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OPTIMAL MANAGEMENT OF NATURAL RESOURCES. ACCOUNTING FOR HETEROGENEITY

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Optimal Management of Natural Resources

Accounting for Heterogeneity

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Chapter 1

General Introduction

1.1 Motivation

Dynamic optimization methods have become increasingly important over the last 30 years in economics, and form the methodological cornerstone for particular economic areas such as macro-economics or environmental and resource economics. Within the dynamic optimization techniques employed, optimal control has emerged as the most powerful tool for the theoretical economic analysis. However, there is the need to advance further and take account that many dynamic economic processes are, in addition, dependent on some other parameter different than time. In other words, the decision variables as well as the state variables may depend not only one argument such as time or space but also on other aspects. One can think of relaxing the assumption of a representative (homogeneous) agent in macro- and micro-economic applications allowing for heterogeneity among the agents. For instance, the optimal adaptation and diffusion of a new technology over time, may depend on income or age of the person that adopted the new technology. Therefore, there are many fields, such as health economics, epidemiology, vintage capital theory, or demography, where the quality of the economic analysis would be enriched substantially by introducing heterogeneity in the economic models.

Likewise, it is easy to imagine that accounting for heterogeneity in natural resources enhances the quality of the economic analysis in a substantial manner. One can think of quality- or size- or age- distributed resources. In this case, it is necessary to take account of the heterogeneity of natural resources in the design of environmental policies. For example, in the case of agriculture, agricultural production is as a major contributor to numerous environmental problems, such as soil erosion, build-up of pesticide resistance, water logging, and the contamination of groundwater and surface waters (EPA, 1998). Most of these problems have a dynamic aspect, therefore, dynamic models have to be introduced to design efficient policies that aim to establish the socially optimal outcome. However, the magnitude of these environmental problems depends also on the distribution of the characteristics of the land within the relevant area, and on the type and intensity of the economic activities at each location. Therefore, policies designed to establish the social outcome need to be simultaneously targeted site-specifically and time-specifically.

Another example can be found on the optimal management of age- or size-dependent populations, such as forests or fishery. The evolution of an heterogeneous population depends not only on time, but on the distribution of the resource, because it influences the growth process of each individual. For example, in a size-distributed forest, the growth rate of trees does depend on the size of the neighboring trees. For instance, given the same size of the biomass, a forest with a high share of big trees leaves less space, light and nutrients in comparison with a forest with a high share of young and small trees. Likewise the reproduction rate of a fish population varies with the age distribution.

Despite the great potential of dynamic optimization problems that are structured in some additional dimension, such as age or space, the existing economic applications are very scarce. The reason could be the high complexity of this type of systems, generally characterized by integral equations or partial differential equations. Moreover, optimization problems where the control variables depend on two or more arguments are difficult to solve analytically, and thus, numerical methods have to be employed to solve these type of problems. Given this context, the objectives of the thesis are defined in the following section.

1.2 Objectives

This thesis intends to accomplish two goals. The first goal is to analyze and revise existing environmental policies that focus on defining the optimal management of natural resources over time, by taking account of the heterogeneity of environmental conditions. Thus, the thesis makes a policy orientated contribution in the field of environmental policy by defining the necessary changes to transform an environmental policy based on the assumption of homogeneity into an environmental policy which takes account of heterogeneity. As a result the newly defined environmental policy will not only be more efficient but most likely also politically more acceptable since it is tailored more specifically to the heterogeneous environmental conditions. Additionally to its policy orientated contribution, this thesis aims making a methodological contribution by applying a new optimization technique for solving problems where the control variables depend on two or more arguments — the so-called two-stage solution approach —, and by applying a numerical method — the Escalator Boxcar Train Method — for solving distributed optimal control problems, i.e., problems where the state variables, in addition to the control variables, depend on two or more arguments.

The next chapter presents a theoretical framework to determine optimal resource allocation over time for the production of a good by heterogeneous producers, who generate a stock externality. It is assumed that producers differ with respect to the quality of the assets. It is analyzed the optimal intertemporal and quality specific combination of abatement strategies at the source (source abatement) given by a change in the intensity (intensive margin) of production, by the choice of technology (extensive margin), and/or by the removal of existing pollution stock (stock abatement). The framework presented in chapter 2 can be utilized in various contexts, for instance, agricultural production where the heterogeneity resides in varying land quality, manufacturing where the heterogeneity is generated by the different characteristics of the capital goods (different machines or different plants) or the energy sector with plants of different characteristics. Chapter 3 illustrates the method in a more specific context, and integrates the aspects of quality and time, presenting a theoretical model that allows to determine the socially optimal outcome over time and space for the problem of waterlogging in irrigated agricultural production. Moreover, the applicability of the theoretical approach is demonstrated by reformulating the mathematical model such that it can be solved with standard mathematical software. For this purpose, an empirical study based on the cotton produced in the San Joaquin Valley in California is presented, and it is determined the socially optimal water price in the presence of a common bad, i.e. waterlogging.

Another issue where the consideration of heterogeneity can have considerable implications is the optimal management of heterogenous populations. In particular one can think of age- or size-distributed populations such as fishes, animal herds, forests, or one can think of it in a broad concept of biodiversity which includes the distribution of age/size of the population. Chapter 4 of this thesis concentrates on forestry resources, where the optimal management of a size-distributed forest is analyzed. The consideration of size-distributed forest growth allows to come up with an yet unavailable economic model to determine the optimal selective cutting regime. A regime which is nowadays gaining more and more importance in forest management due to its lower environmental impact than the previously analyzed clear-cutting regime.

1.3 Methodology

The modern economic analysis of natural resource problems considers the seminal article by Hotelling (1931) as the starting point of its field. It identified the need for an intertemporal approach to exhaustible resource economics. Certainly, most of the studies that intend to find the optimal management of natural resources have a common feature: They are confronted with dynamical systems, that is, systems that evolve over time. The current state of the system results as a consequence of the overall past decisions, and the present decisions will condition its future evolution. Thus, dynamic optimization methods have become increasingly utilized over the last decades in economics, to solve these type of problems. Within the dynamic optimization techniques employed, optimal control, developed by Pontryagin et al. (1961), has turned into the most powerful tool for the theoretical economic analysis.

To describe the methodology utilized in this thesis, a classification scheme for the different optimal control methods employed in dynamic economic analysis is presented in Table 1.1. The set of essential properties or characteristics of a dynamic system is called "the state of the system", where $x = (x_1, x_2, ..., x_n)$ is a vector denoting the n "state variables" of the system. The rate of change of the state variables with respect to time may depend on the present state of the system, as well as on variables that can be modified voluntarily, i.e., they are under control. These variables are called "control variables", and are denoted by $u = (u_1, u_2, ..., u_m)$. Thus, the law of motion or state equation is a differential equation governing the evolution of the system, denoted by $\dot{x}(t) = g(x(\cdot), u(\cdot), \cdot),\ g(\cdot) \in C^{(2)}$. The objective function of the decision maker is to maximize or minimize a certain function $f(x(\cdot), u(\cdot), \cdot),\ f(\cdot) \in C^{(2)}$, which is influenced by the control variables, u, both directly, and/or indirectly through its impact on the evolution of the state of the system, x. In economic analysis, the argument of the state and control variables is often unique, and it stands for calendar time, denoted by t.

Table 1.1: Classification Scheme for Dynamic Optimization Problems*

		Number of arguments of the decision variables: 1	Number of arguments of the decision variables: 2
Number of arguments of the restricted variables which influence the restriction function	1	Optimal Control $\max\limits_{u(t)} \int_0^\infty f(x(t), u(t), t)\, dt$ subject to $\frac{dx(t)}{dt} = g(x(t), u(t), t)$	Two-Stage Optimal Control $\max\limits_{u(t,\epsilon)} \int_0^\infty \int_{\epsilon_0}^{\epsilon_1} f(x(t), u(t,\epsilon), t, \epsilon)\, d\epsilon\, dt$ subject to $\frac{dx(t)}{dt} = \int_{\epsilon_0}^{\epsilon_1} g(x(t), u(t,\epsilon), t, \epsilon)\, d\epsilon$
	2	does not exist	Distributed Optimal Control $\max\limits_{u(t,\epsilon)} \int_0^\infty \int_{\epsilon_0}^{\epsilon_1} f(x(t,\epsilon), u(t,\epsilon), t, \epsilon)\, d\epsilon\, dt$ subject to $\frac{\partial x(t,\epsilon)}{\partial t} = g(x(t,\epsilon), u(t,\epsilon), \frac{\partial x(t,\epsilon)}{\partial \epsilon}, t, \epsilon)$

*Based on Calvo and Goetz (2001).

However, only recently the restriction that physical of bio-physical attributes of the variables of interest are homogeneous has been relaxed. In other words, the decision variables as well as the state variables may depend not only on time but also on other aspects, denoted by ϵ. For instance, space, age or size. In this context, one can distinguish two different cases. In the case where the decision variables depend on two arguments, for instance time and one-dimensional space, but the state variables depend only on one argument one may utilize two stage optimal control (Goetz and Zilberman, 2000). However, often the state variables are also structured over a particular characteristic, which leads to a distribution of the state variables for every moment of time over the characteristic ϵ, e.g., quality, age, size. In the case where both, decision and state variables, depend on two arguments one need to employ distributed optimal control.

Chapter 2 and 3 of this thesis modify the framework initially described by Goetz and Zilberman (2000) to find the optimal management of natural resources when the state variable is an aggregate that only depends on time. In this situation, it is possible to utilize two-stages optimal control to find an analytical solution. A general outline of a two stage optimal control problem is described in Table 1.1. For its analytical solution, let start out from its definition given by

$$\max_{u(t,\epsilon)} \quad \int_0^\infty e^{-\delta t} \int_{\epsilon_0}^{\epsilon_1} f(x(t),\, u(t,\epsilon), t, \epsilon)\, d\epsilon\, dt, \tag{S}$$

subject to

$$\dot{x}(t) \;=\; \int_{\epsilon_0}^{\epsilon_1} g(x(t),\, u(t,\epsilon), t, \epsilon)\, d\epsilon, \quad x(0) = x_0.$$

Utilizing Pontryagin's Maximum Principle, the current Hamiltonian of problem (S) is given by

$$\mathcal{H} \;\equiv\; \int_{\epsilon_0}^{\epsilon_1} f(x(t),\, u(t,\epsilon), t, \epsilon)\, d\epsilon - \mu(t)\Big(\int_{\epsilon_0}^{\epsilon_1} g(x(t),\, u(t,\epsilon), t, \epsilon)\, d\epsilon\Big),$$

The solution of problem (S) has to satisfy the following necessary conditions stated in accordance with Theorem 1, page 276, Seierstad and Sydsæter (1987)

$$\mathcal{H}_u \;\equiv\; f_u - \mu(t) g_u = 0, \tag{1.1}$$

$$\dot{\mu}(t) \;=\; \delta\mu(t) + \mathcal{H}_x = \mu(t)(\delta + g_x) + f_x, \tag{1.2}$$

$$\dot{x}(t) \;=\; \int_{\epsilon_0}^{\epsilon_1} g(x(t),\, u(t,\epsilon), t, \epsilon)\, d\epsilon, \quad x(0) = x_0, \tag{1.3}$$

where the subscript of a function with respect to a variable denotes its partial derivative. The analytical solution of the necessary conditions (1.1) - (1.3) is difficult, since the law of motion is determined by an integral equation. Thus, to solve problem (S), a solution technique in two stages is proposed. Due to the structure of problem (S), one is able to decompose part of the problem into a static, and another part into a dynamic control problem. The static control problem optimizes the use of resources over the heterogeneous characteristic ϵ, for example, vintage or space. The solution to the static problem is then plugged into the dynamic control problem to determine the optimal solution over time. Separating the problem in two stages is possible because the state variable, $x(t)$, is an aggregate function that depends exclusively on time.

