Dynamic diagnosis based on interval analytical redundancy relations and signs of the symptoms

Gabriela Calderón-Espinoza, Joaquim Armengol *, Josep Vehí and Esteban R. Gelso
Institut d’Informàtica i Aplicacions, Universitat de Girona. E-17071 Girona, Catalonia, Spain
E-mail: {gcalder,armengol,vehi,ergelso}@eia.udg.cat

Abstract. A model-based approach for fault diagnosis is proposed, where the fault detection is based on checking the consistency of the Analytical Redundancy Relations (ARRs) using an interval tool. The tool takes into account the uncertainty in the parameters and the measurements using intervals. Faults are explicitly included in the model, which allows for the exploitation of additional information. This information is obtained from partial derivatives computed from the ARRs. The signs in the residuals are used to prune the candidate space when performing the fault diagnosis task. The method is illustrated using a two-tank example, in which these aspects are shown to have an impact on the diagnosis and fault discrimination, since the proposed method goes beyond the structural methods.

Keywords: Consistency-based diagnosis, fault models, model-based diagnosis, uncertain dynamic systems

1. Introduction

Fault detection and diagnosis is an active area of research because of the increasing demand for prevention, safety, and reduction of both faults and their effect. This is specially so in industrial processes where dangerous situations can occur.

One approach to diagnosis is model-based diagnosis (MBD), which is based on comparing observations of the actual behavior of the process and the behavior of a model of the process. Two research communities have used the MBD approach in parallel. DX, from the fields of computer science and artificial intelligence, has made a contribution, among other approaches, using the consistency-based logical approach [11,19], and FDI, from the field of automatic control, has used the analytical redundancy approach [10,14]. In [9] the links among concepts from both approaches are clarified.

Models are representations of knowledge about physical systems. The models used in MBD can be obtained from physical laws, human experience, data from the process, or from combination of the above. Since different levels of abstraction can be needed, properties such as precision, uncertainty or accuracy are reduced, increased or even lost. The properties of the models used in MBD have strong influence on the diagnosis results [12].

In this paper, models of components defined by a set of equations, which includes variables, have been used. Deviations between the actual value of a variable and the predicted value are used in the fault isolation task. Deviations are qualitatively expressed by a sign denoting whether the value of a variable is equal to, greater than, or less than the predicted value and therefore discarding the value of the deviation.

Some work has been carried out in the topic of taking into account deviations in symptoms (as they are called in the DX community) or in residuals (as they are called in the FDI community).

In [13], two approaches were presented: an interval-based method and a sign-based method. The interval-based method uses an anticipated dictionary of faults and gives bounds in measurements to every fault. In the sign-based method, faults are expressed in terms of deviations w.r.t. the nominal values. By studying the sign of the partial derivatives, the faulty parameters provide a signed influence of the parameters on each test.

The approach proposed in [8] consists of deriving semi-automatically, qualitative deviation models from
quantitative models developed in Matlab™. The general objective is to analyze the behavior in terms of: (i) how a variable deviates from its nominal value when a fault occurs, and (ii) how the deviation of the input variables influence the deviation of the output variables.

Another proposal, described in [5], consists of creating a fault library containing qualitative information. Data are obtained from deviations resulting from interval fault detection. Qualitative information about the deviations is used to create rules for faulty behaviors.

In [6] one approach based on parity equations uses the sensitivity of each parity equation with respect to a fault. The fault isolation stage consists of two steps: to find the degree of the fault and to check the consistency of the assumed fault from each parity equation. This approach is restricted to a nominal steady state in a system, so doing a static diagnosis.

This work is an attempt to improve identification of faults in the consistency-based and analytical redundancy approaches. The objective is to reduce adequately the set of diagnoses and to minimize the time required for fault identification. When enough information becomes available, additional information obtained from the fault models and their effect on the system should be incorporated into the fault isolation task.

