bezares (2015) warn about the limitations of the use of quotients in performance measurement, especially when the risk indicator takes a very small value or even negative, as it can be in the case of beta. These authors propose an alternative measure, the penalised internal rate of return, based on linear penalisation of volatility. Bacon (2013) presents a detailed applicability analysis of performance measures. Ferson (2010) provides a theoretical panorama of the state of the art in financial performance analysis. The study of the performance of common stocks inside a portfolio can be applied to the analysis of passive strategies when managers consider portfolio revision, especially when these managers, as pointed out by Sharpe (1991), do not replicate literarily the market index. In fact, although passive strategies are the most logical option in efficient markets, the composition of their market portfolios needs periodical revision to avoid over-priced securities in them, since no market is completely inefficiencies-free. Passive strategies have been praised by many authors (Malkiel, 2015; Sharpe, 1991); but, on the other hand, they have been the object of criticisms for not fulfilling the optimisation criteria (Amenc, Goltz, Lodh, & Martellini, 2012; Markowitz, 2005). Furthermore, the interest of analysing the performance of common stocks is not only for investors, but also for corporate managers. For a corporation, the comparison of the performance of its shares in the stock market with other shares of its same industry and the stock market as a whole constitutes a highly valuable piece of information that enlightens how the market judges its long run strategic decisions and its short run management.

The approach we adopt is centred on the optimal weights of the assets inside a portfolio. Intuition suggests that the higher the weight of an asset, the higher must be its expected performance.

Nevertheless, the available performance indicators are not linked to portfolio weights. This article aims to fulfil this gap. In order to connect optimal weights to performance analysis, especially to the Treynor ratio, we split beta into the parts due to the asset itself and the rest of assets in the market portfolio which we call, respectively, self-generating beta and crossed beta. Relying on this division, we obtain the net risk premium that leads to a ratio, which we call net risk premium ratio, that fulfils the axioms of performance measures, and enables us to answer the following questions:

Q1. Which is the minimum risk premium to be in long position?

Q2. How can we analyse whether the performance of an asset justifies the budget's weight invested in it?

Q3. How can we apply ex-post optimisation to performance analysis?

The relevance of these questions lies in the importance of assets' weights for corporate managers, portfolio managers and investors. A substantial amount of short sales positions clearly constitutes a motive of concern for the managers of these corporations. The knowledge of the financial causes of optimal portfolio weights enlightens investment decisions, and the comparison between expected and actual values benefits from the more detailed analysis that this approach provides. Furthermore, a relationship between weights and market efficiency can be considered as well. Weights can be regarded as a summary of expectations together with prices. To advance in the knowledge of the information embedded in weights and on how the corresponding variables are combined may contribute to study market efficiency from the point of view of optimal weights, although a detailed analysis of this issue goes beyond the scope of this paper. In an efficient market, the price of each security equates its value, i.e. prices are fair. By the same token, each portfolio weight should approach the optimal one.

Methodologically we first centre on the Mean-Variance paradigm, specifically the CAPM, and next we extend our analysis to the Mean-Gini approach to portfolio selection. As pointed out by Yitzhaki (1982) and Shalit and Yitzhaki (1984, 2005, 2010), the Mean-Gini approach has the advantage, with respect to the Mean-Variance approach, of being compatible with the second-order conditions of stochastic dominance. Furthermore, the Gini approach also admits incorporating investors' risk preferences (Yitzhaki & Schechtman, 2013, pp. 99–132). Gerstenberger and Vogel (2015) show that, for heavy-tailed distributions, Gini's mean-difference is more efficient than the standard deviation.

The article is organised as follows. In Section 2 we develop the concepts of crossed beta and net risk premium. Section 3 is centred on building up the net risk premium ratio and analysing its validity as a performance measure. In Section 4 we relate the net risk premium ratio to the optimal weights. Next, in Section 5, we deal with the connections between the Treynor ratio and the net risk premium ratio. Section 6 extends the previous analysis to the Mean-Gini portfolio model. Section 7 centres on the ex-post optimisation. In this we present an empirical illustration, and Section 8 closes the paper.

2. From crossed beta to the net risk premium

A basic concept for the analysis developed in this paper is the *net risk premium*. We define the net risk premium of an asset as the part of its total risk premium generated by the asset itself, namely, the one that is independent from the interaction between the asset and the rest of assets in the portfolio. To obtain the net risk premium of an asset, we split its covariance with the market portfolio into two parts: the one explained by the weight of the asset in the portfolio, and the one explained by its interaction with the rest of assets. Substituting in the covariance between the asset and the market portfolio, $Cov(R_i,\ R_M)$, the rate of return of the market portfolio by the weighted average of its assets $(\sum_{j=1}^{n} x_j R_j)$, we have:

$$Cov(R_i,\ R_M) = Cov\left(R_i,\ \sum_{j=1}^{n} x_j R_j\right) \tag{1}$$

Hence, with the additive property of covariances in mind, we can write:

$$Cov(R_i,\ R_M) = x_i\sigma_i^2 + \sum_{j=1\forall j\neq i}^{n} x_j Cov(R_i, R_j) \tag{2}$$

Rubinstein (2006, p. 174) writes the covariance decomposition of Equation (2) in his proof of the property, introduced by Lintner (1965), about the negligible effects on value of the own variance in large markets.

The first addend of (2) can be called self-covariance, while we may call crossed covariance the second one. We denote it by σ_{Ci}:

$$\sigma_{Ci} = \sum_{j=1\forall j\neq i}^{n} x_j Cov\left(R_i,\ R_j\right) \tag{3}$$

On this basis, we define crossed beta (β_{Ci}) as the part of beta generated by the crossed covariance:

$$\beta_{Ci} = \frac{\sum_{j=1\forall j\neq i}^{n} x_j Cov\left(R_i,\ R_j\right)}{\sigma_M^2} \tag{4}$$

Thus, the net risk premium (π) can be obtained by subtracting from the total risk premium the part of it due to crossed beta:

$$\pi_i = \left(\bar{R}_i - r\right) - \left(\bar{R}_M - r\right)\beta_{Ci} \tag{5}$$

where the term $(R_M - r)\beta_{Ci}$ can be called crossed risk premium because it is the part of the risk premium due to crossed beta.

Any net risk premium has its required counterpart, which consists of the difference between the required risk premium according to the Security Market Line (SML) and the crossed risk premium:

$$\pi_i^* = \left(\bar{R}_M - r\right)\left(\beta_i - \beta_{Ci}\right) \tag{6}$$

Hence, the net risk premium can be written as the addition of Jensen alpha and the required net risk premium (π_i^*):

$$\pi_i = \alpha_i + \pi_i^* \tag{7}$$

Both, alpha and the required net risk premium, have different consequences for the analysis we develop below.

3. The net risk premium ratio as a performance measure

By turning the net risk premium into a ratio that expresses the points of total risk premium per point of crossed risk premium, that we denote by η:

$$\eta_i = \frac{\left(\bar{R}_i - r\right)}{\left(\bar{R}_M - r\right)\beta_{Ci}} \tag{8}$$

we obtain a coherent performance measure that fulfils the axiomatic conditions presented by Schuhmacher and Eling (2012). Let us call it net risk premium ratio.

In Equation (8), the numerator fulfils the properties of positive homogeneity and functional translation invariance for reward measures, as Equations (9) and (10) show:

$$E\left[k\left(R_i - r\right)\right] = k\,E\left(R_i - r\right) \tag{9}$$

$$E\left[\left(R_i - r\right) + c\right] > E\left(R_i - r\right) \tag{10}$$

where $E(\cdot)$ denotes expected value[1]; k and c are positive constants.

The denominator, in turn, fulfils the equivalent conditions for risk measures. Substituting in it β_{Ci} according to (4) and recalling the properties of covariance, we can write:

$$\left(\bar{R}_M - r\right)\frac{\sum_{j=1\forall j\neq i}^{n} x_j Cov\left(kR_i, R_j\right)}{\sigma_M^2} = \left(\bar{R}_M - r\right)k\beta_{Ci} \tag{11}$$

for the positive homogeneity property, and:

$$\left(\bar{R}_M - r\right)\frac{\sum_{j=1\forall j\neq i}^{n} x_j Cov\left(R_i + c, R_j\right)}{\sigma_M^2} = \left(\bar{R}_M - r\right)\beta_{Ci} \tag{12}$$

for the functional translation invariance.[2]

Interpreting these axioms for the present case, we can say that positive homogeneity means that, for an individual who invests the k% of his budget in the asset under analysis (asset i) and the rest of it in the risk-free asset, his expected return and the corresponding required risk premium experience a proportional change, as shown by (9) and (11). Therefore, the net risk premium ratio is neutral with respect to changes of scale. As for the functional translation invariance, if asset i captures an abnormal return, the increase in its return does not change its required risk premium, as shown by (10) and (12).