In the first stage the solution of the static problem is given by the value function $V(z, x)$ defined as:

$$V(z,\, x) \;\equiv\; \max_{u(\epsilon)} \int_{\epsilon_0}^{\epsilon_1} f(x,\, u(\epsilon), \epsilon)\, d\epsilon \tag{S1}$$

subject to

$$z \;=\; \int_{\epsilon_0}^{\epsilon_1} g(x,\, u(\epsilon), \epsilon)\, d\epsilon,$$

where z denotes a predetermined level of the aggregate characteristic over the entire range of ϵ, from ϵ_0 to ϵ_1. Thus, the newly introduced variable z as well as the state variable x of problem (S) turn into parameters in the first stage. Taking account of the constraint leads to the Lagrangian

$$\mathcal{L}1 \equiv \int_{\epsilon_0}^{\epsilon_1} f(x,\, u(\epsilon), \epsilon)\, d\epsilon + \lambda\Big(z - \int_{\epsilon_0}^{\epsilon_1} g(x,\, u(\epsilon), \epsilon)\, d\epsilon\Big)$$

A solution of the problem has to satisfy the following necessary conditions:

$$\mathcal{L}1_u \;\equiv\; f_u - \lambda g_u = 0, \tag{1.4}$$

$$\mathcal{L}1_\lambda \;\equiv\; z - \int_{\epsilon_0}^{\epsilon_1} g(x, u, \epsilon)\, d\epsilon = 0. \tag{1.5}$$

To analyze how the optimal solution is affected over time, the value function V obtained in the first stage is maximized over time. Hence, the dynamic decision problem is given by:

$$\max_{z(t)} \int_0^\infty e^{-\delta t} V(z(t),\, x(t))\, dt \tag{S2}$$

subject to

$$\dot{x}(t) \;=\; z(t), \quad x(0) = x_0.$$

The parameter z of the first-stage problem, which denotes the aggregate characteristic over the entire range of quality ϵ becomes the decision variable in the second stage. The current value Hamiltonian of the second stage is given by: $\mathcal{H}2 = V(z(t), x(t)) - \varphi(t)z(t)$, where φ denotes the costate variable. The first-order conditions read as follows:

$$\mathcal{H}2_z \;\equiv\; V_z - \varphi = 0, \qquad \Rightarrow \lambda = \varphi \tag{1.6}$$

$$\dot{\varphi} \;=\; \delta\varphi + \mathcal{H}2_x = \delta\varphi + V_x = (\delta + f_x)\varphi + g_x, \tag{1.7}$$

$$\dot{x} \;=\; z, \qquad x(0) = x_0, \tag{1.8}$$

where the envelope theorem (Sydsaeter and Hammond, 1995) has been used in equations (1.6) and (1.7). Assume that problems (S1) and (S2) have been solved. That is, the functions u^* and z^* that solve equations (1.4) to (1.7) have been found. Given that:

- Equation (1.4) of problem (S1) is parallel to equation (1.1) of the main problem.

- Equation (1.7) of problem (S2) is equivalent to equation (1.2) of problem (S).

- Equation (1.5) is the definition of the aggregate characteristic that has been introduced and, thus, its substitution into equation (1.8) leads to equation (1.3) of problem (S).

- Equation (1.6) links the two stages. The change in the value function of the static problem (S1) resulting from a marginal increment of the aggregate characteristic z, by the envelope theorem, is given by $V_z = dV/dz = \partial\mathcal{L}1/\partial z = \lambda^*$. Thus, it is obtained that $\lambda^*(t) = \varphi^*(t)$. Furthermore, envelope theorem also allows to calculate the change in the value function resulting from a marginal increment of x, given by $V_x = dV/dx = \partial\mathcal{L}1/\partial x = f_x + \lambda^*\, g_x = f_x + \varphi^*\, g_x$.[1]

It can be concluded that these functions will also maximize the problem (S), with the costate variables $\mu^*(t) = \lambda^*(t) = \varphi^*(t)$.

Chapter 4 presents an optimization model where the state variable, in addition to the control variables, is also structured over a particular characteristic, ϵ. These type of problems include a system of partial differential equations in the necessary conditions, and thus they are difficult to solve analytically. Consequently, it might be necessary to implement numerical methods to find the optimal solutions to these problems. Thus, Chapter 4 utilizes the Escalator Boxcar Train Method developed by de Roos (1988), in Biology, for the study of dynamics of structured populations. This method consists on the partition of the domain of the structuring variable, ϵ — age or size — in small subdomains, such that individuals whose characteristic ϵ pertains to the same subdomain can be considered rather homogeneous and thus they can be grouped into the same cohort. Each cohort is characterized by the number of individuals it contains and its intensity/level of the particular characteristic, that evolve over time. In this way, the PDE governing the restriction function, can be decoupled into a system of Ordinary Differential Equations on the number of individuals an on the characteristic.

[1] It is assumed that V_z and V_x exist.

Chapter 2

Optimal Control of a Stock Externality

2.1 Introduction

Major environmental policy problems, including climate change and water-quality deterioration, are stock externality problems (Farzin, 1996; Baudry, 2000). The accumulating pollutants are frequently the result of inputs (water, chemicals, fossil fuels) by heterogeneous producers. Run-off of both family and corporate farms contribute to the contamination of bodies of water, and emission of small producers and large factories contribute to climate change. Thus, the design of policies to control stock externalities should consider both time and heterogeneity dimensions of these problems and the technologies that affect accumulation of pollutants.

The buildup of the externality stock can be modified either through changes in production practices (source abatement) or, when possible, by removal of existing pollution stock (stock abatement). Source abatement can be achieved by reducing input use levels (control at the intensive margin), by retiring production units (extensive margin), and through adoption of precision technologies and management practices (Khanna and Zilberman, 1997). This is a broad category of technologies that includes conservation technologies (Fuglie and Kascak, 2001) and consists of technologies and practices that improve technical efficiency of variable inputs and reduce the associated pollution. Examples include insulation, fuel-efficient engines and stoves, and improved quality fuels that reduce the pollution intensity of energy generation, transportation, temperature control, and cooking. Drip, sprinkler, and computerized irrigation and high precision chemical applications are agricultural examples of technologies that improve variable input productivity and reduce damaging residues. Stock abatement reduces pollution once it has been generated, for example, by catalytic converters, barriers (e.g., plants, containing walls), wetland buffers and vegetative practices such as filter strips nearby surface waters (Mitsch et al., 1999), or various sequestration activities.

This chapter develops a framework to determine the optimal resource allocation over time for the production of a good by heterogeneous producers who generate accumulating pollutant. The chapter also derives government policies to modify the behavior of competitive producers, which leads to optimality. This framework considers the adoption of precision technologies and use of stock abatement activities, and can be applied to a wide variety of settings. For instance, agricultural pollution where the heterogeneity results from varying land quality, manufacturing where the heterogeneity is generated by the different characteristics of the capital goods (different machines or different plants) employed for distinct production processes[1] or the energy sector with plants of different characteristics and a long economic life. In this way, the chapter expands the results of studies that analyze control of stock externalities of production activities with either precision technologies or stock abatement.

[1] It is assumed that producers consider investment as a fixed variable and only determine the intensity of the production and the extent and form of the use of the capital good. In other words producers follow a putty clay approach and no differential equation describing investment and depreciations is required.

A major contribution of this thesis is the decomposition of the complex social optimization problems that encompass time and heterogeneity dimensions to a manageable two-stage procedure. The two-stage solution allows to derive the qualitative characteristics of the solution in more detail and more easily than a single-stage solution. The first stage consists of static optimization of resource allocation by heterogeneous producers, given aggregate pollution. The results of this optimization feed into an optimal intertemporal resource allocation problem determining aggregate production and pollution accumulation over time. The two stages are linked by a common shadow price of the externality stock that is derived in the temporal optimization and is used to derive individually tailored incentives based on rules determined by solving the temporal optimization problems.

With these techniques, the optimal resource allocation is derived, and it is find that it may be attained by technology-differentiated input taxes and taxes or subsidies on fixed assets that allow optimal adjustments in the intensive and extensive margins. These incentives are designed to favor adopters of precision technologies, and thus trigger their adoption.

To address both heterogeneity and dynamic considerations, it is assumed no production uncertainty. The results show that the optimal interim policy depends on the curvature of the convex abatement cost and damage functions. When the pollution stock accumulation leads to significant increments of the abatement and/or damage cost, it is optimal to decrease the stock abatement and to reinforce source abatement, according to the specific conditions for each producer. In contrast, if the marginal stock abatement cost increases slowly, the optimal intertemporal policy is characterized by high stock abatement. The results also demonstrate that a policy, which is solely targeting reduction of input use, may not be socially optimal since it produces a distortion at the extensive margin and, therefore, it needs to be complemented by incentives based on the technology choice.

The chapter is organized as follows. Section 2.2 reviews the literature, section

2.3 describes the basic features of the model, and section 2.4 establishes the optimal environmental policy, consisting of the optimal static and intertemporal solution. Section 2.5 defines individually tailored and intertemporal policies with respect to input and technology choices that can establish the social optimum. Section 2.6 presents some conclusions.

2.2 Review of the Literature

There is a wide array of cases where aggregate pollution, in stock externality problems, is the result of the input utilization in economic processes by heterogeneous producers. Some examples are summarized in Table 2.1. One can distinguish four different strands in the existing literature. The first relates to the question of whether an when to adopt a precision technology in solving stock externality problems. Examples include the use of efficiency enhancing technologies in the energy-generating sector (Siegel and Temchin, 1991; Khanna and Zilberman, 1999), and energy-saving appliances (Hausman, 1979; Jaffe and Stavins, 1995). Similar benefits of precision technologies have been established in other economic sectors, for example in agriculture, the adoption of modern agricultural irrigation technologies in order to reduce the amount of water applied and hence the leaching of pollutants (Caswell and Zilberman, 1985; Caswell, Lichtenberg, and Zilberman, 1990; Green et al., 1996), and the adoption of soil nitrogen testing to adjust the input application more precisely to crop needs to reduce potential N-losses (Fuglie and Bosch, 1995). Lal et al. (1998) show that the use of minimum tillage increases carbon sequestration rates which, in turn, contribute to moderate global warming. Likewise, in the transportation sector, the use of cleaner fuels demonstrated the advantages of precision technologies in reducing urban pollution and greenhouse gas emissions (Khazzoom, 1995; Nakata, 2000). Most of these studies support the finding that the heterogeneity of producing units is a crucial issue in answering the question of whether or not to adopt a technology.

A second strand of papers focuses on regulations at the extensive margin to reduce the stock of pollutant once it has been generated. These includes the establishment of funds to purchase environmental goods and to conserve natural resources, such as the use of land retirement programs to reduce water contamination. Wu, Zilberman, and Babcock (2001) analyze alternative targeting strategies for resource conservation programs when aimed at reducing pollution, accounting for heterogeneity in the targeted resource. Another example of this strand of literature is Ribaudo, Osborn, and Konyar (1994), where they study crop land retirement as an option for reducing water pollution. Their results show that land retirement as a primary pollution control tool is expensive. However, if it is appropriately tailored to the individual producer, benefits may outweigh costs.