In Section 3, we propose a diagnosis reasoning in which the signs of the partial derivatives are derived from possible conflicts (as they are called in the DX community) or analytical redundancy relations (as they are called in the FDI community) or qualitative information. The signs are integrated into the fault signature matrix, to be compared with the deviations of the residuals calculated using an interval tool. The interval tool called SQualTrack [3], described in Section 2, takes into account uncertainty in models which improves the fault detection task with regard to nominal predictions results.

An application example is presented in Section 4, and finally, some conclusions are given at the end of the paper.

2. Interval dynamic models

One way to detect faults is by comparing the real system behavior with the predicted behavior obtained from a model. Then a fault is detected at a time \( t \) when the predicted behavior from the model, \( y_r(t) \), is different from the corresponding measurement, \( y_m(t) \),

\[
y_r(t) \neq y_m(t). \tag{1}
\]

The consistency of the system behavior with that of the model can be checked at a time \( t \), by determining the difference using

\[
r(t) = y_m(t) - y_r(t), \tag{2}
\]

which is called the residual. When there is no fault, the value of the residual should be zero.

Unfortunately, most of the time, the residual is nonzero, and consequently, a continuous detection of faults occurs. One reason is that in the industrial monitoring of processes, the uncertainty is often present because of the noise in sensors and signals, an imprecise knowledge of the model parameters or a variation of the parameters over time.

Continuous-time systems are typically described using differential equations. Usually, the input, state, and output variables are sampled as time signals defined over a time variable, \( t \), which belongs to a discrete set. All signals are assumed to be sampled synchronously in a fixed sampling period. It is for this reason that discrete models are used. One example of this type of model is shown in Eq. (3), which is an \( n \)-th order SISO (Single Input, Single Output) system, represented by a difference equation, where \( u \) are the inputs, \( T \) is the sampling time, and \( a \) and \( b \) are the parameters of the system,

\[
y_t = \sum_{i=1}^{m+1} a_i y_{t-iT} + \sum_{j=1}^{p+1} b_j u_{t-jT}. \tag{3}
\]

When the uncertainty in the variables and in the parameters is represented by means of intervals, the resulting interval models are less precise but can be accurate.

The prediction of a real-value model produces a trajectory for each output variable which is a curve representing the evolution of the variable of the system with time, \( y_r(t) \). In the case of an interval model, there is a set of models indeed and hence a set of curves (an envelope) represents the evolution of each variable [1]. The limits of the envelope are

\[
Y_r(t) = [\min(y_r(t)), \max(y_r(t))]. \tag{4}
\]

2.1. SQualTrack

To compute the envelope limits, it is necessary to compute the range of a function in a given parameter space at each prediction step. This is a task related to global optimization in which the objective is
Fig. 1. The three zones defined by the internal and external estimations of the exact envelope.

to find the maximum or the minimum value of a function and the combinations of parameters to obtain this value. This task usually requires a high computational effort. Using the SQualTrack [3,20] software package, results can be obtained at a lower cost by calculating iteratively external estimations, \( Y_{rex}(t) \supseteq Y_r(t) \), of the range of the function, which are closer at each iteration. After an infinite number of iterations, the exact range would be calculated, but the algorithm stops when the external estimation is close enough to detect the fault, thus saving much computational effort for the detection of faults. However, if no fault is detected, the algorithm will never stop. This drawback can be overcome by using an internal estimation, \( Y_{rin}(t) \subseteq Y_r(t) \), which is included in the exact envelope. If the measurement is within this envelope, then the fault, if it exists, will not be detected, and so the algorithm will stop iterating.

Sometimes a fault exists but the value of a specific variable can be included in the set of values considered as normal at least for a period of time due to the dynamics of the system, the severity of the fault, etc. In this case the fault can only be detected with a delay or even can not be detected.

