

Erratum to: A Network Epidemic Model with Preventive Rewiring: Comparative Analysis of the Initial Phase

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After a presentation of the paper cited above at a workshop on Dynamic Networks at the Isaac Newton Institute for Mathematical Sciences, Cambridge, Prof. Frank Ball in discussions explained two potential errors in our analysis. After further discussions, this was indeed confirmed. One mistake was an oversight, whereas the second one was more subtle. It turns out that the first mistake has impacts on the results of the paper, whereas the second one can be repaired and hence has no effect on the results.

The oversight appears in Sect. 4.1 where the basic reproduction number R_0^{BA} for the SEIR- ω model is derived, and it only affects the case $\alpha\omega_{EI} > 0$. There the probability for an exposed but not yet infectious individual to rewire away from its infector, and reconnect to a new (susceptible) individual, is computed. The competing events are that the exposed individual rewires (at rate ω_{EI} and only with probability α does the individual reconnect to a new individual), that the individual becomes infectious (when he/she stops rewiring), but also if the infector stops being infectious and recovers,

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because then the exposed individual stops rewiring according to the model. This last possibility was forgotten, and the rate γ at which the infector recovers is missing in the denominator of the probability. So, the last term in Eq. (5) should be $\alpha\omega_{EI}/(\phi + \gamma + \omega_{EI})$, and the correct expression for the basic reproduction number is

$$R_0^{BA} = \frac{\phi\beta}{(\phi + \omega_{SE})(\beta + \gamma + \omega_{SI})} \left(E(\tilde{D}) - 1 + \frac{\alpha\omega_{EI}}{\phi + \gamma + \omega_{EI}} \right). \quad (1)$$

This corrected term $\alpha\omega_{EI}/(\phi + \gamma + \omega_{EI})$ should also replace the old expression (without γ) in the last term on the right of Eq. (7) when deriving the exponential growth rate r (the event that the infector recovers is also left out in the integrals preceding this equation). The corrected version of Eq. (7) is

$$\frac{\beta\phi}{(r + \omega_{SI} + \beta + \gamma)(r + \phi + \omega_{SE})} \left(E(\tilde{D}) - 1 + \frac{\alpha\omega_{EI}}{r + \phi + \gamma + \omega_{EI}} \right) = 1.$$

Finally, according to expression (1) of R_0^{BA} , the right panel of Fig. 2 which compares both basic reproduction numbers now becomes

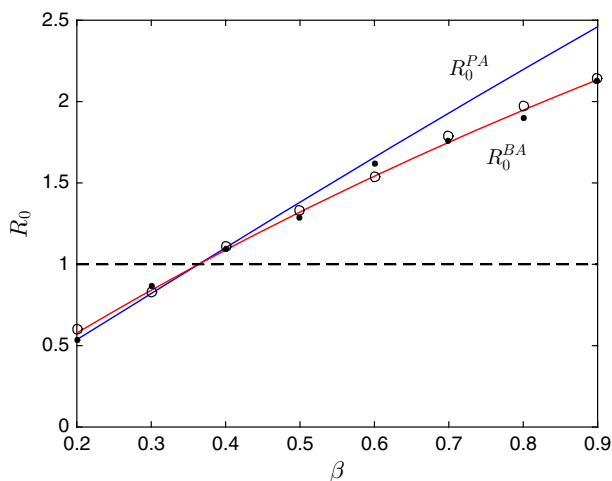


Fig. 2 (Right) Basic reproduction number, as a function of the infection rate β , of an SEIR- ω epidemic with $\omega_{SE} = 0$, $\omega_{EI} = 1$. Dashed line corresponds to $R_0 = 1$. PA pair approximation. BA branching process approximation. Open circles (solid dots) correspond to R_0 computed from stochastic simulations of the epidemic on a Poisson (scale-free) network. Each network has a degree sequence with an average size-biased degree very close to $E(\tilde{D}) = 10$. Parameters: $\phi = 1$, $\gamma = 2$, $\alpha = 1$, and $\omega_{SI} = 1$. Note that, when $\omega_{EI} > 0$, R_0^{BA} is supercritical (i.e. larger than 1) for smaller β than R_0^{PA}

The reason why this mistake has consequences on the results is that after this correction has been made, R_0^{BA} now has the same threshold value as R_0^{PA} , the basic reproduction number obtained from the pair approximation. The two reproduction numbers are not identical, but traverse the threshold value $R_0 = 1$ for the same parameter set-up, as is evident from comparing the new R_0^{BA} with last equation of the “Appendix”. One of the general conclusions of the paper, for example stated in the last sentence of the abstract, is hence wrong. The conclusion is *not* that the two approximations have different thresholds for the SEIR- ω model when $\alpha\omega_{IE} > 0$, but instead that they give the *same* threshold value (although the reproduction numbers remain different from each other in other parts of the parameter space as correctly stated).

Similarly as before, the second mistake applies only to the situation where $\alpha\omega_{IE} > 0$. In Sect. 4, we approximate the initial phase of the epidemic by a branching process assuming a large initial population. A basic assumption for branching processes is that siblings give birth to new children independently. However, in the SEIR- ω model, when $\alpha\omega_{IE} > 0$, this is not the case as the following example shows. Suppose an individual infects two of its neighbours who then are siblings in the approximating branching process. These two siblings then infect their other neighbours independently. However, they each may also infect one individual using the edge of their common infector in the case when they rewire away from their common infector; these two events both depend on the duration of the infectious period of their common infector (if he/she has a very long infectious period it is more likely that both rewire and infect the new neighbour) and are thus *not* independent. This implies that the defined limiting process is not really a branching process.

However, it is possible to approximate the initial phase of the epidemic by a multi-type branching process (where the type is the number of neighbours at the end of the latent period). This process has a next generation matrix, say $M = (m_{ij})$, which has rank 1 since $m_{ij} = im_{1j}$ for all i, j . As a consequence, it can be shown that the basic reproduction number of the “correct” multitype branching process equals the basic reproduction number R_0^{BA} given in Eq. (1) above.

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