Another strand of papers focuses on economics of reduction of pollution stocks. For example, Zhao and Zilberman (2001) identify conditions when the use of minimum tillage increases carbon sequestration rates, which, in turn, contribute to moderate global warming.

A forth line of research analyzes the optimal combination of source abatement versus stock abatement. Shah, Zilberman, and Lichtenberg (1995) present a dynamic framework to analyze the optimal combination of on-farm and off-farm pollution abatement strategies with respect to the problem of water logging, however, assuming constant land quality. Ribaudo et al. (2001) evaluate empirically the impacts of alternative strategies to achieve reduced nitrogen concentrations in the Mississippi Basin. Similarly Farzin (1996) in a more general context develops a dynamic framework to analyze in which way static policy instruments need to be modified in the presence of a stock externality.

The existing papers either establish the optimal intertemporal policy assuming homogeneity with respect to the production units, or neglect the intertemporal

Table 2.1: Examples of Stock Externalities

		Decision Variables			
Source	Type of Problem	Technology	Products	Inputs	State Variable
Agriculture	Soil erosion and siltation of rivers, canals, reservoirs	Abatement technology capital (terracing, machinery with special tires, dams); cultivation techniques (minimum tillage, conventional tillage, working along isohight lines)	Crops, cover crops	Intensity of tillage	Sedimentation layer
	Water logging	Irrigation technologies (furrow irrigation, drip irrigation, sprinkler irrigation)	Crops	Water use	Height of the water table, conc. of salt
	Phosphorus and Nitrogen emissions	Cultivation techniques(N min method of fertilization, frequency of N applications, application machinery); type of fertilizer (organic, inorganic)	Crops, number and types of animals, catch crops	Nitrogen and Phosphorus	Concentration of Nitrogen or Phosphorus
	GHG emissions	Cultivation techniques (minimum tillage, conventional tillage)	Agriculture (cropland, permanent grass land), afforestation,	Intensity of tillage	Concentration of GHG in the atmosphere
Households	GHG emissions	Building structure (thermal insulation, chromogenic glazings, reduction of air leakage), refrigerators and freezers (vacuum insulation panels), lighting (high efficiency fluorescent lamps, compact fluorescent and halogen lamps, electronic ballasts), type of fuel		Energy use	Concentration of GHG in the atmosphere
	Indoor air pollution from cooking stoves	Biomass based fuels (wood, charcoal) electricity, kerosene, gas	Type of meals	Fuel use	Concentration of CO and Benzopyrene
Vehicles	GHG emissions	Type of fuel (gas, ethanol, electric vehicles)		Fuel use	Concentration of GHG in the atmosphere
Electricity generation	GHG emissions	Natural gas, clean coal technologies		Fuel use	Concentration of GHG in the atmosphere
	SO_2, NOx	Low sulphur fuels, low NOx boilers and emission control catalysts		Fuel use	Concentration of SO_2 and NOx in the atmosphere
Paper	GHG emissions	Dry sheet forming, new pressing techniques (press drying, condensing belt drying, impulse drying, air impingement drying), latent heat recovery systems (steam impingement drying, airless drying)	Writing paper, tissue, paperboard	Fuel use	Concentration of GHG in the atmosphere
Iron and steel	GHG emissions	Type of furnace (blast furnace, basic oxygen furnace, electric arc furnace), technologies of scrap melting	Spray casted products	Fuel use	Concentration of GHG in the atmosphere

aspect of the pollution problem if they consider heterogeneity of the production units. In this chapter both aspects are integrated, and a model that incorporates simultaneously heterogeneity and time is developed. In considering heterogeneity, the effects of a change in the quality of the fixed asset at the extensive and intensive margins will be determined. In contrast to the previous literature, for example, by Pan and Hodge (1994) or Glaeser and Shleifer (2001), it is shown that regulations at the extensive and intensive margins should not be considered as a substitute but, rather, as complements. Moreover, the dynamic framework, makes it possible to evaluate the incentives for source abatement versus stock abatement. As a result, this chapter present the formulation of individually tailored, dynamic policies that act separately or in combination at the intensive and extensive margin, to induce socially optimal behavior by the individual agents.

2.3 The Modelling Framework

Consider a competitive industry made of heterogeneous production units (farm, plants), which produce a good using fixed assets (land, machines) and variable inputs (water, chemicals, fossil fuels) with one of two technologies. The production units differ by the quality of asset they use, when $\epsilon \in [\epsilon_0, \epsilon_1]$ is a measure of quality (where higher ϵ corresponds to higher quality). Total assets with quality ϵ are denoted by $X(\epsilon)$.

Let i be a technology indicator, $i = 1$ for the precision technology and $i = 2$ for a traditional technology. The amount of assets of quality ϵ allocated to technology i at time t is denoted by $x_i(t, \epsilon)$. Each technology is assumed to have constant returns-to-scale production function using the fixed asset and variable input. Let $u_i(t, \epsilon)$ be variable input per unit of fixed asset (pesticides per acre, fuel per unit of machine capacity). Output per unit of fixed asset with technology i is $y_i = h_i(\epsilon) f(\beta_i u_i(t, \epsilon))$, $i = 1, 2$. It is assumed that asset quality and precision technology affect productivity through a multiplicative fixed asset effect, represented by $h_i(\epsilon)$, where $h(\cdot)$ is C^2, $h_i(\epsilon_0) = 0$, $h_i(\epsilon_1) = 1$. With each technology, assets of higher quality are more productive ($dh_i/d\epsilon > 0$) and adoption of precision technology tends to increase fixed asset productivity ($h_1(\epsilon) > h_2(\epsilon)$). It is also assumed that precision technology increases the productivity of variable input, represented by β_i, $\beta_1 > \beta_2$.[2]

The product and input price of the industry are p and c, respectively. Each technology requires a per asset fee for the technology, denoted by I_i (rent or annualized cost of the technology). It is assumed that $I_1 > I_2$, that is, the precision technology is more costly.

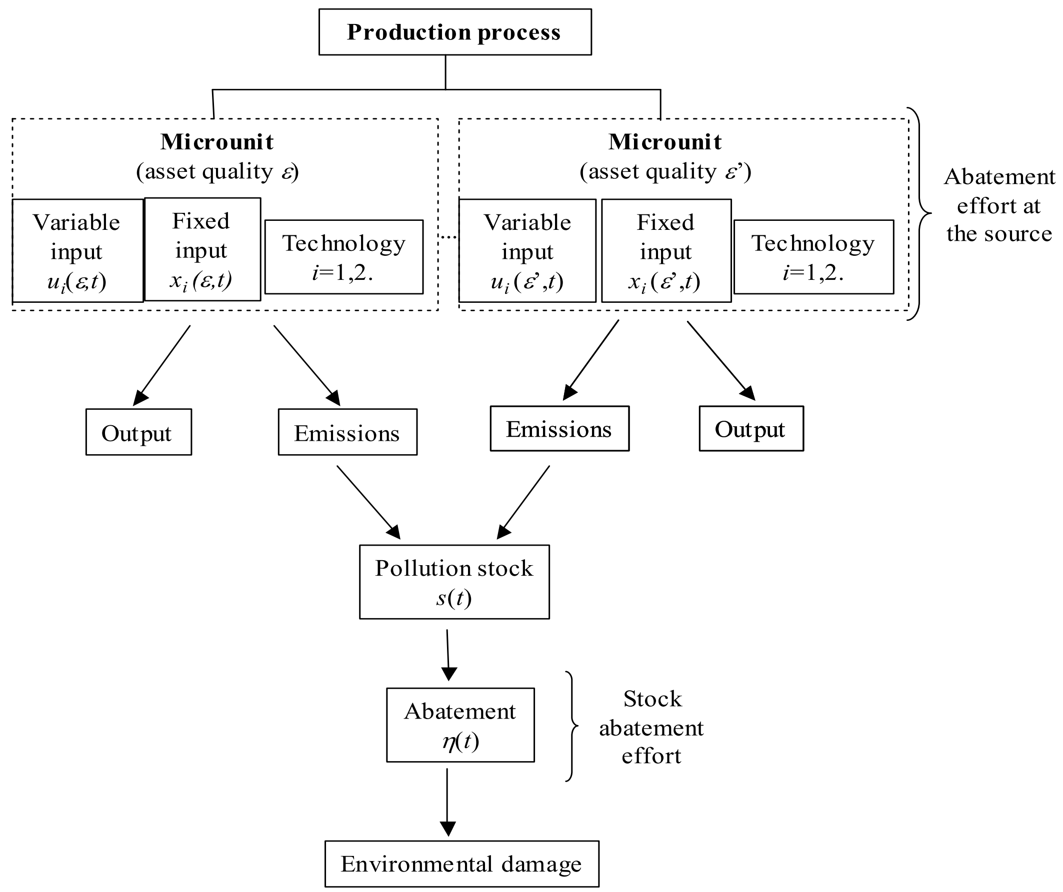
The pollutant is assumed to emanate from variable input use. Potential pollution per asset per unit of time is $g(u_i(t, \epsilon))$, a convex function of applied input, $g_{u_i} > 0$, $g_{u_i u_i} \geq 0$. High quality asset and/or precision technology consumes much of the variable input and reduces leakage, so actual pollution per asset unit per unit of time is $\alpha_i(\epsilon) g(u_i(t, \epsilon))$, where the pollution coefficient $\alpha_i(\epsilon)$ is a fraction of potential pollution that is realized. It is assumed (i) that no pollution occurs with highest quality asset, that is, $\alpha_i(\epsilon_1) = 0$, (ii) the pollution coefficient is declining with quality, i.e., $d\alpha_i/d\epsilon < 0$, and (iii) that precision technology has lower pollution coefficient than the traditional one, i.e., $\alpha_2(\epsilon) > \alpha_1(\epsilon)$ for $\epsilon < \epsilon_1$.

The aggregate pollution stock at time t is $s(t)$, and the temporal economic loss of pollution stock per period is given by the damage function $d(s(t))$ with $d(0) = 0$, $d_s > 0$ and $d_{ss} > 0$. The pollution stock may be reduced by various abatement activities (e.g., carbon sequestration by forestry and/or in deep oceans). Let $\eta(t)$ denote stock abatement at time t and $k(\eta(t), s(t))$ be stock abatement cost. It is assumed that marginal cost of stock abatement is positive and increasing with η, $k_\eta > 0$, $k_{\eta\eta} > 0$, but decreasing with the pollutant stock, $k_{\eta s} < 0$. A higher pollutant stock is assumed to increase total abatement cost, $k_s > 0$, and $k_{ss} > 0$.

[2]For example, drip irrigation may have 0.95 irrigation efficiency, while irrigation efficiency of traditional methods is 0.6 (Hanemann et al., 1987).

Figure 2.1 presents a possible scheme of the key variables and their relationships in the production process.

Figure 2.1: Scheme of the Production Process



The dynamics of the pollutant stock can now be stated as

$$\dot{s}(t) = \int_{\epsilon_0}^{\epsilon_1} \left(\sum_{i=1}^{2} \alpha_i(\epsilon) g(u_i(t,\epsilon)) x_i(t,\epsilon) \right) d\epsilon - \eta(t) - \zeta s(t), \tag{2.1}$$

where a dot over a variable denotes the operator $\frac{d}{dt}$. Following Clasen et al. (1989), the concentration of the pollutant over time is described as a linear function in s. The parameter $\zeta,\ 0 < \zeta < 1$ represents the decay rate of the pollutant stock.