From (6) we can write the required net risk premium ratio (η^*) as:

$$\eta_i^* = \frac{\left(\bar{R}_M - r\right)\beta_i}{\left(\bar{R}_M - r\right)\beta_{Ci}} \tag{13}$$

Subtracting (13) from (8), the difference between the actual and the required net risk premia ratio, that we will call excess net risk premium ratio (ϖ), turns out to be:

$$\varpi_i = \eta_i - \eta_i^* = \frac{\alpha_i}{(\bar{R}_M - r)\beta_{Ci}} \tag{14}$$

i.e. the units of alpha per unit of crossed risk premium.

The excess net risk premium ratio can be applied as well to the performance analysis for short positions. In this case, we invert the difference between the required and the expected net risk premium ratio, and write the excess net risk premium ratio for short positions as:

$$\omega_i^S = \eta_i^* - \eta_i = \frac{-\alpha_i}{\left(\bar{R}_M - r\right)\beta_{Ci}} \tag{15}$$

4. Relating the net risk premium to the optimal weights: The short sales barrier

The net risk premium embeds the condition that an asset must fulfil in order to be held in long position in the optimal portfolio. As we show next, this condition can be obtained from the SML. Sharpe (1970, Chap. 5, p. 89) in his deduction of the SML connects the parameters of a single security with the market portfolio, arriving at:

$$\bar{R}_i - r = \frac{\bar{R}_M - r}{\sigma_M^2}\sigma_{iM} \tag{16}$$

where σ_{iM} is the abridged notation for $Cov(R_i,\ R_M)$.

Substituting σ_{iM} in (16) according to (2), the optimal weight of the asset under analysis can be written as:

$$x_i = \frac{(\bar{R}_i - r)}{(\bar{R}_M - r)}\frac{\sigma_M^2}{\sigma_i^2} - \frac{1}{\sigma_i^2}\sum_{j=1\forall j\neq i}^{n} x_j Cov(R_i,\ R_j) \tag{17}$$

In this expression, the weight of the asset under analysis is also part of the market parameters $(\bar{R}_M,\ \sigma_M)$. Therefore, strictly, it is an implicit equality. Nevertheless, as pointed out by Lintner (1965) and Rubinstein (2006, p. 174), among others, if the capital market is large, and, thus, the weight of each asset in it is small, the effect of any asset in the market parameters is negligible. According to this argument, the presence of the asset in the market parameters does not hinder the CAPM as a theoretical model. Nevertheless, the fact that the weight of an asset in the market can be very small does not make it irrelevant. What finally matters for a corporation is the total value of its assets, which is the product of the weight by the total value of the market. As the market grows, the weight of each one of its assets diminishes, but the product of a specific weight by the market value may remain constant or even increase. Even if the weight approaches zero when the market value approaches infinity, the product between both obviously does not approach zero.

Multiplying and dividing the second addend of (17) by σ_M^2, and substituting $\sum_{j=1\forall j\neq i}^{n} x_j Cov\left(R_i, R_j\right)/\sigma_M^2$ by β_{Ci} we have:

$$x_i = \frac{\left(\bar{R}_i - r\right)}{\left(\bar{R}_M - r\right)}\frac{\sigma_M^2}{\sigma_i^2} - \frac{\sigma_M^2}{\sigma_i^2}\frac{\sum_{j=1\forall j\neq i}^{n} x_j Cov\left(R_i, R_j\right)}{\sigma_M^2} = \frac{\sigma_M^2}{\sigma_i^2}\left[\frac{\left(\bar{R}_i - r\right)}{\left(\bar{R}_M - r\right)} - \beta_{Ci}\right] \tag{18}$$

and multiplying and dividing β_{Ci} by, we arrive $(\bar{R}_M - r)$ at:

$$x_i = \frac{\sigma_M^2}{\sigma_i^2}\left[\frac{\left(\bar{R}_i - r\right) - \left(\bar{R}_M - r\right)\beta_{Ci}}{\bar{R}_M - r}\right] \tag{19}$$

which, taking (6) into account, is equivalent to:

$$x_i = \frac{\sigma_M^2}{\sigma_i^2}\left[\frac{\pi_i^*}{\bar{R}_M - r}\right] \tag{20}$$

Let us define the total risk ratio for asset *i* as:

$$\mathrm{TRR}_i = \frac{\sigma_i^2}{\sigma_M^2} \tag{21}$$

and the required net risk premium/market ratio for the same asset $\left(\pi_{Mi}^*\right)$ as:

$$\pi_{Mi}^* = \frac{\pi_i^*}{\bar{R}_M - r} \tag{22}$$

Then, we can say that the optimal weight is expressed through the quotient between the requite net risk premium/market ratio and the total risk ratio $\left(\pi_{Mi}^*/\mathrm{TRR}_i\right)$.

It stems from (19) that a positive value of the optimal weight requires a total asset risk premium, $(\bar{R}_i - r)$, greater than the crossed risk premium defined in (5):

$$(\bar{R}_i - r) > (\bar{R}_M - r)\beta_{Ci} \tag{23}$$

Thus, the crossed risk premium is the minimum value of the expected risk premium that justifies a long position of the asset. For this reason, it can be regarded as the short sales barrier. In other words, if the asset risk premium is greater than the crossed risk premium, the asset qualifies for a long position in the optimal portfolio, while, if the sign of this difference is inverted, the asset's position turns into a negative one. On this basis, with (8) in mind, we can interpret the net risk premium ratio as the number of risk premium units per unit of the short sales barrier. Furthermore, the difference between the risk premium and the crossed risk premium that stems from inequality (23) measures the asset's distance to the short position.

The condition for a long position can also be written as a function of the net risk premium ratio. In effect, multiplying and dividing (20) by crossed beta, and recalling that, as it stems from (2) and (3), crossed beta consists of the ratio σ_{Ci}/σ_M^2, we can write:

$$x_i = \frac{\sigma_M^2}{\sigma_i^2}\frac{\sigma_{Ci}}{\sigma_M^2}\left[\frac{\pi_i^*}{(\bar{R}_M - r)\beta_{Ci}}\right] = \frac{\sigma_{Ci}}{\sigma_i^2}\left[\frac{\pi_i^*}{(\bar{R}_M - r)\beta_{Ci}}\right] \tag{24}$$

Substituting π_i^* according to (6), we obtain:

$$x_i = CRR_i\left[\frac{(R_M - r)(\beta_i - \beta_{Ci})}{(\bar{R}_M - r)\beta_{Ci}}\right] = CRR_i\left[\frac{(R_M - r)\beta_i}{(\bar{R}_M - r)\beta_{Ci}} - 1\right] \tag{25}$$

which, taking into account the expression of the required net risk premium ratio stated in (13), can be written as:

$$x_i = \frac{\sigma_{Ci}}{\sigma_i^2}(\eta_i^* - 1) \tag{26}$$

In this equation, the ratio between the crossed covariance and variance (σ_{Ci}/σ_i^2) expresses the proportion of crossed risk on total risk, while, as stated, the term $(\eta_i^* - 1)$ can be interpreted as the short sales distance.

Equation (26) is central for answering Q1 and Q2. Strictly, the point of view of this equation is the ex-ante analysis because it explains how the parameters of the model justify the weight of an asset in the optimal portfolio. Nevertheless, it can also be applied to study the performance of the assets from the ex-post point of view. In this case, the weights of the assets are given and we examine whether the ex-post performance has fulfilled the expectation. To proceed to the ex-post performance analysis through (26), first of all we calculate the actual and the required values of the net risk premium ratio. Substituting the required net risk premium ratio (η_i^*) in (26), we simply confirm the weight of the asset in the portfolio known as an ex-ante information (data) in the present context of ex-post (performance) analysis. Next, we examine the value of the actual net risk premium ratio from the perspective of Equation (26), and we realise that this ratio has two barriers to overcome, given by the following values:

(1) A value greater than 1 to justify a long position in the asset, i.e. $x_i > 0$.
(2) A value equal or greater than its required equivalent $(\eta_i \geq \eta_i^*)$ to justify the weight of the budget invested in the asset.

Thus, therefore we can hold that (a) answers Q1, and (b) answers Q2. In case of negative crossed beta, the sign of (a) is inverted.

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Comparing (20) and (26), we observe that the optimal weights can be explained from two different points of view: the net risk premium, adopted in (20), and the net risk premium ratio, adopted in (26). Both show that the optimal weight is directly proportional to the distance to the short sales barrier embedded in the net risk premium. In (20) the distance is expressed by the net risk premium itself. In (26), we find the distance to the short sales barrier, which consists of the difference between the net risk premium ratio and 1. Both equalities also show, from different perspectives, the impact of total risk on the optimal weights. The optimal weight, as we can see in (20), is inversely proportional to the total risk ratio while, as (26) shows, it is directly proportional to the crossed risk ratio. Both perspectives shed light on the impact of total risk on the optimal weights: the higher the asset volatility with respect to market volatility, the lower the weight; but, the higher the crossed covariance with respect to the asset volatility, the higher the weight. Therefore, we can say that the optimal weight of each asset depends on its total risk, and, thus, on its specific risk. This property shows that, although the specific risk is neutral with respect to the required rate of return, it is not neutral with respect to the optimal weight. If the specific risk of a corporation increases, and this corporation aims to maintain the optimal weight of its shares in a Mean-Variance portfolio, the corporation is compelled to obtain a higher net risk premium that compensates the increase of specific risk.