The internal and external estimations of the exact envelope, which are depicted in Fig. 1, define three zones. The fault detection system guarantees that a fault exists when the measurement is out of the external envelope (outer zone), so in this way, eliminating any false alarms. However, if the measurement is in the intermediate zone or in the inner zone there can be missed alarms.

Actually, the consistency between the interval model and the real process is performed using interval measurements, which are obtained from the measurements taking into account the uncertainties (noise, bias...) of the sensors. Therefore a fault is detected when the interval measurement does not intersect with the external estimation.

Any measurement belonging to a past time point can be used as initial state to compute the envelopes at the current time point. The time interval from this initial time point to the current one is called time window. If the window used at each prediction step has always the same length, then a sliding time window is being considered [2].

The number of missed alarms is reduced by using several window lengths simultaneously, as a fault is detected when there is an inconsistency in a time window. Therefore, this method maximizes the detection of faults and, at the same time, minimizes the number of computations required.

The iterative computation of the external and the internal estimations of the exact envelope is made by a branch-and-bound algorithm which is very efficient because it uses the Modal Interval Analysis [21]. Moreover, the use of the Modal Interval Analysis guarantees that this method does not generate any false alarms. If there were false alarms, they would indicate that either the interval model does not represent the system adequately, or that the interval measurements do not represent the true values of the variables.

This fault detection method has been implemented in the SQualTrack software package and has been applied to several real processes [4] within the European project CHEM [7]. SQualTrack can be used either offline or online.

When a fault is detected, the measured value is either larger or smaller than the predicted value. This information, combined with qualitative information from the model, is proposed in the following for fault diagnosis.

3. Fault diagnosis using the sign of the symptom

Consistency-based diagnosis (CBD) is one of the most widely used approaches in model-based diagnosis within the artificial intelligence community. Possible conflicts are used for the CBD. Each possible conflict represents a subsystem within the system description containing a minimal analytical redundancy which is capable of becoming a conflict. In [18] a set of possible conflicts is obtained by an offline analysis of a set of equations in a model without prior knowledge of the device fault modes.

Each possible conflict has an associated model, \( pc \), which is used for fault detection.

In [9,18], it is demonstrated that possible conflicts and Analytical Redundancy Relations (ARR) are equivalent for a set of given assumptions.
The elementary analytical relations of a model can include information on how possible faults can influence the process. These faults may be represented as unknown extra inputs acting on the system (additive faults), or as changes in some plant parameters (multiplicative faults) [10].

The constraints in each possible conflict can be rewritten by including knowledge about the device fault modes into the model. Additive faults can represent plant leaks or biases in sensors, for example, and multiplicative faults can represent clogging or deterioration of plant equipment, for example.

This new model that is associated with each possible conflict has a computational form \( (pe^{\text{comp}}) \) and an internal form \( (pe^{\text{int}}) \). The computational form is based on known variables, and the internal form is based on faults, known variables and parameters. The internal form is not computable because the value of the fault is not known, but it does allow to abstract information about the way in which a fault can act.

Some information is lost when the DX and FDI approaches are applied to dynamic systems. A binary codification is used in the results of the fault detection test associated with a given ARR or possible conflict [17]. In particular, the sign of the symptom and the sensitivity of the symptom with respect to each fault is not considered. This work proposes to perform this analysis using the internal form of the residuals.

As in [6], the sensitivity \( S \) is an \( m \times n \) matrix (where \( m \) is the number of possible conflicts, and \( n \) is the number of faults) with the entry \( s_{ji} \). The term \( s_{ji} \) can be viewed as being the sensitivity of the model associated with the \( j \)th possible conflict, \( (pe_j) \), with respect to the \( i \)th fault, \( (f_i) \). Mathematically, this is expressed as

\[
s_{ji} = \frac{\partial pe_i^{\text{int}}}{\partial f_j}.
\]

The function \( s_{ji} \) depends on the process measurements and system parameters. Then, the sign of the sensitivities, \( \text{sgn}(s_{ji}) \), can be inferred for the domain of the parameters and the measured signals (the function \( \text{sgn}(s_{ji}) \) takes the value of +1 when \( s_{ji} > 0 \), −1 when \( s_{ji} < 0 \), and 0 when \( s_{ji} = 0 \)).