2.4 Dynamics of the pollution stock problem

It is assumed that a social planner exists and maximizes the present discounted value of the net benefits from production, while taking into account the social economic losses due to the accumulation of the pollutant.[3] Thus, the social planner's decision problem is given by

$$\max_{u_i(t,\epsilon),\, x_i(t,\epsilon),\, \eta(t)} \quad \int_0^\infty e^{-\delta t}\Big[\int_{\epsilon_0}^{\epsilon_1}\Big(\sum_{i=1}^{2}\big(ph_i(\epsilon)f(\beta_i u_i(t,\epsilon)) - \\ -cu_i(t,\epsilon) - I_i\big)x_i(t,\epsilon)\Big)d\epsilon - \Big(d(s(t)) + k(\eta(t), s(t))\Big)\Big]\,dt, \tag{S}$$

[3]Given the regional focus of the analysis, it is assumed that the product prices are not influenced by regional production decisions and, therefore, they are taken as given. Thus, the output price is also not influenced by the production of the externality. It is assumed that there are no transportation costs, and the utility function of the consumers is quasilinear with respect to the traded goods and the externality. Thus, the optimal level of the externality is independent of the consumers' expenditures, and it is possible to derive a utility function which depends only on the externality s (Mas-Colell, Whinston, and Green, 1995). To discuss the results of our model in a practical setting, it is proposed that the derived utility function can be represented by the damage function $d(s(t))$ and the abatement cost function $k(\eta(t), s(t))$. Additionally, it is also assumed that there are no cost of public funds, and lump-sum transfers are available to redistribute income so that input taxes, technology taxes, or subsidies are not distortionary (Sandmo, 1995). The assumptions made with respect to the quasilinearity of the utility function and the existence of costless public funds help to keep the model simple. It allows to concentrate the analysis on answering the question of whether or not it is socially optimal to abate at the source or to abate the pollution stock and which is the optimal policy to achieve the socially optimal outcome.

subject to

$$
\begin{aligned}
\dot{s}(t) &= \int_{\epsilon_0}^{\epsilon_1} \Big(\sum_{i=1}^{2} \alpha_i(\epsilon) g(u_i(t,\epsilon)) x_i(t,\epsilon) \Big)\, d\epsilon - \eta(t) - \zeta s(t), \\
& s(0) = s_0, \quad u_i(t,\epsilon) \geq 0,\ i = 1,2, \quad x_i(t,\epsilon) \geq 0,\ i = 1,2, \\
& x_1(t,\epsilon) + x_2(t,\epsilon) \leq X(\epsilon) \quad \eta(t) \in [0, s(t)],
\end{aligned}
$$

where s_0 denotes the amount of pollution stock at the initial point of calendar time, and the parameter $\delta > 0$ denotes the social discount rate.

Utilizing Pontryagin's Maximum Principle, the current Hamiltonian of the optimal pollution restoration strategy (S) is given by

$$
\begin{aligned}
\mathcal{H} \equiv & \int_{\epsilon_0}^{\epsilon_1} \Big(\sum_{i=1}^{2} \big(p h_i(\epsilon) f(\beta_i u_i(t,\epsilon)) - c u_i(t,\epsilon) - I_i\big) x_i(t,\epsilon) \Big)\, d\epsilon \\
& - \Big(d(s(t)) + k(\eta(t), s(t)) \Big) \\
& - \mu(t) \Big(\int_{\epsilon_0}^{\epsilon_1} \Big(\sum_{i=1}^{2} \alpha_i(\epsilon) g(u_i(t,\epsilon)) x_i(t,\epsilon) \Big)\, d\epsilon - \eta(t) + \zeta s(t) \Big).
\end{aligned}
$$

To facilitate the interpretations of the costate variable μ, it has been multiplied by minus one. In this way μ has a positive value. The arguments ϵ and t of the variables and the Lagrange multipliers ω_i, $i = 1, \cdots, 7$, to be introduced later, will be suppressed to simplify the notation unless it is required for an unambiguous notation. Taking account of the constrains on the control variables leads to the Lagrangian: $\mathcal{L} \equiv \mathcal{H} + \omega_1 u_1 + \omega_2 u_2 + \omega_3 x_1 + \omega_4 x_2 + \omega_5 (X - x_1 - x_2) + \omega_6 \eta + \omega_7 (s - \eta)$, where $\omega_1, \ldots, \omega_7$ denote Lagrange multipliers. The solution of problem (S) has to satisfy the following necessary conditions stated in accordance with Theorem 1, page 276, Seierstad and Sydsæter (1987)

$$
\begin{aligned}
\mathcal{L}_{u_1} &\equiv (ph_1\beta_1 f_{u_1} - c - \mu\alpha_1 g_{u_1})x_1(t,\epsilon) + \omega_1 = 0, && (2.2)\\
\mathcal{L}_{u_2} &\equiv (ph_2\beta_2 f_{u_2} - c - \mu\alpha_2 g_{u_2})x_2(t,\epsilon) + \omega_2 = 0, && (2.3)\\
\mathcal{L}_{x_1} &\equiv py_1 - cu_1(t,\epsilon) - I_1 - \mu\alpha_1 g(u_1(t,\epsilon)) + \omega_3 - \omega_5 = 0, && (2.4)\\
\mathcal{L}_{x_2} &\equiv py_2 - cu_2(t,\epsilon) - I_2 - \mu\alpha_2 g(u_2(t,\epsilon)) + \omega_4 - \omega_5 = 0, && (2.5)\\
\mathcal{L}_{\eta} &\equiv -k_\eta + \mu + \omega_6 - \omega_7 = 0, && (2.6)\\
\dot{\mu}(t) &= \delta\mu + \mathcal{L}_s = \mu(\delta+\zeta) - d_s - k_s + \omega_7, && (2.7)\\
\dot{s}(t) &= \int_{\epsilon_0}^{\epsilon_1}\Big(\sum_{i=1}^{2}\alpha_i g(u_i(t,\epsilon))x_i(t,\epsilon)\Big)\,d\epsilon - \eta(t) + \zeta s(t), &&\\
& \quad s(0) = s_0. && (2.8)
\end{aligned}
$$

The analytical solution of the necessary conditions (2.2) - (2.8) is difficult. Thus, to solve problem (S), it is proposed a solution technique in two stages described in the following proposition.

Proposition 1: *The dynamic optimization problem S is equivalent to specifying and solving two sequential problems, $S1$ and $S2$. In the first stage (problem $S1$), the model is maximized over ϵ subject to a prespecified level of aggregate emissions z, obtaining the optimal trajectories of $u(\epsilon)$ and $x(\epsilon)$. In the second stage (problem $S2$), the parameter z becomes a decision variable; and the optimal trajectories of $z(t)$ and, consequently, of the functions $u(t,\epsilon)$, $x(t,\epsilon)$, and $\eta(t)$ over time are obtained.*

Following the optimization procedure described in the introduction — two stages optimal control, Proposition 1 can be verified. Due to the structure of problem (S), one is able to decompose part of the problem into a static, and another part into a dynamic control problem. The static control problem optimizes the use of resources over the heterogeneous characteristic of the production units, for example, vintage or space. The solution to the static problem is then plugged into the dynamic control problem to determine the optimal combination of source abatement, leading to a particular level of aggregate emissions and stock abatement.

Separating the problem in two stages is possible because the state variable is an aggregate function that depends exclusively on time. In the first stage the socially optimal solution over ϵ is determined, i.e., for every quality ϵ, it is determined the individually tailored optimal level of input applied, and the individually tailored optimal choice of technology for each production unit, including the option of not to produce. In the second stage the obtained optimal static solution is maximized with respect to time, by determining the optimal combination of source abatement versus stock abatement.

2.4.1 The Solution to the Static Optimization Problem

In the first stage the solution of the social planner's decision problem is given by the value function $V(z)$ defined as:

$$V(z) \equiv \max_{u_i(\epsilon),\, x_i(\epsilon)} \int_{\epsilon_0}^{\epsilon_1} \Big(\sum_{i=1}^{2} \big(ph_i(\epsilon) f(\beta_i u_i(\epsilon)) - cu_i(\epsilon) - I_i\big) x_i(\epsilon) \Big)\, d\epsilon \tag{S1}$$

subject to

$$z = \int_{\epsilon_0}^{\epsilon_1} \Big(\sum_{i=1}^{2} \alpha_i(\epsilon) g(u_i(\epsilon)) x_i(\epsilon) \Big)\, d\epsilon,$$

$$u_i(\epsilon) \geq 0,\ i = 1, 2, \quad x_i(\epsilon) \geq 0,\ i = 1, 2, \quad x_1(\epsilon) + x_2(\epsilon) \leq X(\epsilon),$$

where z denotes the aggregate emissions over the entire range of ϵ, from ϵ_0 to ϵ_1. The argument ϵ of the variables and the Lagrange multipliers υ_i, $i = 1, \cdots, 5$, to be introduced later, will be suppressed to simplify the notation unless it is required for an unambiguous notation. Taking account of the constraints on the control variables leads to the Lagrangian

$$
\begin{aligned}
\mathcal{L}1 \quad \equiv \quad & \int_{\epsilon_0}^{\epsilon_1} \Big(\sum_{i=1}^{2} \big(ph_i f(\beta_i u_i) - cu_i - I_i\big) x_i \Big)\, d\epsilon \\
& + \lambda \Big(z - \int_{\epsilon_0}^{\epsilon_1} \Big(\sum_{i=1}^{2} \alpha_i g(u_i) x_i \Big)\, d\epsilon \Big) \\
& + \upsilon_1 u_1 + \upsilon_2 u_2 + \upsilon_3 x_1 + \upsilon_4 x_2 + \upsilon_5 \big(X - x_1 - x_2 \big).
\end{aligned}
$$

To facilitate the interpretations of the costate variable λ, it has been multiplied by minus one, leading to a positive value for λ.

A solution of the problem has to satisfy the following necessary conditions:

$$\mathcal{L}1_{u_1} \equiv (ph_1 \beta_1 f_{u_1} - c - \lambda \alpha_1 g_{u_1}) x_1 + \upsilon_1 = 0, \tag{2.9}$$

$$\mathcal{L}1_{u_2} \equiv (ph_2 \beta_2 f_{u_2} - c - \lambda \alpha_2 g_{u_2}) x_2 + \upsilon_2 = 0, \tag{2.10}$$

$$\mathcal{L}1_{x_1} \equiv py_1 - cu_1 - I_1 - \lambda \alpha_1 g(u_1) + \upsilon_3 - \upsilon_5 = 0, \tag{2.11}$$

$$\mathcal{L}1_{x_2} \equiv py_2 - cu_2 - I_2 - \lambda \alpha_2 g(u_2) + \upsilon_4 - \upsilon_5 = 0, \tag{2.12}$$

$$\mathcal{L}1_{\lambda} \equiv z - \int_{\epsilon_0}^{\epsilon_1} \Big(\sum_{i=1}^{2} \alpha_i g(u_i) x_i \Big)\, d\epsilon = 0. \tag{2.13}$$

The Lagrange multiplier λ is interpreted as the shadow costs of the prespecified level of aggregate emissions, z. Please note that z does not depend on ϵ, and thus λ is constant over ϵ. For an interior solution, given quality ϵ and given a particular technology, necessary conditions (2.9) and (2.10) indicate that the value of the marginal product of applied input per unit of asset should equal the sum of the marginal cost of input use and the marginal cost of pollution per unit of asset. In the case of a boundary solution, the Lagrange multiplier of the binding constraint reflects the difference between the value of the marginal product and the sum of the marginal costs. The necessary conditions (2.11) and (2.12) indicate that the marginal net benefits of production per unit of asset with quality ϵ, given a particular technology, should equal the marginal cost of pollution per unit of asset. However, since both the production and emission functions are linear in the fixed asset, the technology that leads to a higher social quasirent, defined as $\Pi_i^* \equiv py_i - cu_i - I_i - \lambda\alpha_i g(u_i)$, will be completely preferred to the technology with the lower quasirent, implying that the entire asset with quality ϵ should be combined in the production process with the technology that yields the highest quasirent. Hence, boundary solutions are exclusively obtained for every quality ϵ, given by $x_1(\epsilon) = X(\epsilon)$, $x_2(\epsilon) = X(\epsilon)$, or $x_1(\epsilon) = x_2(\epsilon) = 0$. In this case, the Lagrange multiplier of the binding constraint reflects the difference between the value of the marginal net benefits and the marginal pollution cost. However, the maximal quasirent for technology i, Π_i^* depends on the asset quality and, thus, it will change over ϵ.