On basis of (26) we can express the optimal weight of any asset as the addition of the effects of the net risk premium ratio (first addend of (27)) and the correction that the crossed risk ratio introduces in it (second addend of (27)).

$$x_i = (\eta_i^* - 1) + (\eta_i^* - 1)\left(\frac{\sigma_{Ci}}{\sigma_i^2} - 1\right) \tag{27}$$

This result can be regarded as the answer to Q2 because it explains the relationship of the asset's weight and its performance, measured through the net risk premium ratio. As a matter of fact, Equation (27) shows that in order to justify the budget's weight invested in an asset, this asset must reach the expected values of the net risk premium ratio, and the crossed total risk ratio (σ_{Ci}/σ_i^2).

5. Net risk premium ratio vs. Treynor ratio

The net risk premium ratio is closely related to the Treynor (1966) ratio. Not only they have a parallel meaning, but also they are functionally related. Bodie et al. (2009, p. 764) point out that two assets that belong to a well-diversified portfolio cannot be compared solely on basis of alphas because this would mean to put aside their systematic risks. As these authors show, in this case the appropriate measure to compare them is the Treynor ratio (*T*) that, in addition to its classical presentation as the quotient between the risk premium and beta, can be written also as a function of alpha:

$$T_i = \frac{\alpha_i}{\beta_i} + T_M \tag{28}$$

where T_M denotes the Treynor ratio of the market portfolio, namely the market risk premium since its beta equates 1. A similar argument can be used for preferring the net risk premium ratio to the net risk premium when we focus on these measures. In fact, the former can be written as a following function of the latter:

$$\eta_i = \frac{\pi_i}{(R_M - r)\beta_{Ci}} + 1 \tag{29}$$

The functional relationship between the net risk premium ratio and the Treynor ratio is obtained by multiplying and dividing the right-hand side term of (8) by β_i, and rearranging the resulting equality. Then, we realise that the net risk premium ratio equates the product of the proportion between the Treynor ratios of the asset and the market multiplied by the ratio between beta and crossed beta:

$$\eta_i = \frac{T_i}{T_M}\frac{\beta_i}{\beta_{Ci}} \tag{30}$$

This result shows that the net risk premium ratio of an asset is an increasing function of the quotient between its Treynor ratio and the market Treynor ratio, and, at the same time, of the ratio between its beta and its crossed beta.

6. The Mean-Gini approach

The Mean-Gini portfolio model successfully overcomes some of the limitations of the CAPM. Mainly, it fulfils the requirements of the second-order conditions of stochastic dominance, as pointed out in the introduction. Furthermore, its formal presentation shows a productive parallelism with the CAPM that preserves the conceptual clarity of the latter. To obtain the crossed covariance for the Mean-Gini portfolio model (*CC*) a previous development, displayed in Appendix A, is necessary. The result at which we arrive in (A5) is:

$$CC_i = Cov(R_i,\ F_M^G) - x_i^G Cov(R_i, F_i) \tag{31}$$

where *F* denotes the cumulative distribution function.[3] In this section the superscript *G* denotes that the variable is referred to the Mean-Gini model. In this context, the total risk consists of the covariance between the asset or the portfolio with its cumulative distribution. Therefore, the total risk ratio turns out to be $\left[Cov(R_M^G,\ F_M^G)/Cov(R_i, F_i)\right]$. In the Mean-Gini model the beta coefficient is expressed as:

$$\beta_i^G = \frac{Cov(R_i,\ F_M^G)}{Cov(R_M^G,\ F_M^G)} \tag{32}$$

From (31) we can express crossed beta as:

$$\beta_{Ci}^G = \frac{CC_i}{Cov(R_M^G,\ F_M^G)} \tag{33}$$

The required net risk premium, that, by definition, excludes crossed beta, can, in turn, be written as:

$$\pi^{*G} = (\bar{R}_M^G - r)(\beta_i^G - \beta_{Ci}^G) \tag{34}$$

The actual net risk premium can be obtained by adding the value of Jensen alpha, adapted to the Mean-Gini model, to the required net risk premium:

$$\pi_i^G = \alpha_i^G + \pi_i^{*G} \tag{35}$$

Therefore, the net risk premium ratio and its required value turn out to be:

$$\eta_i^G = \frac{(\bar{R}_i - r)}{(\bar{R}_M^G - r)\beta_{Ci}^G} \tag{36}$$

$$\eta_i^{*G} = \frac{(\bar{R}_M^G - r)\beta_i^G}{(R_M^G - r)\beta_{Ci}^G} \tag{37}$$

Substituting in (34) β_i^G according to (32) and β_{Ci}^G according to (33), we have:

$$\pi^{*G} = (\bar{R}_M^G - r)\left(\frac{Cov(R_i,\ F_M^G) - CC_i}{Cov(R_M^G,\ F_M^G)}\right) \tag{38}$$

and substituting CC_i according to (31) and simplifying:

$$\pi_i^{*G} = (\bar{R}_M^G - r)\frac{x_i^G Cov(R_i,\ F_i)}{Cov(\bar{R}_M^G,\ F_M^G)} \tag{39}$$

Clearing x_i^G in this equation, we obtain:

$$x_i^G = \left[\frac{Cov(R_M^G,\ F_M^G)}{Cov(R_i,\ F_i)}\right]\left[\frac{\pi_i^{*G}}{(\bar{R}_M^G - r)}\right] \tag{40}$$

This result, similar to (20), expresses the optimal risk premium as the product between the inverse of the total risk ratio and the required net risk premium/market ratio for the Gini case. In the Gini framework, the total risk ratio consists of the quotient $Cov(R_M^G,\ F_M^G)/Cov(R_i,\ F_i)$, while the net risk premium/market ratio becomes $\pi_i^{*G}/(R_M^G - r)$.

Substituting (34) in (40):

$$x_i^G = \left[\frac{Cov(R_M^G,\ F_M^G)}{Cov(R_i,\ F_i)}\right]\left[\frac{(\bar{R}_M^G - r)(\beta_i^G - \beta_{Ci}^G)}{(\bar{R}_M^G - r)}\right] \tag{41}$$

Multiplying and dividing this expression by β_{Ci}, we have:

$$x_i^G = \left[\frac{Cov(R_M^G,\ F_M^G)}{Cov(R_i,\ F_i)}\beta_{Ci}^G\right]\left[\frac{(\bar{R}_M^G - r)(\beta_i^G - \beta_{Ci}^G)}{(\bar{R}_M^G - r)\beta_{Ci}^G}\right] \tag{42}$$

Substituting (37) in the right-hand side of (42), we obtain:

$$x_i^G = \left[\frac{Cov(\bar{R}_M^G,\ F_M^G)}{Cov(R_i, F_i)}\beta_{Ci}^G\right](\eta_i^{*G} - 1) \tag{43}$$

And, substituting β_{Ci}^G according to (33), we arrive:

$$x_i^G = \left(\frac{CC_i}{Cov(R_i,\ F_i)}\right)(\eta_i^{*G} - 1) \tag{44}$$

which can be written as:

$$x_i^G = (\eta_i^{*G} - 1) + (\eta_i^{*G} - 1)\left(\frac{CC_i}{Cov(R_i,\ F_i)} - 1\right) \tag{45}$$

where $\left[CC_i/Cov(R_i,\ F_i)\right]$ expresses the crossed total risk ratio for the Gini case.

From (36), (40) and (45) we have obtained the answers to Q1 and Q2 from the point of view of the Mean-Gini model. In effect, (44) shows that the condition for a long position is again a net risk premium ratio greater than 1. Relating (44) to (36), we realise that this condition is equivalent to a risk premium greater than the required one according to crossed beta. As for the optimal weights, the parallelism between the Mean-Variance and Mean-Gini models can be seen by comparing (40) with (20), and (45) with (27).