However, in some cases, \( \text{sgn}(s_{ji}) \) can change according to the values of the measurements. Therefore, the corresponding cell of the table of \( \text{sgn}(S) \) cannot be completed beforehand without calculating the sensitivity at each moment.

The possible sign of a fault, \( f_i \), can be:
- \( \pm 1 \), when \( f > 0 \) or \( f < 0 \), e.g. the fault is a bias in a sensor; or
- \( +1 \) (when \( f > 0 \)) or \( -1 \) (when \( f < 0 \)), e.g. the fault is a leak in a tank.

For the sake of clarity, in this work, faults with a sign of ±1 or +1 are considered.

A new fault signature matrix can be constructed by multiplying the sign of the sensitivity by the sign of the corresponding fault. Then the elements can be: \( \{0, +1, -1, \pm 1\} \).

- 0, if the fault does not affect the possible conflict
- +1, when a fault \( f > 0 \) affects the symptom with a positive sign
- −1, when a fault \( f > 0 \) affects the symptom with a negative sign
- ±1, when the sign of a fault is ±1. If \( f > 0 \), the fault affects the symptom with a positive sign, and if \( f < 0 \), the fault affects the symptom with a negative sign
- ±1, when the sign of a fault is ±1. If \( f > 0 \), the fault affects the symptom with a negative sign, and if \( f < 0 \), the fault affects the symptom with a positive sign.

The diagnostic process will incrementally generate a set of candidates when a new possible conflict is confirmed, without providing a transient erratic diagnosis. Based on [9], the DX approach follows a row view of the fault signature matrix, considering each line separately corresponding to a confirmed possible conflict, and isolating the possible conflicts before searching for a common explanation. A fault signature matrix with the sign of the symptom helps to discard some diagnostics.

SQualTrack guarantees that a fault exists when the intersection between the interval measurement and the external envelope is void. Therefore, there are two possibilities for analyzing the internal and computational forms of the model associated with a possible conflict: either the external envelope is greater than the interval measurement, and the sign of the symptom would be +1, or the external envelope is smaller than the interval measurement, and the sign would be −1. This can be seen in Fig. 2, where the interval measurement is shown by the solid lines, and the inner and outer envelope are shown by the dotted and dashed lines, respectively.
4. An application to coupled water tanks

4.1. System description

A well-known dynamical example of a system based on two coupled water tanks [15] will be used to explain the obtainment of additional information from the model for diagnostic reasoning. Figure 3 shows a schematic drawing of the system.

The system is composed of two tanks, T1 and T2, a valve, V1, and a controller, PI1, which receives the current level of T1 as the input, and controls a valve, V1, which regulates the flow of water to T1.

4.2. Model equations

The system is described by the elementary analytical relations (EAR) shown in Table 1.

The terms \( q_v \), \( q_{s1} \) and \( q_{s2} \) denote the volumetric flows, \( x_1 \) and \( x_2 \) are the heights of the water in tanks T1 and T2, respectively, and \( u \) is the output signal of the controller. The variables \( u, q_v, q_{s1}, q_{s2}, x_1, \) and \( x_2 \) are unknown, \( \tilde{u}, \tilde{x}_1, \) and \( \tilde{x}_2 \) are known variables obtained from sensors, and \( k, k_{s1}, \) and \( k_{s2}, \) and \( S \) are the constant parameters of the system.

This model, which is linearized, has been taken from [15] and [16]. In this paper, it is used for comparison purpose and to simplify the explanation of the proposed strategy for fault diagnosis, but notice that SQualTrack and the proposed method can also be used for nonlinear models.

All the variables and parameters are considered as intervals for the consistency test using SQualTrack.