The next proposition explains how the optimal level of the key variables changes with a change in quality ϵ.

Proposition 2: *For a given technology, an increase in the quality of the asset leads to an increase in the input use and, consequently, to an increase in the output and in the quasirent.*

$$\frac{\partial u_i^*}{\partial \epsilon} > 0, \qquad \frac{\partial y_i^*}{\partial \epsilon} > 0, \qquad \frac{\partial \Pi_i^*}{\partial \epsilon} > 0.$$

Proof: To determine the effect of a change in asset quality on the level of applied input, equations (2.9) and (2.10) are differentiated with respect to ϵ and solve for $\partial u_i/\partial\epsilon$. Hence, it results in:

$$\frac{\partial u_i}{\partial \epsilon} = \frac{-(ph_i'\beta_i f_{u_i} - \lambda\alpha_i' g_{u_i})}{ph_i\beta_i^2 f_{u_i u_i} - \lambda\alpha_i g_{u_i u_i}} > 0. \tag{2.14}$$

Since the production function has regular, neoclassical properties, it is obtained:

$$\frac{\partial y_i^*}{\partial \epsilon} = h_i' f(\beta_i u_i^*) + f_{u_i}\frac{\partial u_i^*}{\partial \epsilon} > 0. \tag{2.15}$$

The changes in the allocation of the technologies are determined by differentiating equations (2.11) and (2.12) with respect to ϵ:

$$\Pi^*_{i_\epsilon} = [ph'_i f(\beta_i u_i)] - [\lambda \alpha'_i g(u_i)] > 0, \tag{2.16}$$

where the first term in brackets presents the value of the change in production per unit of asset due to an increase in the productivity of the asset, and the second term in brackets measures the monetary value of the changes in the actual emissions per unit of asset. Since $h'_i > 0$, $f(\beta_i u_i) > 0$, and $\lambda \alpha'_i g(u_i) < 0$, the quasirent of activity i, $i = 1, 2$, is upward sloping with an increase in ϵ. ■

The concavity/convexity of the quasirent is shown in the second derivative, given by

$$\Pi^*_{i_{\epsilon\epsilon}} = \Big[ph''_i f(\beta_i u_i) - \lambda \alpha''_i g(u_i)\Big] - \Big[(ph'_i \beta_i f_{u_i} - \lambda \alpha'_i g_{u_i})\frac{\partial u_i}{\partial \epsilon}\Big] \gtreqless 0. \tag{2.17}$$

Equation (2.17) shows that the quasirent is more likely to be convex $\Pi^*_{i_{\epsilon\epsilon}} > 0$ in low qualities due to the observation that, if the input use is low, production and pollution are low while the derivatives f_{u_i} and g_{u_i} are large. As the asset quality increases, the first term in brackets increases while the second decreases and, therefore, the quasirent is more likely to be concave, i.e., $\Pi^*_{i_{\epsilon\epsilon}} < 0$.

Equation (2.14) implies that an increase in the asset quality increases the optimal intensity of input use of either technology, since it increases the productivity of the marginal unit of the applied input and decreases the marginal pollution level. When the effectiveness of input use, β_i, depends on ϵ, as in Caswell and Zilberman (1986), the sign of $\partial u^*_i / \partial \epsilon$ will depend on the elasticity of marginal productivity.

In a parallel manner, equation (2.16) shows that the quasirent for each technology increases with an increase in the quality ϵ, since it increases the asset productivity and decreases the amount of generated emissions. The adoption of the precision technology is optimal when its associated quasirent is positive and higher than that of the traditional technology. The difference in quasirent per unit of asset with quality ϵ with the modern and traditional technologies can be written as:

$$\Pi_1^* - \Pi_2^* = p\triangle y^* - c\triangle u^* - \triangle I - \lambda(\alpha_1 g(u_1^*) - \alpha_2 g(u_2^*)), \qquad (2.18)$$

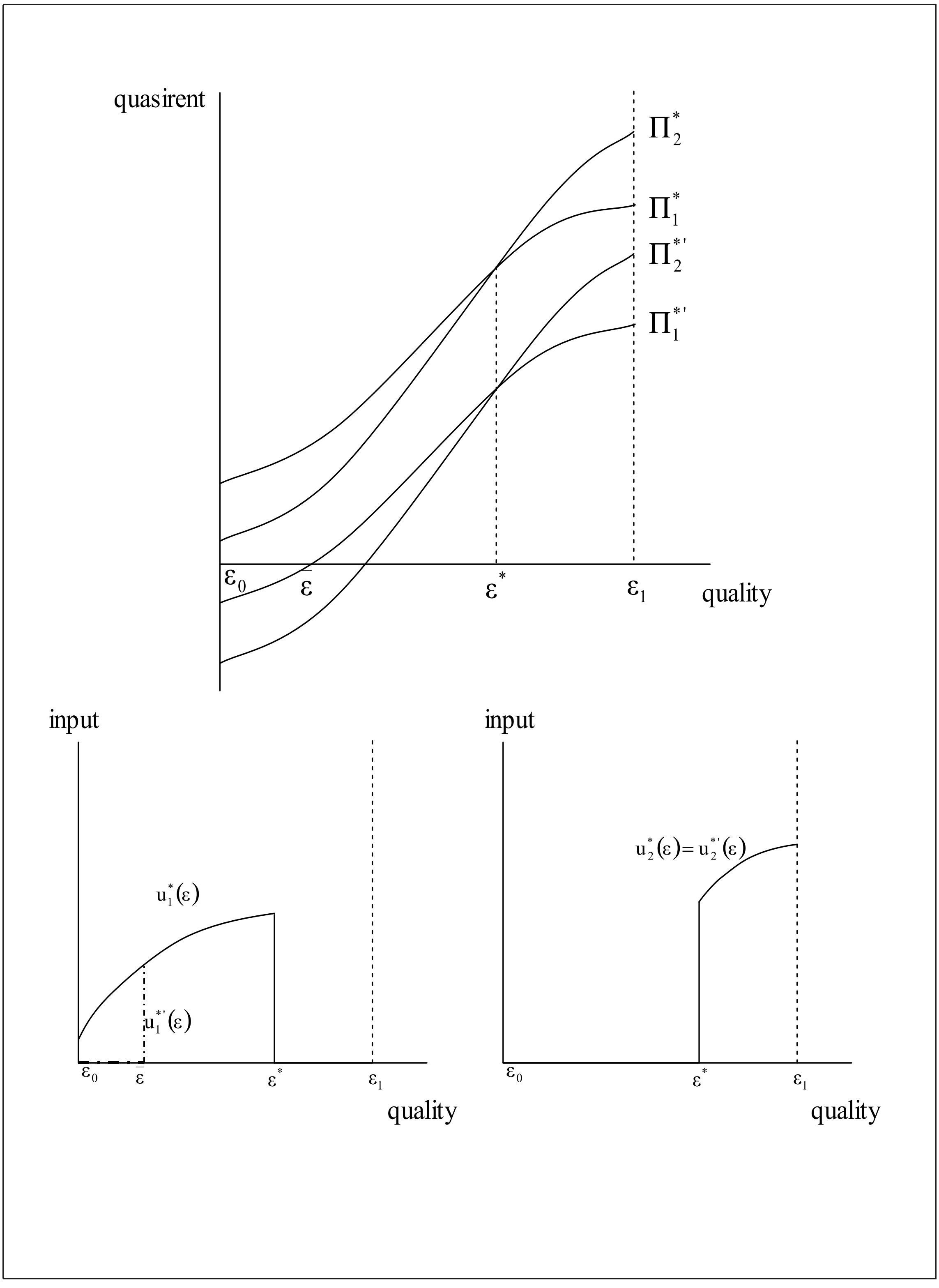
where $\triangle$ represents the difference in the level of the different variables with the two technologies. Equation (2.18) implies that the precision technology should be adopted if the resulting increase in output, decrease in the level of input use, and decrease in pollution compensates the higher fixed cost required to implement the new technology, that is, the higher annual costs per unit of asset.

In order to gain more insight into the analysis, let us assume that the new technology only has the input quality augmenting effect, but it does not increase the productivity of the fixed asset, i.e., $\beta_1 = \beta_2 = 1$. Although this assumption does not change the dynamic aspect of the model, it will be useful to derive some characteristics of the adoption pattern. With quality at its maximum, ϵ_1, the traditional technology will be preferred to precision technology since it has lower costs of adoption ($\Pi_1^*(\epsilon_1) < \Pi_2^*(\epsilon_1)$). As the asset quality declines, the precision technology becomes socially more profitable because activity 2 exacerbates the potential emissions. When the order of the inequality is reversed, we know that the social quasirent functions intersect, and it is optimal to diversify the technology choice. The switching point is given by ϵ^* where $\Pi_1^*(\epsilon^*) = \Pi_2^*(\epsilon^*)$. This case is depicted in Figure 2.2 for the quasirents Π_1^* and Π_2^*, where the traditional technology is optimal for higher levels of ϵ.[4] The advantage of the traditional technology is the lower annualized fixed costs of adoption per unit of asset. However, this advantage gets lost as quality declines and, therefore, potential emissions increase. Thus, for $\epsilon < \epsilon^*$, the precision technology will be optimal due to higher productivity and lower emissions.

Additionally, Figure 2.2 also presents the case where both quasirents $\Pi_1^{*\prime}$ and $\Pi_2^{*\prime}$ turn negative for qualities below $\bar{\epsilon}$, $\bar{\epsilon} \in [\epsilon_0, \epsilon_1]$. Hence, no production will take

[4]Even though $X(\epsilon)$ presents the total of the asset of quality ϵ, and it varies over ϵ, the graphical presentation of the results is simplified by considering $X(\epsilon)$ to be constant over ϵ.

Figure 2.2: Optimal Technology Choice when It Is Optimal to Diversify the Use of the Fixed Asset



place in the interval between ϵ_0 and $\bar{\epsilon}$.

If the order of the inequality ($\Pi_1^*(\epsilon_1) < \Pi_2^*(\epsilon_1)$) is not reversed if evaluated at ϵ_0, the quasirents of activities 1 and 2 do not intersect and, therefore, it is optimal not to diversify the technology choice. Figure 2.3 presents the case where the traditional technology will be preferred over the entire range of quality because the lower costs of adoption are not outweighed by an increase in the social cost of pollution and output of the precision technology. Like in Figure 2.2, Figure 2.3 shows that there may exist an $\bar{\epsilon}$ below that no production takes place.