7. Ex-post optimisation

Often, the performance of common stocks is analysed individually. Nevertheless, by applying portfolio optimisation techniques to the ex-post portfolio data, we may find which would have been the optimal portfolio if the expectations had equated the ex-post data. In other words, this calculation answers the question: which would be the optimal composition of a portfolio to be built up now if the future expectations were equal to the current ex-post data? To obtain sound results, short positions are to be discarded. Sharpe (1970, p. 85) points out that the market portfolio, i.e. the optimal portfolio, becomes inefficient from the ex-post perspective because, if investors had been able to forecast exactly the market evolution, they would have invested their whole budget in the asset with the greatest return. Our use of the ex-post data is, nevertheless, different because we estimate which would be the optimal portfolio for the next period if the expectations for this period equated the current ex-post data. The relevance of this optimisation lies on ranking the assets according to the weights they will reach in the optimal portfolio by extrapolating the current data. The weights we obtain after this optimisation can be analysed through the net risk premium ratio and the weights' decomposition we have presented before. The results of this analysis show the performance properties of the asset in the extrapolated portfolio. Furthermore, we may compare the different weights obtained from Gini and Mean-Variance. In order to compare these weights, we subtract (20) from (33). Nevertheless, a preliminary step will clarify this difference. Let us define the differential crossed risk ratio (dc) as:

$$dc_i = \left(\frac{\sigma_{Ci}}{\sigma_i^2} - 1 \right) \tag{46}$$

And the Gini's differential crossed risk ratio (g) as:

$$g_i = \left(\frac{CC_i}{Cov(R_i,\ F_i)} - 1 \right) \tag{47}$$

Introducing these notations in the difference between (33) and (20), we obtain:

$$x_i^G - x_i = \left(\eta_i^{*G} - \eta_i \right) + \left[g_i \left(\eta_i^{*G} - 1 \right) - c_i \left(\eta_i^* - 1 \right) \right] \tag{48}$$

which shows that the difference between the optimal weights of both models consists of the difference between their required net risk premia ratio, $(\eta_i^{*G} - \eta_i^*)$, minus the difference between the risk corrections. As we can see, each risk correction consists of the product between each differential risk ratio (g and c, respectively) by the corresponding short sales distance.

8. Empirical illustration: The net risk premium ratio and the optimal weights in performance analysis

An empirical illustration complements the theoretical part of this paper. Its goal is to show how it enables us to answer, in practice, the three questions asked in the introduction. This illustration is based on the thirty stocks that constituted the DJIA at the beginning of 2014. We study, from the Mean-Variance and Mean-Gini points of view, their performance during 2014, the most recent year, before 2016, in which the DJIA has captured a positive risk premium. To this end, we have built up a portfolio constituted by these stocks weighting them according to their capitalisation. In other words, this illustration is based on the DJIA, but changing its price-weighting method by the most usual capitalisation method. Table 4 shows the detail of this calculation. The interest rate is set out to zero due to the low interest rates of this period.[4] In case of positive interest rates, the interest rate would be subtracted from the rates of return in order to obtain the stocks' and market risk premia. The data-set of this illustration consists of the weekly rates of return obtained from the adjusted prices, i.e. dividend and splits corrected, downloaded from Yahoo!Finance. The period of this data goes from 31th December 2013 to 31th December of 2014. Each rate of return has been calculated from the logarithm of the ratio between the corresponding price at the beginning and at the end of each week. Table 1 contains the answers to Q1 and Q2, while Table 2 complements it by showing the relationship between the performance measures and the weights of the assets. Table 3, answers Q3. Table 4, in Appendix B, shows the basic calculations that support the previous tables.

Table 1 shows the values of the Treynor ratio, the actual risk premium ratio, the required net risk premium ratio, and their difference, the excess net risk premium ratio. The calculations have been made for the Mean-Variance and the Mean-Gini models. As seen in Equation (26), the condition that justifies a long position in the portfolio is a net risk premium ratio greater than 1, which answers Q1. This condition is fulfilled by sixteen stocks, for both the Mean-Variance and the Mean-Gini criteria. For instance, NKE risk premium is 5.05 times the short sales barrier according to Mean-Variance, and

Table 1. Net risk premium ratio

Asset	Mean-Variance								Mean-Gini							
	Treynor (%)	Rank	η	Rank	η^*	Rank	(ϖ)	Rank	Treynor (%)	Rank	η	Rank	η^*	Rank	(ϖ)	Rank
AXP	0.07	21	0.45	21	1.043	24	−0.592	21	0.07	22	0.429	22	1.027	25	−0.597	22
BA	−0.05	26	−0.31	26	1.064	21	−1.376	26	−0.05	26	−0.306	26	1.033	23	−1.339	26
CAT	0.07	22	0.44	22	1.045	23	−0.601	22	0.08	20	0.464	20	1.023	28	−0.559	20
CSCO	0.56	5	3.48	6	1.069	20	2.408	6	0.60	4	3.654	4	1.047	17	2.607	4
CVX	−0.09	29	−0.58	29	1.150	5	−1.727	28	−0.09	29	−0.546	29	1.069	10	−1.615	29
DD	0.27	15	1.61	15	1.031	29	0.577	15	0.28	13	1.620	15	1.017	29	0.603	15
DIS	0.51	8	3.13	8	1.071	18	2.058	8	0.52	7	3.186	7	1.052	16	2.134	7
GE	−0.08	28	−0.52	28	1.118	9	−1.639	27	−0.08	28	−0.495	28	1.079	8	−1.573	27
GS	0.16	17	0.95	17	1.040	27	−0.088	17	0.16	17	0.950	17	1.024	26	−0.074	17
HD	0.56	6	3.45	7	1.070	19	2.384	7	0.49	8	2.948	8	1.037	21	1.910	8
IBM	−0.43	30	−3.07	30	1.239	2	−4.313	30	−0.45	30	−2.943	30	1.127	2	−4.071	30
INTC	0.65	4	4.08	4	1.084	14	2.991	4	0.64	3	3.838	3	1.045	19	2.793	3
JNJ	0.30	12	1.91	12	1.100	10	0.815	12	0.31	11	1.952	11	1.078	9	0.874	11
JPM	0.12	18	0.79	18	1.121	7	−0.329	18	0.12	18	0.767	18	1.066	11	−0.299	18
KO	0.18	16	1.19	16	1.134	6	0.059	16	0.17	16	1.082	16	1.079	7	0.003	16
MCD	0.02	25	0.09	25	1.041	26	−0.950	25	0.02	25	0.105	25	1.045	18	−0.940	25
MMM	0.29	13	1.71	14	1.036	28	0.675	14	0.29	12	1.695	13	1.023	27	0.672	12
MRK	0.34	10	2.16	10	1.099	11	1.057	10	0.35	10	2.156	10	1.058	13	1.098	10
MSFT	0.55	7	3.77	5	1.196	3	2.579	5	0.52	6	3.380	6	1.116	3	2.264	6
NKE	0.81	1	5.05	1	1.075	16	3.970	1	0.97	1	5.883	1	1.055	15	4.828	1
PFE	0.09	19	0.58	19	1.099	12	−0.515	19	0.09	19	0.551	19	1.059	12	−0.508	19
PG	0.65	3	4.14	3	1.096	13	3.045	3	0.56	5	3.549	5	1.091	5	2.458	5
T	0.04	23	0.28	23	1.121	8	−0.844	23	0.04	23	0.270	23	1.082	6	−0.813	23
TRV	0.42	9	2.44	9	1.011	30	1.433	9	0.41	9	2.393	9	1.009	30	1.384	9
UNH	0.71	2	4.25	2	1.043	25	3.210	2	0.66	2	3.909	2	1.027	24	2.883	2
UTX	0.08	20	0.50	20	1.050	22	−0.549	20	0.08	21	0.454	21	1.033	22	−0.578	21
V	0.28	14	1.76	13	1.082	15	0.678	13	0.27	14	1.645	14	1.038	20	0.608	14
VZ	0.02	24	0.14	24	1.071	17	−0.933	24	0.02	24	0.147	24	1.057	14	−0.910	24
WMT	0.31	11	2.09	11	1.171	4	0.923	11	0.27	15	1.750	12	1.105	4	0.645	13
XOM	−0.07	27	−0.47	27	1.257	1	−1.730	29	−0.07	27	−0.452	27	1.137	1	−1.589	28

Notes: The values of the actual Treynor ratio, the actual net risk premium ratio (η), the required net risk premium ratio (η^*) and the excess net risk premium/market ratio(ϖ_M) for the Mean-Variance and the Mean-Gini models. It shows as well the rankings that stem from these calculations, which are commented in the text of the article.

5.88 times for Mean-Gini. The same ratios for AXP are 0.45 and 0.429, showing that its risk premium has fallen down to the short sales barrier. The required net risk premium ratio is the one that justifies the weight that the asset has in the portfolio, while the actual value of this ratio expresses the performance of the asset with respect to the required value that, in fact, is its benchmark. For this reason, and since it is a portfolio without short sales, all required net risk premium ratios are higher than 1. The highest η^* corresponds to XOM, although its actual net risk premium ratio turns out to be negative. The difference between the actual and required ratios constitutes, as seen, the excess net risk premium ratio (ϖ) that measures how many excess points, i.e. which alpha, the asset has captured per each point required to justify its position in the portfolio, which answers Q2. Observing the values of η and ϖ, we can classify the assets in three groups that, respectively, include sixteen, nine and five assets:

(1) Assets with positive η and ϖ (CSCO, DD, DIS, HD, INTC, JNJ, KO, MMM, MRK, MSFT, NKE, PG, TRV, UNH, V, WTT).
(2) Assets with negative ϖ but positive η (AXP, CAT, GS, JPM, MCD, PFE, T, UTX, VZ).
(3) Assets with negative ϖ and negative η (BA, CVX, GE, IBM, XOM).