4.3. Faults

Nine possible fault scenarios are considered.

- Sensor \( x_1 \):
  - \( f_1 \): an additive fault (bias), with \( f_1 > 0 \) or \( f_1 < 0 \).
  - \( f_2 \): a multiplicative fault, with \( f_2 > 0 \).

- Sensor \( x_2 \):
  - \( f_3 \): an additive fault (bias), with \( f_3 > 0 \) or \( f_3 < 0 \).
  - \( f_4 \): a multiplicative fault, with \( f_4 > 0 \).

- D/A converter:
  - \( f_5 \): an additive fault (bias), with \( f_5 > 0 \) or \( f_5 < 0 \).

- Tank T1 and its output pipe:
The columns of the table correspond to the elementary relationships shown in Table 2.

**4.4. Model with faults**

Extending the model to include faults provides the relationships shown in Table 2.

**4.5. Consistency-based diagnosis**

Two different possible conflicts were obtained [15] with the structural analysis, which are minimal with respect to the set of constraints used in the model (Table 3). The columns of the table correspond to the elementary analytical relation described in Table 1. The number “1” indicates that the corresponding EAR is involved in a pc.

The computational forms of the models associated with the possible conflicts are

$$pc_1^{\text{comp}} = S\dot{x}_1 - k_u\dot{x}_1 + k_s\dot{x}_1,$$

$$pc_2^{\text{comp}} = S\dot{x}_2 - k_s\dot{x}_1 + k_s\dot{x}_2,$$

and the corresponding internal forms are:

$$pc_1 = q_v = k_u$$

$$S\dot{x}_1 = q_v - q_{s1} - f_6$$

$$q_{s1} = k_s x_1 (1 - f_7)$$

$$S\dot{x}_2 = q_{s1} - q_{s2} - f_8$$

$$q_{s2} = k_s x_2 (1 - f_9)$$

$$\dot{u} = u + f_5$$

$$\dot{x}_1 = x_1 (1 - f_2) + f_1$$

$$\dot{x}_2 = x_2 (1 - f_4) + f_3.$$

**Table 2**

<table>
<thead>
<tr>
<th>Elementary relationships</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e)</th>
<th>(f)</th>
<th>(g)</th>
<th>(h)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>q_v = k_u</td>
<td>S\dot{x}<em>1 = q_v - q</em>{s1} - f_6</td>
<td>q_{s1} = k_s x_1 (1 - f_7)</td>
<td>S\dot{x}<em>2 = q</em>{s1} - q_{s2} - f_8</td>
<td>q_{s2} = k_s x_2 (1 - f_9)</td>
<td>\dot{u} = u + f_5</td>
<td>\dot{x}_1 = x_1 (1 - f_2) + f_1</td>
<td>\dot{x}_2 = x_2 (1 - f_4) + f_3</td>
</tr>
</tbody>
</table>

**Table 3**

Possible conflicts of the system

<table>
<thead>
<tr>
<th>pc</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
<th>g</th>
<th>h</th>
</tr>
</thead>
<tbody>
<tr>
<td>pc_1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>pc_2</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

* f_6: a constant leak, with f_6 > 0.
* f_7: a clogging fault in the output pipe, with f_7 > 0.

- Tank T2 and its output pipe:
  * f_8: a constant leak in tank T2, with f_8 > 0.
  * f_9: a clogging fault in the output pipe, with f_9 > 0.