Next, the previous assumption of $\beta_1 = \beta_2 = 1$ will be relaxed, and it is assumed that the new technology enhances the productivity of the fixed asset, to see how the neutral multiplicative effect can affect the adoption pattern. Given the highest quality, precision technology will be preferred to the traditional technology if the decrease in the input use exceeds the higher costs of adoption, that is, if

$$\left(\frac{\beta_2}{\beta_1} - 1\right) c u_2^* > I_1 - I_2. \tag{2.19}$$

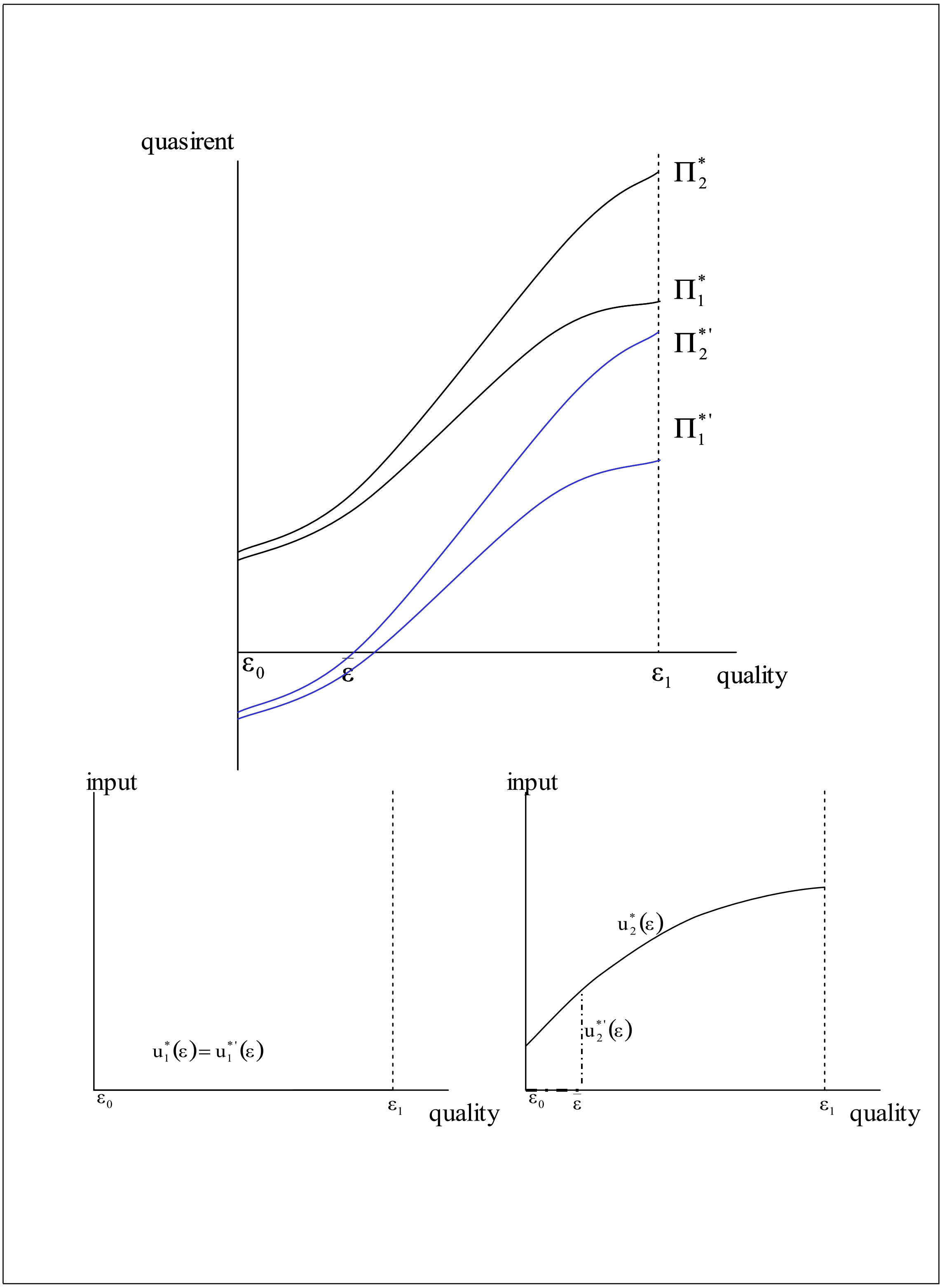
In this case precision technology will be preferred over the entire range of ϵ.

2.4.2 The Optimal Dynamic Solution

In the first stage, the socially optimal, quality-differentiated, however static, solution was derived, given by the optimal allocation of the technologies and the optimal level of input use. To analyze how the optimal solution is affected over time, the value function V obtained in the first stage is maximized over time. Hence, the social planner's decision problem is given by:

$$\max_{z(t),\, \eta(t)} \quad \int_0^\infty e^{-\delta t}\Big(V(z(t)) - d(s(t)) - k(\eta(t), s(t))\Big)\, dt \tag{S2}$$

Figure 2.3: Optimal Technology Choice when It Is Not Optimal to Diversify the Use of the Fixed Asset



subject to

$$\begin{aligned} \dot{s}(t) &= z(t) - \eta(t) - \zeta\, s(t), \\ & \qquad s(0) = s_0, \quad \eta(t) \in [0, s(t)], \quad z(t) \geq 0. \end{aligned}$$

The parameter z of the first-stage problem, which denotes aggregate emissions over the entire range of quality ϵ becomes the decision variable in the second stage. However, it now depends on t. Thus, the decision variables in the intertemporal allocation problem are given by the aggregate emissions $z(t)$ and the stock abatement effort $\eta(t)$. Hence, it will be able to analyze the optimal mix of abatement effort at the source versus stock abatement. The current value Hamiltonian of the second stage is given by: $\mathcal{H}2 = V(z(t)) - d(s(t)) - k(\eta(t), s(t)) - \varphi(t)\Big(z(t) - \eta(t) - \zeta s(t)\Big)$, where φ denotes the costate variable. The first-order conditions for an interior solution read as follows:

$$\begin{aligned} \mathcal{H}2_z &\equiv V_z - \varphi = 0, \qquad \Rightarrow \lambda = \varphi && (2.20) \\ \mathcal{H}2_\eta &\equiv -k_\eta + \varphi = 0, && (2.21) \\ \dot{\varphi} &= \delta\varphi + \mathcal{H}2_s = \big(\delta + \zeta\big)\varphi - d_s - k_s, && (2.22) \\ \dot{s} &= z - \eta - \zeta s, \qquad s(0) = s_0, && (2.23) \end{aligned}$$

where the dynamic envelope theorem has been used in equation (2.20). This equation states that the marginal value of the aggregate emissions of the production units, should equal its marginal shadow cost φ, which, in turn, is equal to the shadow cost of the static allocation problem λ. Equation (2.21) indicates that the marginal cost of stock abatement should equal the shadow cost of pollution stock. Equation (2.22) suggests that the cost of a one-period delay in the generation of marginal unit of pollutant stock will be the extra discounting and depreciation cost $(\delta+\zeta)\varphi$ minus the temporal marginal social cost of the pollutant stock d_s plus the marginal effect of pollutant stock on stock abatement costs $-k_s$.

For a sustainable environmental policy, the social planner is particularly interested in the achievement of a steady state, defined by equations (2.22) and (2.23) by $\dot{\varphi} = \dot{s} = 0$. Assuming an interior solution, equations (2.20) and (2.21) can be solved globally and uniquely by using Theorem 6 in Gale and Nikaidô (1965) for $z = \hat{z}(\varphi, s)$ and $\eta = \hat{\eta}(\varphi, s)$. In order to analyze the effect of a change in the shadow cost or in the stock of pollution on aggregate emissions (source abatement) and stock abatement a comparative static analysis is conducted. Hence, by the implicit function theorem, it is obtained

$$\begin{pmatrix} \mathcal{L}2_{zz} & \mathcal{L}2_{z\eta} \\ \mathcal{L}2_{\eta z} & \mathcal{L}2_{\eta\eta} \end{pmatrix} \begin{pmatrix} \frac{\partial \hat{z}}{\partial \varphi} & \frac{\partial \hat{z}}{\partial s} \\ \frac{\partial \hat{\eta}}{\partial \varphi} & \frac{\partial \hat{\eta}}{\partial s} \end{pmatrix} + \begin{pmatrix} \mathcal{L}2_{z\varphi} & \mathcal{L}2_{zs} \\ \mathcal{L}2_{\eta\varphi} & \mathcal{L}2_{\eta s} \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}. \tag{2.24}$$

The application of Cramer's rule yields that

$$\frac{\partial \hat{z}}{\partial \varphi} = \frac{1}{V_{zz}} \leq 0, \qquad \frac{\partial \hat{z}}{\partial s} = 0, \qquad \frac{\partial \hat{\eta}}{\partial \varphi} = \frac{1}{k_{\eta\eta}} \geq 0, \qquad \frac{\partial \hat{\eta}}{\partial s} = -\frac{k_{\eta s}}{k_{\eta\eta}} \geq 0. \tag{2.25}$$

For the purposes of a qualitative analysis, the necessary conditions (2.20) - (2.23) are reduced to a pair of differential equations in φ and s by substituting $z = \hat{z}(\varphi, s)$ and $\eta = \hat{\eta}(\varphi, s)$ into (2.22) and (2.23) to obtain

$$\dot{\varphi} = \big(\delta + \zeta\big)\varphi - d_s - k_s(\hat{\eta}(\varphi, s), s), \tag{2.22 $'$}$$

$$\dot{s} = \hat{z}(\varphi, s) - \hat{\eta}(\varphi, s) - \zeta s, \qquad s(0) = s_0. \tag{2.23 $'$}$$

A linearization of the canonical system of differential equations around the steady-state values of φ and s results in

$$\begin{pmatrix} \dot{\varphi} \\ \dot{s} \end{pmatrix} = \begin{pmatrix} \frac{\partial \dot{\varphi}}{\partial \varphi} & \frac{\partial \dot{\varphi}}{\partial s} \\ \frac{\partial \dot{s}}{\partial \varphi} & \frac{\partial \dot{s}}{\partial s} \end{pmatrix} \begin{pmatrix} \varphi - \varphi^{\infty} \\ s - s^{\infty} \end{pmatrix}. \tag{2.26}$$

The implicit function theorem is also used to calculate the elements of the Jacobian matrix evaluated at the steady-state equilibrium with $\dot{\varphi} = \dot{s} = 0$, leading to

$$\tilde{J} = \begin{pmatrix} \frac{\partial \dot{\varphi}}{\partial \varphi} = \delta + \zeta - k_{\eta s}\frac{\partial \eta}{\partial \varphi} > 0 & \frac{\partial \dot{\varphi}}{\partial s} = -k_{\eta s}\frac{\partial \eta}{\partial s} - d_{ss} - k_{ss} \gtreqless 0 \\ \frac{\partial \dot{s}}{\partial \varphi} = \frac{\partial z}{\partial \varphi} - \frac{\partial \eta}{\partial \varphi} < 0 & \frac{\partial \dot{s}}{\partial s} = -\zeta - \frac{\partial \eta}{\partial s} < 0 \end{pmatrix}. \tag{2.27}$$

Since the trace of the Jacobian matrix, trJ is equal to $\delta > 0$, employing the fact that trJ equals the sum of its eigenvalues assures that at least one eigenvalue is positive. First, it is considered the case that $\frac{\partial \dot{\varphi}}{\partial s} < 0$. In this case, the determinant of the Jacobian matrix is negative and, thus, the eigenvalues have opposite signs and the steady-state equilibrium is locally characterized by a saddle point. However, in the case that $\frac{\partial \dot{\varphi}}{\partial s} > 0$, the determinant of the Jacobian matrix may be negative or positive. If it is negative, the steady-state equilibrium is locally characterized by a saddle point. However, in the case that the expression $\frac{\partial \dot{\varphi}}{\partial s} > 0$ leads to a positive determinant of $\tilde{J}$, the steady state is characterized by an unstable equilibrium. Nevertheless, the abatement and damage cost functions need to take on very extreme values in order to lead to a positive determinant of $\tilde{J}$. Thus, this particular case is unlikely to happen in practice and, therefore, the analysis concentrates on the first two cases.