All assets of group b) have captured a positive risk premium, but lower than the required one, i.e. their alphas are negative. Furthermore, although the combination $\eta > 1$ with $\varpi < 0$ is theoretically possible, it does not appear in this sample. This case would show the combination of a risk premium higher than the short sales barrier with a negative alpha. Since assets in group (a) present a positive excess net risk premium, we can say, following (14), that their actual risk premium ratios are greater than their required ones. Therefore, they justify the weights invested in them, as shown in (26). The performances of the assets in groups (b) and (c) have not reached the necessary level to justify the corresponding weights. To sum up, Table 1 contains the answers to Q1 and Q2 for the analysed portfolio.

The rankings for the Treynor ratio and the excess net risk premium ratio turn out to be very similar, because the maximum difference in these rankings consists of two positions. Switching to the Mean-Gini parameters, we also observe a remarkable similarity in the rankings with no differences greater than two positions again. We can interpret this fact as a sign of the robustness of the Treynor ratio and the Mean-Variance model. Nevertheless, it does not change neither the potential interest of the information added by the net risk premium ratio nor the stronger statistical analysis provided by the Gini methodology. Being more specific, we can observe that CSCO, with respect to its Treynor ratio rank, goes down one position from the point of view of the Mean-Variance net risk premium, and goes up one position from the Mean-Gini approach, being 5-6-4 its evolution in these ranks. In contrast, the HD evolution is 6-7-8. NKE clearly dominates all rankings in the first position. In this sample, the introduction of the benchmark for the net risk premium does not change any ranking, in other words, the rankings that stem from the net risk premium ratio (η) and their corresponding net risk premia ratio equivalents (ϖ) coincide.

Table 2 complements Table 1 by showing how the weights of the assets are linked to the risk and return parameters that stem from the sample under analysis. The weights of this table are the outcome of the capitalisation criterium that we have introduced in order to build up the portfolio of this illustration, assuming that they represent the optimality generated by the market itself. Thus, all short sales distance are positive, while all risk corrections are negative due to the values of the total risk ratios lower than 1. This approach shows how the expected performance of the net risk premium ratio is turned into the corresponding optimal weights through the risk correction. The decomposition to which we submit the weights shows their components in case they were optimal. Nevertheless, as expected, the ex-post data present deviations from optimality. In the analysis we have developed, the effects on non-optimality are concentrated in the actual net risk premium and, more precisely, in the difference between the actual net risk premium ratio and its required counterpart, i.e. in the excess net risk premium ratio (ϖ) shown in Equation (14) and calculated in Table 1.

Table 2. Breaking down the portfolio weights

Asset	Mean-Variance					Mean-Gini					
	TRR	π_M (%)	SD (%)	CRR	RSKC (%)	TRR	π_M (%)	SD (%)	CRR	RSKC (%)	%
AXP	2.43	4.94	4.2680	0.4754	−2.2389	1.59	3.22	2.6539	0.7646	−0.6248	2.0291
BA	3.61	7.78	6.4123	0.3362	−4.2564	1.86	4.02	3.2562	0.6621	−1.1004	2.1558
CAT	4.13	5.03	4.4520	0.2735	−3.2345	2.01	2.45	2.2680	0.5368	−1.0506	1.2174
CSCO	2.26	5.69	6.8726	0.3668	−4.3520	1.47	3.71	4.7103	0.5351	−2.1896	2.5207
CVX	3.89	19.63	15.0009	0.3366	−9.9517	1.90	9.58	6.9285	0.7288	−1.8793	5.0492
DD	2.98	3.77	3.1145	0.4065	−1.8486	1.62	2.05	1.7046	0.7426	−0.4388	1.2659
DIS	2.09	5.71	7.1242	0.3832	−4.3942	1.49	4.07	5.1766	0.5274	−2.4467	2.7299
GE	2.07	12.31	11.7627	0.5067	−5.8020	1.46	8.68	7.8613	0.7582	−1.9006	5.9608
GS	2.66	4.49	3.9548	0.4270	−2.2662	1.65	2.78	2.4411	0.6917	−0.7525	1.6887
HD	2.48	6.05	6.9789	0.3492	−4.5417	1.56	3.81	3.7498	0.6500	−1.3125	2.4372
IBM	2.65	11.00	23.9282	0.1737	−19.7712	1.47	6.11	12.7189	0.3268	−8.5619	4.1570
INTC	3.15	8.52	8.3824	0.3233	−5.6722	1.79	4.86	4.4969	0.6027	−1.7866	2.7103
JNJ	1.67	9.08	9.9607	0.5451	−4.5307	1.29	7.00	7.8373	0.6929	−2.4072	5.4301
JPM	3.11	14.36	12.0897	0.3822	−7.4689	1.75	8.10	6.6030	0.6998	−1.9823	4.6207
KO	2.35	9.01	13.4266	0.2856	−9.5923	1.53	5.86	7.9151	0.4844	−4.0808	3.8343
MCD	1.00	2.02	4.0689	0.4987	−2.0396	0.95	1.93	4.5046	0.4505	−2.4753	2.0293
MMM	2.34	4.58	3.6117	0.5414	−1.6563	1.50	2.93	2.2889	0.8543	−0.3335	1.9554
MRK	2.70	8.31	9.9031	0.3110	−6.8233	1.57	4.83	5.7558	0.5351	−2.6761	3.0798
MSFT	2.27	14.83	19.6184	0.3327	−13.0919	1.50	9.78	11.5843	0.5634	−5.0579	6.5264
NKE	2.48	3.64	7.4834	0.1961	−6.0156	1.56	2.29	5.4802	0.2678	−4.0124	1.4678
PFE	2.48	10.37	9.9020	0.4214	−5.7294	1.57	6.55	5.9090	0.7061	−1.7365	4.1726
PG	0.86	4.02	9.6441	0.4823	−4.9928	0.95	4.43	9.1075	0.5107	−4.4562	4.6513
T	2.10	8.17	12.0760	0.3224	−8.1828	1.47	5.72	8.2418	0.4724	−4.3486	3.8932
TRV	1.34	0.93	1.0742	0.6450	−0.3814	1.13	0.78	0.8820	0.7856	−0.1891	0.6928
UNH	2.23	3.49	4.2633	0.3668	−2.6996	1.52	2.38	2.6700	0.5857	−1.1062	1.5637
UTX	2.00	4.40	4.9885	0.4395	−2.7958	1.45	3.17	3.2650	0.6716	−1.0724	2.1927
V	3.90	9.23	8.2362	0.2875	−5.8686	1.90	4.50	3.7513	0.6311	−1.3837	2.3676
VZ	1.80	5.31	7.1252	0.4149	−4.1691	1.34	3.97	5.6684	0.5215	−2.7123	2.9561
WMT	1.90	10.19	17.0563	0.3138	−11.7044	1.40	7.52	10.5147	0.5090	−5.1628	5.3519
XOM	2.97	27.61	25.7375	0.3610	−16.4451	1.65	15.37	13.7090	0.6778	−4.4167	9.2923

Notes: TRR = Total risk ratio; π_M = Required Net risk premium/market ratio; SD = Short sales distance; CRR = Crossed risk ratio; RSKC = Risk correction. This table explains the portfolio weights as the addition of the net risk premium ratio effect and the risk correction, following Equations (20), (26) and (27) for the Mean-Variance model, and Equations (40), (44) and (45) for the Mean-Gini model.

Each weight can be alternatively explained by Equations (20), (26) and (27) for the Mean-Variance model, and by Equations (40), (44) and (45) for the Mean-Gini model. These equations provide three different approaches to the generation of the weights:

(1) From the perspective of (20) and (40), we can see that the weights depend on the total risk, specifically on the total risk ratio. In this case, the risk premium effect is represented by the net risk premium/market ratio that, in turn, embeds the impact of crossed beta.
(2) From the perspective of (26) and (44), the weights are explained by the short sales distance multiplied by the crossed total risk ratio. This ratio, as stated, is interpreted as the proportion of total risk that has not been absorbed by crossed risk.
(3) The perspective of (27) and (45) shows how the weights additively depend on the short sales distance and on the proportion of total risk not absorbed by crossed risk.