**Table 4**

Influence of the faults in possible conflicts

<table>
<thead>
<tr>
<th>pc</th>
<th>f_1</th>
<th>f_2</th>
<th>f_3</th>
<th>f_4</th>
<th>f_5</th>
<th>f_6</th>
<th>f_7</th>
<th>f_8</th>
<th>f_9</th>
</tr>
</thead>
<tbody>
<tr>
<td>pc_1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>pc_2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

**Table 5**

Sign of the sensitivity of a symptom with respect to each fault

<table>
<thead>
<tr>
<th>pc</th>
<th>f_1</th>
<th>f_2</th>
<th>f_3</th>
<th>f_4</th>
<th>f_5</th>
<th>f_6</th>
<th>f_7</th>
<th>f_8</th>
<th>f_9</th>
</tr>
</thead>
<tbody>
<tr>
<td>pc_1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>+1</td>
<td>+1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>pc_2</td>
<td>+1</td>
<td>-1</td>
<td>-1</td>
<td>+1</td>
<td>0</td>
<td>0</td>
<td>+1</td>
<td>+1</td>
<td>-1</td>
</tr>
</tbody>
</table>

$$pc_1^{\text{int}} = -f_1 k_{s1} + f_2 k\dot{u} + f_5 k + f_6 - f_7 k_{s1} \dot{x}_1$$

$$pc_2^{\text{int}} = f_1 k_{s1} - f_2 (S\dot{x}_2 + k_s \dot{x}_2) - f_3 k_{s2} + f_4 k_{s1} x_1 + f_5 k_{s1} x_1 + f_8 - f_9 k_{s2} \dot{x}_2$$

In this work, only single faults are considered, f_i f_j = 0, \forall i \neq j, and so some terms in the internal forms can be simplified.

$$pc_1^{\text{int}} = -f_1 k_{s1} + f_2 k\dot{u} + f_5 k + f_6 - f_7 k_{s1} \dot{x}_1.$$  \hspace{1cm} (10)

$$pc_2^{\text{int}} = f_1 k_{s1} - f_2 (S\dot{x}_2 + k_s \dot{x}_2) - f_3 k_{s2} + f_4 k_{s1} x_1 + f_5 k_{s1} x_1 + f_8 - f_9 k_{s2} \dot{x}_2.$$  \hspace{1cm} (11)

By inspecting the internal forms, the structure of the influence of the faults in the residuals can be concluded to be as shown in Table 4. A number “1” in row j and column i of the table denotes that fault i influences the possible conflict j ideally.

The signs of s_{ji} are shown in Table 5. For example, the sensitivity of the possible conflict pc_1 with respect to the fault f_7 is:

$$s_{17} = \frac{\partial pc_1^{\text{int}}}{\partial f_7} = -k_{s1} \dot{x}_1.$$  \hspace{1cm} (12)
Table 6
Possible conflicts and their related fault modes using the sign of the symptom

<table>
<thead>
<tr>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_3$</th>
<th>$f_4$</th>
<th>$f_5$</th>
<th>$f_6$</th>
<th>$f_7$</th>
<th>$f_8$</th>
<th>$f_9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$pc_1$</td>
<td>+1</td>
<td>+1</td>
<td>0</td>
<td>0</td>
<td>±1</td>
<td>+1</td>
<td>−1</td>
<td>0</td>
</tr>
<tr>
<td>$pc_2$</td>
<td>±1</td>
<td>−1</td>
<td>+1</td>
<td>0</td>
<td>0</td>
<td>+1</td>
<td>−1</td>
<td>−1</td>
</tr>
</tbody>
</table>

Table 7
Diagnosis using and not using signs

<table>
<thead>
<tr>
<th>Symptoms</th>
<th>$sgn(pc_1)$</th>
<th>$sgn(pc_2)$</th>
<th>$sgn(pc_1)$ using signs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1, f_2, f_7$</td>
<td>+1</td>
<td>+1</td>
<td></td>
</tr>
<tr>
<td>$f_1, f_2$</td>
<td>−1</td>
<td>−1</td>
<td></td>
</tr>
<tr>
<td>$f_1, f_2, f_3, f_4, f_5, f_6, f_7$</td>
<td>0</td>
<td>±1</td>
<td></td>
</tr>
<tr>
<td>$f_1, f_3, f_4, f_7, f_8, f_9$</td>
<td>0</td>
<td>−1</td>
<td></td>
</tr>
</tbody>
</table>

The sign of each fault is analyzed for each case in Table 6. For example, for $pc_1$ and $f_1$, if $f_1$ is positive, then the symptom of $pc_1$ will be negative. Consequently, if $f_1$ is negative, then the symptom of $pc_1$ will be positive. This behavior is reflected in the table using ±1.