For any initial value of s within the neighborhood of s^{∞}, where the superscript ∞ indicates the steady-state equilibrium value, it is possible to find a corresponding value of the shadow cost, which assures that the optimal environmental abatement policy leads toward the long-run optimum.[5]

The sign of $\partial\dot{\varphi}/\partial s = -k_{\eta s}\frac{\partial \eta}{\partial s} - d_{ss} - k_{ss}$ of equation (2.27) depends on the effect of an increase in one unit of the pollutant stock on the marginal damage cost, and on the marginal abatement cost. By the given definitions and by equation (2.25) the first term of $\partial\dot{\varphi}/\partial s$ is positive, while the second and third terms are negative. Thus, the overall sign of $\partial\dot{\varphi}/\partial s$ depends on the magnitude of these opposing effects. Given this setup, the following proposition can be formulated.

[5] This results holds only for values within a certain neighborhood of the steady state as the steady-state analysis has local character.

Proposition 3: *The sign of the slope of the stable path depends on the curvature of the convex abatement cost and damage functions. When the direct effect in the form of an increase in the marginal abatement cost and marginal damage exceeds the indirect effect, i.e.,* $k_{ss} + d_{ss} > |k_{\eta s}\frac{\partial \eta}{\partial s}|$*, the shadow cost* φ *increases with an increase in the stock of pollution. In the opposite case, where the indirect effect exceeds the direct effect, i.e.,* $k_{ss} + d_{ss} < |k_{\eta s}\frac{\partial \eta}{\partial s}|$*, the shadow cost decreases with an increase in the stock of pollution.*

Proof: The first step in analyzing a system of differential equations is to find the eigenvalues and eigenvectors of the Jacobian matrix

$$\tilde{J} = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix}, \tag{2.28}$$

where the elements of $\tilde{J}$ are given in equation (2.27). The eigenvalues of $\tilde{J}$ are determined by solving $|\tilde{J} - \gamma I| = 0$, leading to

$$\gamma_1, \gamma_2 = \frac{trJ \pm \sqrt{trJ^2 - 4detJ}}{2}, \tag{2.29}$$

where trJ is the trace of $\tilde{J}$ and $detJ$ denotes the determinant of the Jacobian matrix. According to Angel de la Fuente (2000) p. 476, the slope of the stable path leading to the steady state is the same as the slope of the eigenvector $e_2 = (e_{12}, e_{22})$ associated to the stable root, i.e., the negative eigenvalue γ_2. The eigenvector e_2 is given by $\tilde{J}e_2 = \gamma_2 e_2$, normalizing its second component to 1, one has that $J_{21}e_{21} + J_{22} = \gamma_2$, implying that the slope of the stable path is determined by the sign of the following equation

$$e_{21} = \frac{\gamma_2 - J_{22}}{J_{21}}. \tag{2.30}$$

In the model, $J_{21} = \partial \dot{s}/\partial \varphi < 0$, thus, the slope of the stable path leading to the steady state will be positive when:

$$\gamma_2 - J_{22} < 0. \tag{2.31}$$

Substituting the expression of γ_2 presented in equation (2.29) into inequality (2.31) leads to

$$\frac{trJ - \sqrt{trJ^2 - 4detJ}}{2} - J_{22} < 0. \tag{2.32}$$

Knowing that $trJ = J_{11} + J_{22}$, it is obtained

$$\sqrt{trJ^2 - 4detJ}2 > J_{11} - J_{22}, \tag{2.33}$$

and after some transformations it is obtained that the slope of the shadow path will be positive if $J_{12}J_{21} > 0$. Thus, $J_{12} = \partial\dot{\varphi}/\partial s$ has to be negative given the fact that $J_{21} = \partial\dot{s}/\partial\varphi$ is also negative. On the contrary, when $J_{12} = \partial\dot{\varphi}/\partial s$ is positive, the slope of the stable path leading to the steady state is negative. ■

Proposition 3 is illustrated in Figures 2.4 and 2.5. If $d_{ss} + k_{ss} > |k_{\eta s}\frac{\partial\eta}{\partial s}|$, the slopes of the $\dot{\varphi} = 0$ and $\dot{s} = 0$ isoclines of the phase diagram in the (s, φ) space are given by

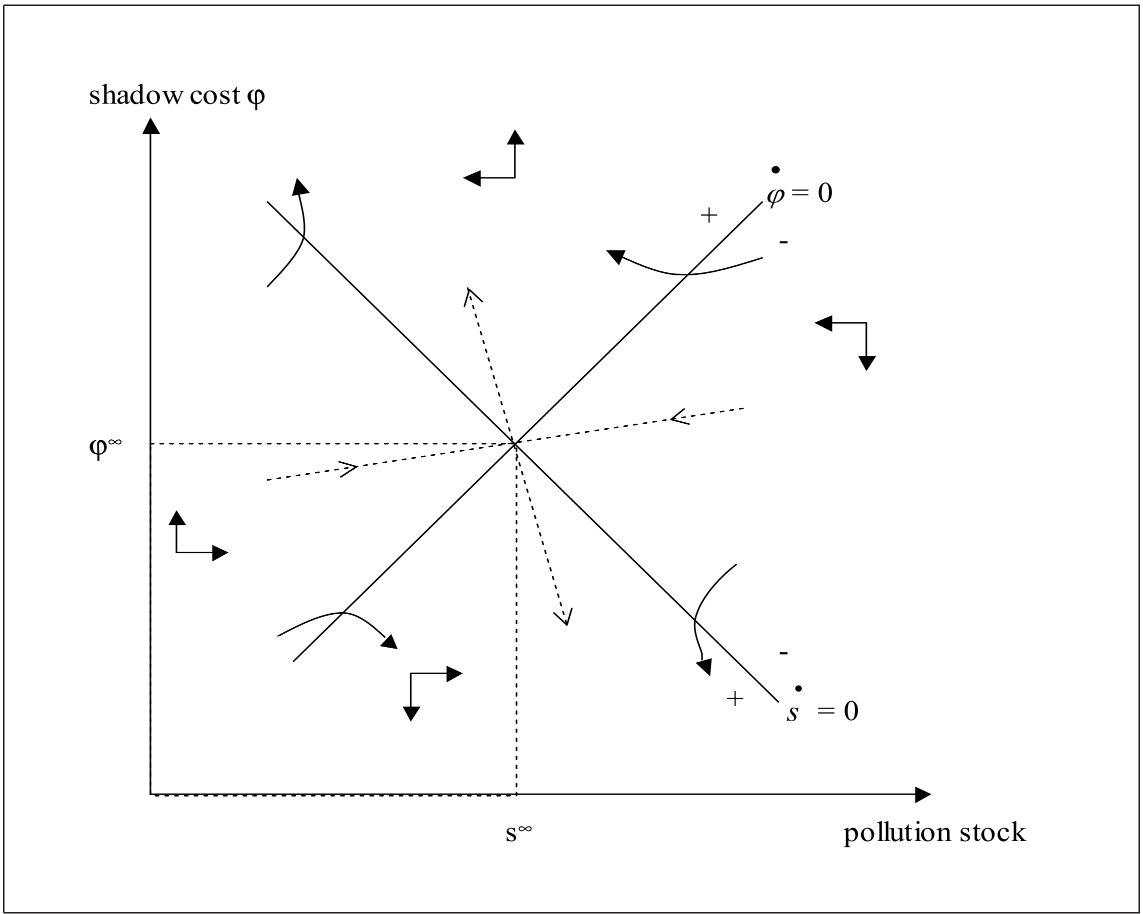
$$\left.\frac{d\varphi}{ds}\right|_{\dot{\varphi}=0} = -\frac{\frac{\partial\dot{\varphi}}{\partial s}}{\frac{\partial\dot{\varphi}}{\partial\varphi}} > 0, \qquad \left.\frac{d\varphi}{ds}\right|_{\dot{s}=0} = -\frac{\frac{\partial\dot{s}}{\partial s}}{\frac{\partial\dot{s}}{\partial\varphi}} < 0. \tag{2.34}$$

The resulting phase diagram, depicted in Figure 2.4, shows that the stable path leading to the steady state is upward sloping, while the unstable path is downward sloping. In this case the pollution stock and its shadow cost evolve in the same direction. Therefore, any pollution abatement policy is characterized by a decrease in the shadow cost.

On the contrary, if $d_{ss} + k_{ss} < |k_{\eta s}\frac{\partial\eta}{\partial s}|$, the pollution stock and its shadow cost evolve in the opposite direction. Therefore, any pollution abatement policy can be depicted by an increase in the shadow cost. In this case the slopes of the $\dot{\varphi} = 0$ and $\dot{s} = 0$ isoclines are both negatives, and it holds that

$$\left.\frac{d\varphi}{ds}\right|_{\dot{\varphi}=0} = -\frac{\frac{\partial\dot{\varphi}}{\partial s}}{\frac{\partial\dot{\varphi}}{\partial\varphi}} \qquad > \qquad \left.\frac{d\varphi}{ds}\right|_{\dot{s}=0} = -\frac{\frac{\partial\dot{s}}{\partial s}}{\frac{\partial\dot{s}}{\partial\varphi}}. \tag{2.35}$$

Figure 2.4: The Phase Diagram in the (s, φ) Space, where the Stable Path Is Upward Slopping (The direct effect dominates the indirect effect)