Let us focus on how the weight of CSCO has been generated according to (a), (b) and (c). The required net risk premium/market ratio for CSCO is 5.03%. This would have been its percentage in the portfolio if its total risk had equated the portfolio variance, and thus, the total risk ratio equated 1. Nevertheless, the value of the total risk ratio turns out to be 2.26. Thus, the optimal weight becomes 2.5207%, which is the ratio between 5.03% and 2.26. The same percentage is reached by multiplying the crossed risk ratio (0.3668) by the short sales distance (0.068726). Finally, the optimal weight can also be broken down into the addition of the short sales distance with the risk correction (−0,04352). The latter is obtained by multiplying the proportion of total risk not absorbed by crossed risk (risk crossed ration minus 1) by the sort sales distance. Again, the calculations for Mean-Gini parallel the Mean-Variance ones.

Table 3 presents the results of the ex-post optimisation for the empirical illustration developed in this paper. The optimisation we have performed follows the works by Markowitz (1952, 1959), Merton (1972) and Huang and Litzenberger (1988), for the Mean-Variance model, developed through *Excel* and *Mathematica*. For the Mean-Gini model, we apply the algorithm developed by Cheung, Kwan, and Miu (2007) through *Excel*. To obtain these portfolios we have submitted to optimisation the data above for 2014 in order to answer the question of which would be the optimal portfolios, for Mean-Variance and Mean-Gini, if the expected data equated the actual ones. The table displays the weights of the new optimisations together with the values of the parameters involved in them. The interest of the ex-post optimisation lies in the study of the interaction among the assets under the caeteris paribus hypothesis about the future. The results can be read from the performance point of view because they can be interpreted as the optimal portfolio that the stocks under analysis would create by extrapolating to the future the actual data and their randomness.

Table 3 is divided in three sections: Mean-Variance, Mean-Gini, and the analysis of the differences in the weights obtained through these methods. The Mean-Variance and Mean-Gini sections explain the generation of the optimal weights. The explanatory relationships are the same that we have applied in Table 2, but now in an optimisation framework. For instance, the weight of CSCO is explained by the ratio between π_M and TRR (0.5118/3.1327) and also by the algebraic addition of its short sales distance and the risk correction for the proportion of total risk not absorbed by crossed risk, weighted again with the short sales distance (0.9617–0.7983). Although this performance is excellent, let us ask ourselves how it could be improved because this question could be applied to any asset. Observing the figures of this table and also recalling Equations (20), (26) and (27), we can say that an increase in the weight can be generated by: an increase in the short sales distance, a reduction in the difference between total risk and crossed risk, and a reduction in the total risk ratio. Comparing the weights of PG and UNH, we observe that UNH has a higher short sales distance, and, consequently, a higher net risk premium/market ratio (0.7634 > 0.3709), which causes an exchange in their position in the rankings: UNH precedes PG in the short sales distance, but PG reaches a higher optimal weight because the effect of the total risk ratio.

Table 3. Ex-post optimal weights

Mean-Variance									
Asset	**Weight**	**β**	**β_c**	**TRR**	**π_M**	**η^***	**CRR**	**SD**	**Risk correction**
CSCO	16.34%	1.0440	0.5322	3.1327	0.5118	1.9617	0.1699	0.9617	−0.7983
HD	3.89%	1.0855	0.9516	3.4440	0.1339	1.1408	0.2763	0.1408	−0.1019
INTC	10.34%	1.5021	1.0510	4.3620	0.4511	1.4292	0.2409	0.4292	−0.3258
MSFT	2.99%	1.0342	0.9401	3.1509	0.0941	1.1001	0.2984	0.1001	−0.0702
NKE	10.82%	0.8893	0.5173	3.4385	0.3720	1.7190	0.1505	0.7190	−0.6108
PG	30.97%	0.6252	0.2544	1.1975	0.3709	2.4581	0.2124	1.4581	−1.1484
UNH	24.66%	1.2619	0.4986	3.0958	0.7634	2.5311	0.1611	1.5311	−1.2845
	100.00%	1		RM	24.84%		σM	9.29%	
Mean-Gini									
CSCO	19.04%	1.0487	0.8490	1.0487	0.1997	1.2352	0.8096	0.2352	−0.0448
HD	2.04%	1.1188	1.0960	1.1188	0.0228	1.0208	0.9796	0.0208	−0.0004
INTC	10.10%	1.5060	1.3540	1.5060	0.1521	1.1123	0.8990	0.1123	−0.0113
MSFT	7.57%	1.0327	0.9545	1.0327	0.0782	1.0820	0.9243	0.0820	−0.0062
NKE	12.40%	0.8789	0.7699	0.8789	0.1089	1.1415	0.8760	0.1415	−0.0175
PG	28.08%	0.6283	0.4519	0.6283	0.1764	1.3904	0.7192	0.3904	−0.1096
UNH	20.77%	1.2605	0.9986	1.2605	0.2618	1.2622	0.7923	0.2622	−0.0545
	100%	1		RM			Gini		

Analysing differences						
	cd	g	g-cd	Distance differences	Risk correction	Difference
CSCO	−0.8301	−0.1904	0.6397	−0.7265	0.7536	2.70%
HD	−0.7237	−0.0204	0.7033	−0.1199	0.1014	−1.85%
INTC	−0.7591	−0.1010	0.6581	−0.3169	0.3145	−0.24%
MSFT	−0.7016	−0.0757	0.6259	−0.0181	0.0640	4.59%
NKE	−0.8495	−0.1240	0.7256	−0.5775	0.5933	1.58%
PG	−0.7876	−0.2808	0.5068	−1.0677	1.0388	−2.89%
UNH	−0.8389	−0.2077	0.6312	−1.2689	1.2300	−3.88%

Notes: The ex-post optimal portfolio weights for Mean-Variance and Mean-Gini models, with short sales not allowed. The first two sections also show the values of the parameters that explain the optimal weights, following the same path than Table 2, but with the new figures. The last section presents the analysis of the differences between the weights that stem from both models (Mean-Gini minus Mean-Variance) according to Equation (48).

As the outcome of this optimisation, the relevant assets have been reduced from 30 to 7, being the same for Mean-Variance and Mean-Gini. These seven stocks coincide with the ones in the first seventh positions in the ranking of the Mean-Variance net risk premium, while for the Mean-Gini approach the asset in the seventh position (DIS) is substituted by the one in the eighth position (HD). As shown in Equations (26) and (44) the weights are explained by the quotient between the required net premium ratio and the total risk ratio. Another enlightening way of explaining the weights is through the addition of the short sales distance and the risk correction as shown in Equations (27) and (45). In any case, the differences are expressed as the Mean-Gini weight minus the Mean-Variance weight. The analysis of the differences between the Mean-Gini and Mean-Variance weights is shown in the lower part of the table. There, we calculate the risk factors according to (46) and (47). Next, we calculate, according to Equation (48), the risk-corrected distances for both models. The differences between these distances explain the differences between their weights. All in all, the interest of the analysis displayed in this table lies in how it captures the capacity to create performance embedded in the actual data or, in other words, which would be the stocks' performance inside an optimal portfolio under a caeteris paribus assumption. While the short sales distance and the net risk premium ratio stress the relevance of risk-adjusted returns, the risk correction stresses the relevance of the ratio between crossed risk and total risk.

9. Conclusions

This paper has been focused on enlarging the performance information about the assets inside a portfolio provided by the Treynor ratio. Methodologically, we have based the research on the definition of crossed beta and on two concepts that stem from it: the net risk premium, and the net risk premium ratio that fulfils the axioms of performance measures. We have developed these concepts for the Mean-Variance and Mean-Gini models in order to combine the conceptual clarity of the former with the more accurate statistical precision of the latter.

As stated in the introduction, the analysis of this paper has been directed to answer three questions: which is the risk premium that an asset must reach for being in long position? (Q1); how its weight on the optimal portfolio can be explained (Q2); and, which conclusions can be drawn from ex-post optimisation? (Q3). Throughout the paper we have shown that the condition for an asset to be in long position is provided by the net risk premium ratio, the value of which must be greater than 1 to justify this position (answer to Q1). As for the weight of the budget invested in an asset, it is justified if the difference between the net risk premium ratio and 1, corrected by the crossed total risk ratio, equates or overcomes the asset's weight. The crossed total risk ratio captures the influence of total risk in the optimal weights: a crossed covariance factor lower than 1 reduces the weight, while if it turns to be greater than 1, increases the weight (answer to Q2). The ex-post optimisation analysis, in turn, provides a complete performance panorama of the portfolio under analysis by showing which would be the optimal weights in case of equality between the future expected values and the actual ones. We have shown that each weight can be broken down into the addition of its distance to short sales plus a risk correction (answer to Q3). The comparison between the optimal weights that stem from the Mean-Variance and Mean-Gini models has led us to find the impacts of short sales distance, risk, and crossed effect on their differences. The differences between their weights are explained through the differences in their distances to short sales corrected by the crossed risk factor.