4.6. Diagnosis results

Table 7 shows the sets of possible diagnostics obtained using the DX approach when the sign of the residuals is used, and when it is not used. The signs help with discarding some diagnostics, and so the sets are reduced.

4.7. Simulation results

A faulty scenario involving a clogging fault in the output pipe of $T_1, f_7$, is considered. The values of the variables are represented by intervals to take into account any associated uncertainty in the measurements. The parameters of the model are also taken as intervals for the same reason.

The discrete forms of the possible conflicts are used in SQualTrack. Then, the computational forms of $pc_1^{comp}$ and $pc_2^{comp}$ are introduced as follows.

$$\tilde{x}_1(k) = \tilde{x}_1(k - 1) - \frac{T}{S}(-k\tilde{u}(k - 1) + k_s1\tilde{x}_1(k - 1)),$$

$$\tilde{x}_2(k) = \tilde{x}_2(k - 1) - \frac{T}{S}(-k\tilde{u}_1(k - 1) + k_s2\tilde{x}_2(k - 1)) + k_s2\tilde{x}_2(k - 1).$$

Figures 4 and 5 show a window from SQualTrack for the models of $pc_1$ and $pc_2$, respectively.

The upper graphs show the envelopes of the output variable (the inner envelope in dotted lines, and the outer envelope in dashed lines), and the interval measurements in solid lines.

The lower graphs indicate a “1” when a fault is detected.

In all the graphs, the time is expressed in samples, and the sample time is 10 s. The fault begins at sample 200. For the $pc_1$ case, the fault is detected from sample 221, and for the $pc_2$ case, the fault is detected from sample 214.
Table 8: Diagnostics for the fault scenario

<table>
<thead>
<tr>
<th>Sample</th>
<th>214</th>
<th>221</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symptoms</td>
<td>(\text{sgn}(p_{c1}) = 0)</td>
<td>(\text{sgn}(p_{c1}) = -1)</td>
</tr>
<tr>
<td>Diagnosis</td>
<td>(f_1 \lor f_2 \lor f_3 \lor f_4 \lor f_7\lor f_8)</td>
<td>(f_1 \lor f_2 \lor f_7\lor f_8)</td>
</tr>
</tbody>
</table>

Using signs \(f_1 \lor f_3 \lor f_4 \lor f_7\lor f_8\) \(f_1 \lor f_7\lor f_8\)

Table 8 illustrates the diagnostics obtained either by considering or not the signs corresponding at the times in which each symptom appear.

5. Conclusions

In this work, knowledge about signs obtained from partial derivatives in a quantitative model, is suggested to improve the task of diagnosis. The advantages of the interval tool can be exploited to evaluate the consistency between a model and a system for fault diagnosis.

The residual signs have been analyzed directly in consistency relations and this information has been integrated in the fault signature matrix.

By comparing the fault signature matrix with the qualitative deviation resulting from the interval detection tool, the set of diagnoses has been reduced.

Future work will consist of analyzing multiple fault deviations, the magnitude of the deviations, and the sensitivity of faults in conflicts.

Acknowledgements

This work has been partially funded by the European Regional Development Fund and European Social Fund of the European Union, the Spanish Government (Plan Nacional de Investigación Científica, Desarrollo e Innovación Tecnológica, Ministerio de Ciencia y Tecnología) through the coordinated research project grant No. DPI2003-07146-C02-02, by the grant No. 2005SGR00296 and the Departament d’Universitaris, Recerca i Societat de la Iniciació of the Government of Catalonia, and by the Mexican National Council for Science and Technology (CONACyT).

References