These findings enable investors to deepen into the study of the performance of single assets inside a portfolio. In effect, they enable corporations to identify the causes of the weights of their shares in optimal portfolios: risk premium, crossed beta and total risk. The measures proposed in this paper do not substitute the Treynor ratio or other related indicators, but enlarge the performance analysis of single assets. We have verified in the empirical illustration that the rankings based on the net risk premium may differ from the Treynor's ratio rankings, but, at the same time, we have observed that the differences are very small in our case. This fact shows again the robustness of the Mean-Variance analysis, previously mentioned in the introduction. In brief, the analysis based on crossed beta widens the information furnished by the Treynor ratio, but only in extreme cases may contradict it.

For these reasons, the performance analysis of stocks' performance inside a portfolio has proved to be relevant for portfolio managers, investors and corporate managers. Passive portfolio managers, who accept market efficiency as the motto of their management, should nevertheless drop from their portfolios poorly performing assets. The identification of assets that, according to their current performance qualify for being held in short position, challenges active managers for taking short positions in these assets if they estimate that the market is going to correct their overvalued prices in the short run. Corporate managers, obviously interested in the market performance of their corporations, may find in the net risk premium ratio and its complementary indicators a set of measures that contribute to explain the weights of the companies that they are managing in diversified portfolios. The role of total risk in the determination of the optimal weights shows, in turn, the relevance of this neglected indicator for corporations who aim to enlarge the weights of their stocks in diversified portfolios.

The main further research question this paper suggests consists of extending the weights analysis to other performance indicators, although a necessary condition is that they must be connected to a portfolio optimisation method. The empirical analysis can also be enlarged and applied to other markets and portfolios. The necessary connection with portfolio optimisation also contributes to relate the measures developed in this paper to the state of the art of performance measurement: conveniently adapted, they may enlarge and complement the information obtained from any performance indicator that fulfils the twofold condition of being thought for single assets and that can be associated with portfolio optimisation.

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Notes

1. In the rest of the paper, expected and average values are both denoted by the usual bar over the variable; for instance, $\bar{R}$.
2. In Schuhmacher and Eling (2012), positive homogeneity is stated in Equation (3) for risk measures, and Equation (7) for reward measures. Functional translation invariance, in turn, is stated in Equation (6) for risk measures, and Equation (8) for reward measures.
3. We simplify notations by writing F for $F(R)$ as stated in Appendix A.
4. The average yearly return of the 1-Year Treasury Bill in the secondary market has been 0.11% yearly, which is equivalent to a 0.002171% weekly.
5. We substitute X_1 and X_2 in Yitzhaki and Schechtman by R_i and R_j.
6. Unless otherwise stated, we adopt the notations used in Yitzhaki and Schechtman. These authors, denote by Δ the Gini Mean Difference and by Δ^* the outcome of multiplying it by 0.5 used in portfolio optimisation (2013, p. 388, eq. 18.1). Let us clarify that in the body of the paper, where the notation Δ^* is not used, the superscript * denotes "required".
7. Cheung et al. (2007) present the numerical algorithm that has become the classical optimisation tool for the Mean-Gini portfolio selection.

[i] The underlying research data and calculations for this article can be accessed at https://figshare.com/articles/CalculationsRepository_xlsx/4495124.

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Appendix A

Obtaining the Mean-Gini crossed covariance

The aim of this appendix is to obtain the crossed covariance for the Mean-Gini portfolio model. Now, the covariance between any asset and the rate of return of the market portfolio, that we have found in the CAPM, is replaced by the covariance between the asset and the cumulative distribution of the market portfolio. This covariance cannot be split directly into the parts due to the asset under analysis itself (self-generated covariance), and its interaction with the rest of assets (crossed covariance). Instead, to arrive at the crossed covariance for the Mean-Gini model requires some previous operations.

A.1 Central features of the Mean-Gini portfolio model

Before developing the operations that lead to crossed covariance, we summarise the central features of the Mean-Gini portfolio model. The Mean-Gini portfolio theory has a greater stochastic dominance efficiency than the Mean-Variance paradigm, because it is compatible with the second degree conditions of stochastic dominance (Shalit & Yitzhaki, 1984; Yitzhaki & Schechtman, 2013, p. 389). As know, Gini's Mean Difference (Δ) consists of a dispersion measure based on the absolute differences between two random variables or a random variable with itself (Yitzhaki & Schechtman, 2013, p. 13).[5] Being R_i and R_j the two random variables, the basic expression of the Gini coefficient consists of the mathematical expectation of the absolute values of the differences between all pairs of both variables. Nevertheless, an equivalent approach smooths the calculation of the Gini coefficient.

In effect, the Gini coefficient is also equal to four times the covariance between the random variable (R) and its cumulative probability distribution: $\Delta_j = 4Cov[R_j,\ F(R_j)]$ or, for the crossed Gini coefficient with the cumulative distribution function of another variable, i.e. $\Delta_{ij} = 4Cov[R_i,\ F(R_j)]$ (Shalit & Yitzhaki, 1984; Yitzhaki & Schechtman, 2013, p. 18), where $F(\cdot)$ denotes the cumulative distribution function. In its application to portfolio optimisation, in order to facilitate the optimisation (Shalit & Yitzhaki, 1984; Yitzhaki & Schechtman, 2013, p. 388), the risk measure is taken as half of the Gini coefficient. Thus, the risks of asset i, portfolio M^G and asset i with respect to portfolio M^G are, respectively, expressed as:[6] $\Delta_i^* = 2Cov[R_i,\ F(R_i)]$, $\Delta_{M^G}^* = 2Cov[R_{M^G},\ F(R_{M^G})]$ and $\Delta_{iM^G}^* = 2Cov[R_i,\ F(R_{M^G})]$. Then, the Gini's efficient frontier optimisation problem consists of minimising the portfolio's Mean-Gini difference $(\Delta_{M^G}^*)$ subject to a specified value of the rate of return's mean $\left(\sum_{i=1}^{n} x_i{}^G R_i = R^G\right)$, and to the budget restriction $\left(\sum_{i=1}^{n} x_i^G = 1\right)$. We denote the optimal weight of asset i in the Gini optimal portfolio by x_i^G in order to distinguish it from its counterpart in the Mean-Variance paradigm.[7] Henceforth, we adopt the simplified notation F for $F(R)$. The beta coefficient in the Mean-Gini model consists of the ratio $Cov(R_i,\ F_M)/Cov(R_M,\ F_M)$, while the Security Market Line (SML) parallels the one of the CAPM substituting the corresponding parameters by the Mean-Gini ones. Yitzhaki and Schechtman (2013, pp. 33–49) show a complete development of the Gini regression coefficient, and Shalit and Yitzhaki (1984, p. 1460) present the details of the Gini SML, detailing the correlation equivalent and the total risk measures for both, asset and market. Their interaction, leads to the beta expression that we have mentioned.

A.2 Obtaining the crossed covariance

The covariance between asset under analysis (i) and the cumulative distribution function (F) of the Gini market portfolio, taking into account that $\sum_{j=1}^{n} x_j^G = 1$, can be written as:

$$Cov(R_i,\ F_M^G) = Cov\left(R_i, \sum_{j=1}^{n} x_j^G F_M^G\right) = x_i^G Cov(R_i, F_M^G) + \sum_{j=1 \forall j \neq i}^{n} x_j^G Cov(R_i, F_M^G) \tag{A1}$$

Table 4. Basic calculations

	General data				Mean-Variance						Mean-Gini					
Ticker	**Company name**	**Cap 31/12/2013**	**Weights (%)**	**R (%)**	**σ**	**β**	**β_c**	**π^* (%)**	**Crossed π^* (%)**	**α (%)**	**Δ_i^* (%)**	**β^G**	**β_C^G**	**π^{G*} (%)**	**Crossed π^{G*} (%)**	**α^G (%)**
	Portfolio	4,757,616.10	100.00	0.1732	1.5163	1.00	0.00	0.1732	0.1732	0.0000	0.8369	1.00	0.00	0.0000	0.0000	0.0000
AXP	American Express Co	96,536.7	2.03	0.0903	2.3654	1.21	1.16	0.0085	0.2003	−0.1186	1.3290	1.25	1.21	0.0056	0.2102	−0.1255
BA	Boeing Co.	102,565.8	2.16	−0.0655	2.8804	1.29	1.21	0.0135	0.2101	−0.2890	1.5599	1.27	1.23	0.0070	0.2137	−0.2861
CAT	Caterpillar Inc.	57,920.6	1.22	0.0868	3.0828	1.18	1.13	0.0087	0.1957	−0.1176	1.6842	1.10	1.08	0.0042	0.1870	−0.1045
CSCO	Cisco Systems	119,924.6	2.52	0.4987	2.2790	0.89	0.83	0.0099	0.1435	0.3454	1.2327	0.83	0.79	0.0064	0.1365	0.3558
CVX	Chevron Corp.	240,223.5	5.05	−0.1307	2.9901	1.51	1.31	0.0340	0.2266	−0.3913	1.5884	1.48	1.38	0.0166	0.2395	−0.3868
DD	DuPont	602,26.3	1.27	0.3372	2.6175	1.25	1.21	0.0065	0.2097	0.1209	1.3548	1.22	1.20	0.0035	0.2082	0.1254
DIS	The Walt Disney Co.	129,880	2.73	0.4342	2.1926	0.86	0.80	0.0099	0.1387	0.2856	1.2490	0.83	0.79	0.0071	0.1363	0.2908
GE	General Electric	283,589.9	5.96	−0.0945	2.1791	1.17	1.05	0.0213	0.1812	−0.2970	1.2181	1.19	1.10	0.0150	0.1911	−0.3006
GS	Goldman Sachs Group	80,339.7	1.69	0.1873	2.4739	1.18	1.14	0.0078	0.1968	−0.0172	1.3776	1.17	1.14	0.0048	0.1972	−0.0146
HD	Home Depot	115,953.4	2.44	0.5186	2.3896	0.93	0.87	0.0105	0.1502	0.3579	1.3082	1.05	1.02	0.0066	0.1759	0.3361
IBM	IBM Corp.	197,772.1	4.16	−0.2448	2.4671	0.57	0.46	0.0191	0.0796	−0.3434	1.2297	0.54	0.48	0.0106	0.0832	−0.3385
INTC	Intel Corp.	128,943.3	2.71	0.7176	2.6892	1.10	1.02	0.0148	0.1761	0.5268	1.4995	1.13	1.08	0.0084	0.1870	0.5222
JNJ	Johnson & Johnson	258,341.3	5.43	0.3020	1.9603	1.00	0.91	0.0157	0.1578	0.1285	1.0791	0.96	0.89	0.0121	0.1547	0.1352
JPM	JPMorgan Chase & Co.	219,837.4	4.62	0.1628	2.6731	1.33	1.19	0.0249	0.2057	−0.0677	1.4664	1.31	1.23	0.0140	0.2123	−0.0635
KO	The Coca Cola Co.	182,421.8	3.83	0.1387	2.3244	0.76	0.67	0.0156	0.1162	0.0069	1.2784	0.80	0.74	0.0101	0.1281	0.0004
MCD	McDonald's Corp.	96,547.8	2.03	0.0078	1.5136	0.52	0.50	0.0035	0.0860	−0.0818	0.7967	0.45	0.43	0.0033	0.0743	−0.0698
MMM	3 M Company	93,028.7	1.96	0.3755	2.3200	1.31	1.27	0.0079	0.2194	0.1481	1.2536	1.31	1.28	0.0051	0.2216	0.1489
MRK	Merck & Co.	146,522.7	3.08	0.3131	2.4903	0.92	0.84	0.0144	0.1452	0.1535	1.3122	0.89	0.84	0.0084	0.1453	0.1595
MSFT	Microsoft Corp.	310,503	6.53	0.4941	2.2856	0.90	0.76	0.0257	0.1309	0.3375	1.2540	0.94	0.84	0.0169	0.1462	0.3310
NKE	Nike, Inc.	69,832.3	1.47	0.4248	2.3877	0.52	0.49	0.0063	0.0842	0.3343	1.3033	0.44	0.42	0.0040	0.0722	0.3487
PFE	Pfizer Inc.	198,515.2	4.17	0.1058	2.3900	1.15	1.05	0.0179	0.1813	−0.0934	1.3147	1.17	1.11	0.0113	0.1921	−0.0976
PG	Procter & Gamble	221,291.2	4.65	0.2987	1.4090	0.46	0.42	0.0070	0.0721	0.2196	0.7964	0.53	0.49	0.0077	0.0841	0.2068
T	AT&T Inc.	185,222.9	3.89	0.0324	2.1971	0.76	0.68	0.0142	0.1172	−0.0989	1.2305	0.75	0.69	0.0099	0.1203	−0.0977

(*Continued*)

Table 4. (*Continued*)

	General data				Mean-Variance						Mean-Gini					
Ticker	**Company name**	**Cap 31/12/2013**	**Weights (%)**	**R (%)**	**σ**	**β**	**β_c**	**π^* (%)**	**Crossed π^* (%)**	**α (%)**	**Δ_i^* (%)**	**β^G**	**β_C^G**	**π^{G*} (%)**	**Crossed π^{G*} (%)**	**α^G (%)**
TRV	The Travelers Companies	32,962.7	0.69	0.3665	1.7571	0.88	0.87	0.0016	0.1500	0.2149	0.9423	0.89	0.88	0.0014	0.1532	0.2120
UNH	United Health Group Inc.	74,396.4	1.56	0.6029	2.2656	0.85	0.82	0.0060	0.1418	0.4551	1.2726	0.91	0.89	0.0041	0.1542	0.4445
UTX	United Technologies	104,318.9	2.19	0.0764	2.1469	0.93	0.88	0.0076	0.1526	−0.0837	1.2111	1.00	0.97	0.0055	0.1683	−0.0973
V	Visa Inc.	112,642.7	2.37	0.3418	2.9944	1.21	1.12	0.0160	0.1941	0.1317	1.5910	1.24	1.20	0.0078	0.2078	0.1263
VZ	Verizon Communica-tions	140,638.7	2.96	0.0178	2.0323	0.80	0.75	0.0092	0.1290	−0.1204	1.1233	0.74	0.70	0.0069	0.1212	−0.1102
WMT	Wal-Mart Stores	254,622.9	5.35	0.2167	2.0928	0.70	0.60	0.0177	0.1035	0.0956	1.1756	0.79	0.72	0.0130	0.1238	0.0799
XOM	Exxon Mobil Corp.	442,093.6	9.29	−0.0878	2.6137	1.35	1.07	0.0478	0.1857	−0.3213	1.3842	1.27	1.12	0.0266	0.1941	−0.3085

Notes: The results of the basic calculations that display Tables 1–3. First, it shows the corporations listed in DJIA at the end of 2013. For each one of them we have calculated its capitalisation multiplying the number of shares by the share price. Next, it shows the weight of each corporation that stems from this capitalisation. The rates of return that follow are the average of the weekly rates of return of each corporation. The same data has been used to estimate the standard deviation, beta and crossed beta. Next, the table displays the values of the net risk premium (π), required net risk premium (π^*) and alpha. The Gini-Mean differences (Δ_i^*) have been calculated as twice the assets' rates of return and their cumulative distribution function, approached through the rankings of their rates of return. Next, the Table shows the values of beta, crossed beta, actual and required net risk premia, and Gini's alpha.

Now, we need to incorporate into the analysis the covariance between the asset and its cumulative distribution function, that we write as $Cov(R_i, F_i)$. Adding and subtracting $x_i^G Cov(R_i, F_i)$ into the right-hand side of (A1), we obtain:

$$Cov\left(R_i, F_M^G\right) = x_i^G Cov(R_i, F_i) + \left[x_i^G Cov\left(R_i, F_M^G\right) - x_i^G Cov(R_i, F_i)\right] + \sum_{j \neq i}^{n} x_j^G Cov\left(R_i, F_M^G\right) \quad \text{(A2)}$$

The cumulative distribution function F_M^G is the outcome of interacting the whole assets in the market portfolio. If all assets had the same risk than asset *i*, then the cumulative probability of the portfolio would be the one of this asset, i.e. F_i. Thus, the part of the covariance between the asset and the market that can be attributed to the asset itself, namely the self-covariance, is:

$$SC_i = x_i^G Cov(R_i, F_i) \quad \text{(A3)}$$

Therefore, after (A2) and (A3), the crossed covariance of asset *i* can be obtained by detaching the self-covariance from (A2), which leads to:

$$CC_i = \left[x_i^G Cov\left(R_i, F_M^G\right) - x_i^G Cov(R_i, F_i)\right] + \sum_{j \neq i}^{n} x_j^G Cov\left(R_i, F_M^G\right) \quad \text{(A4)}$$

Summing the first and third addends of the right-hand side of this equation we obtain $Cov\left(R_i, F_M^G\right)$. Thus, the crossed covariance can be written as:

$$CC_i = Cov\left(R_i, F_M^G\right) - x_i^G Cov(R_i, F_i) \quad \text{(A5)}$$

Appendix B

Basic calculations

Table 4 displays the basic calculations of the numerical illustration. It shows the composition of the portfolio, the weights of its assets, and the outcome of the calculations of the weekly average rates of return, their corresponding standard deviations and Gini coefficients, together with beta, crossed beta, alpha, actual and expected net risk premia for Mean-Variance and Mean-Gini models. The shadowed part of the table corresponds to the calculations for Mean-Gini.