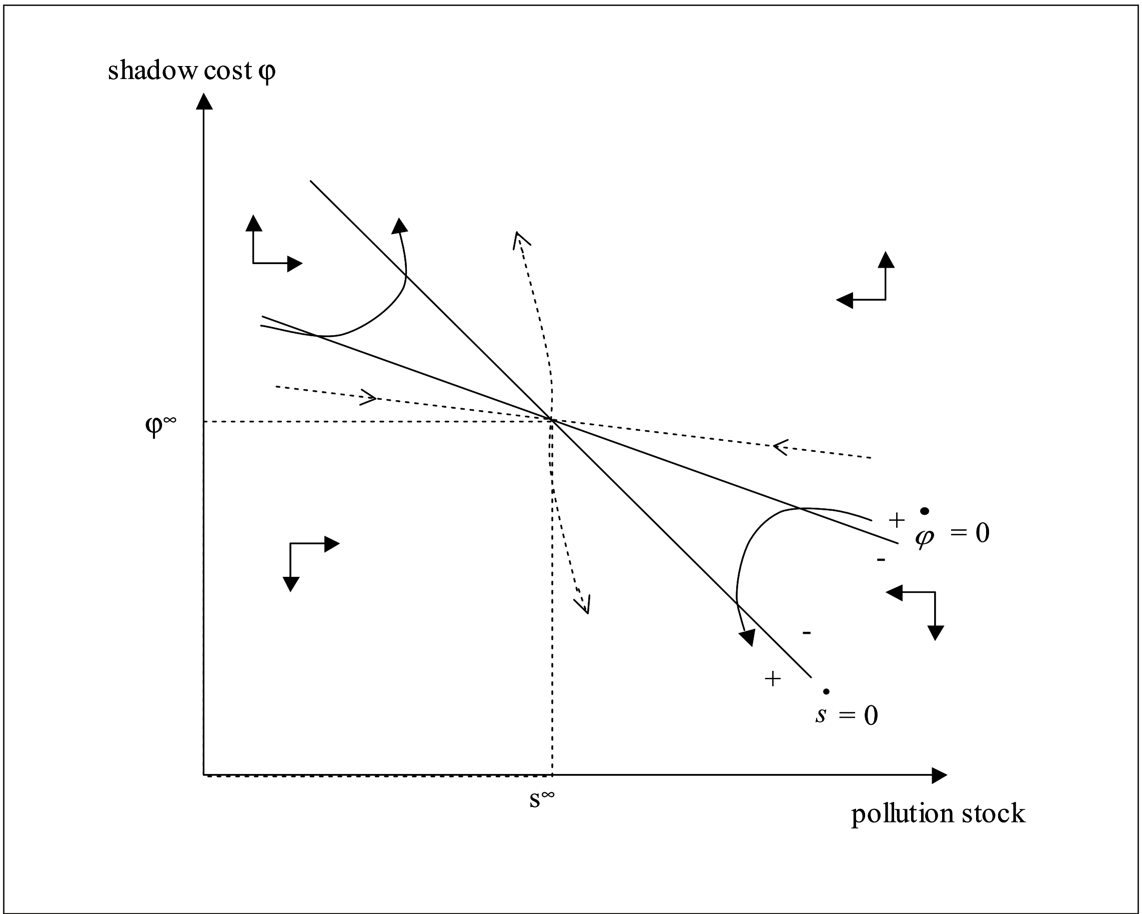


The resulting phase diagram is depicted in Figure 2.5. It shows that the stable path leading to the steady state is downward sloping.

In the case where stock abatement is not possible ($\eta(t) = 0,\ \forall t$) or the abatement cost does not depend on the pollution stock ($k = k(\eta)$), the phase diagram is exclusively given by Figure 2.4.

As the curvature of the convex damage and abatement cost functions determines the slope of the stable path, it also establishes the intertemporal optimal

Figure 2.5: The Phase Diagram in the (s, φ) Space, where the Stable Path Is Downward Sloping (The indirect effect dominates the direct effect)



combination of source abatement versus stock abatement. Moreover, it allows to determine the evolution of the optimal input demand function over time. Hence, one can derive the optimal relationship between short-run and long-run input demand functions. The results are summarized in the following proposition.

Proposition 4: *The evolution of aggregate emissions $z(t)$ and abatement $\eta(t)$ in the optimal dynamic policy is determined by the curvature of the damage and abatement cost functions and the initial stock of pollution s^0 compared to its steady state value s^∞. The following cases can be distinguished, summarized in Table 2.2.*

1. *If $d_{ss} + k_{ss} > |k_{\eta s}\frac{\partial\eta}{\partial s}|$ and the initial stock of pollution s^0 is greater (smaller) than s^∞, the optimal dynamic policy consists in the initial choice of aggregate emissions z^0 and the initial choice of the input use u_i^0, $i = 1, 2$, below (above) their steady-state values z^∞ and u_i^∞, and in their gradual increase (decrease) until the steady-state values are reached. The initial stock abatement η^0 is above (below) its steady-state value η^∞, and it decreases (increases) gradually until the steady-state value is reached.*

2. *If $d_{ss} + k_{ss} < |k_{\eta s}\frac{\partial\eta}{\partial s}|$ and the initial stock of pollution s^0 is greater (smaller) than s^∞, the optimal dynamic spatially differentiated policy consists in the initial choice of aggregate emissions z^0 and the initial choice of the input use u_i^0, $i = 1, 2$, above (below) their steady-state values z^∞ and u^∞, and in their gradual decrease (increase) until the steady-state values are reached. However, the sign of $d\hat{\eta}/dt$ is undetermined.*

Table 2.2: Optimal Trajectories of the Decision Variables

	$d_{ss} + k_{ss} > \lvert k_{\eta s}\frac{\partial\eta}{\partial s}\rvert$	$d_{ss} + k_{ss} < \lvert k_{\eta s}\frac{\partial\eta}{\partial s}\rvert$
$s^0 > s^\infty$	$z^0 < z^\infty$ $u_i^0 < u_i^\infty$ $\eta^0 > \eta^\infty$	$z^0 > z^\infty$ $u_i^0 > u_i^\infty$ $\eta^0 \lesseqqgtr \eta^\infty$
$s^0 < s^\infty$	$z^0 > z^\infty$ $u_i^0 > u_i^\infty$ $\eta^0 < \eta^\infty$	$z^0 < z^\infty$ $u_i^0 < u_i^\infty$ $\eta^0 \gtreqqless \eta^\infty$

Proof: To find the optimal intertemporal path of $z(t)$ and $\eta(t)$, the variables are totally differentiated with respect to time, obtaining

$$\frac{d\hat{z}}{dt} = \frac{\partial\hat{z}}{\partial\varphi}\frac{d\varphi}{dt} + \frac{\partial\hat{z}}{\partial s}\frac{ds}{dt} = \frac{1}{V_{zz}}\frac{d\varphi}{dt}, \tag{2.36}$$

$$\frac{d\hat{\eta}}{dt} = \frac{\partial\hat{\eta}}{\partial\varphi}\frac{d\varphi}{dt} + \frac{\partial\hat{\eta}}{\partial s}\frac{ds}{dt} = \frac{1}{k_{\eta\eta}}\frac{d\varphi}{dt} - \frac{k_{\eta s}}{k_{\eta\eta}}\frac{ds}{dt}. \tag{2.37}$$

Additionally, a comparative static analysis is conducted to determine the effect of a change in the shadow cost on the level of input use. Since neither V nor λ depend on ϵ, it is assumed that the technologies are located optimally and the amount of pollution is chosen optimally, that is, one is moving along the optimal path. The sign of $\partial u_i^*/\partial\lambda$ can be determined by solving the first-order equations (2.9) and (2.10) for $u_i = u_i^*(\lambda)$, $i = 1, 2$. Hence by the implicit function theorem, it is obtained

$$\begin{pmatrix} \mathcal{L}1_{u_1u_1} & \mathcal{L}1_{u_1u_2} \\ \mathcal{L}1_{u_2u_1} & \mathcal{L}1_{u_2u_2} \end{pmatrix} \begin{pmatrix} \frac{\partial u_1^*}{\partial\lambda} \\ \frac{\partial u_2^*}{\partial\lambda} \end{pmatrix} + \begin{pmatrix} \mathcal{L}1_{u_1\lambda} \\ \mathcal{L}1_{u_2\lambda} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{2.38}$$

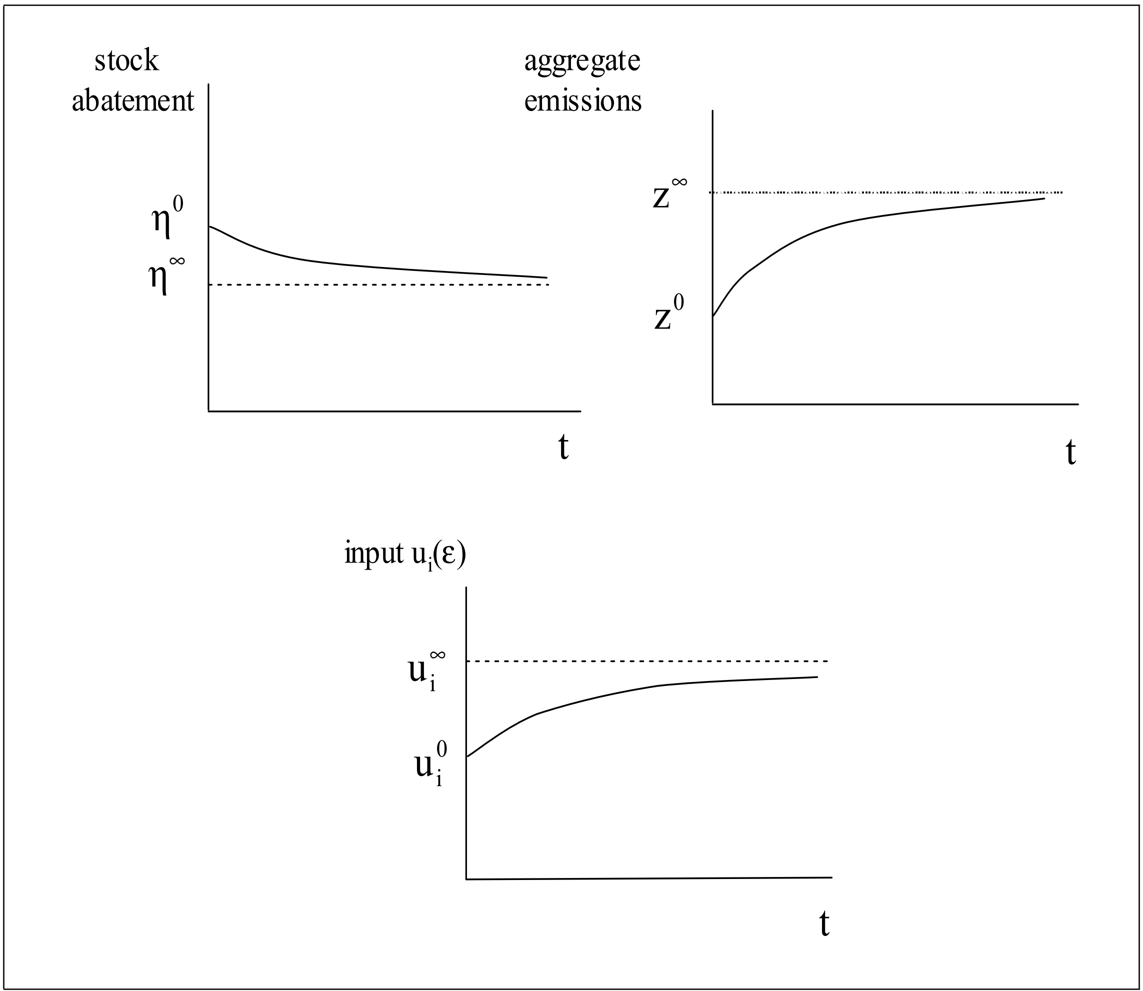
Like before, applying Cramer's rule it is obtained

$$\frac{\partial u_1^*}{\partial\lambda} = \frac{\alpha_1 g_{u_1}}{ph_1 f_{u_1u_1} - \lambda\alpha_1 g_{u_1u_1}} < 0, \qquad \frac{\partial u_2^*}{\partial\lambda} = \frac{\alpha_2 g_{u_2}}{ph_2 f_{u_2u_2} - \lambda\alpha_2 g_{u_2u_2}} < 0. \tag{2.39}$$

Using equations (2.36), (2.37), (2.39), the fact that $\lambda = \varphi$, and that the slope of the stable path is determined by the sign of $\partial\dot{\varphi}/\partial s$, Proposition 4 can be verified.$\blacksquare$

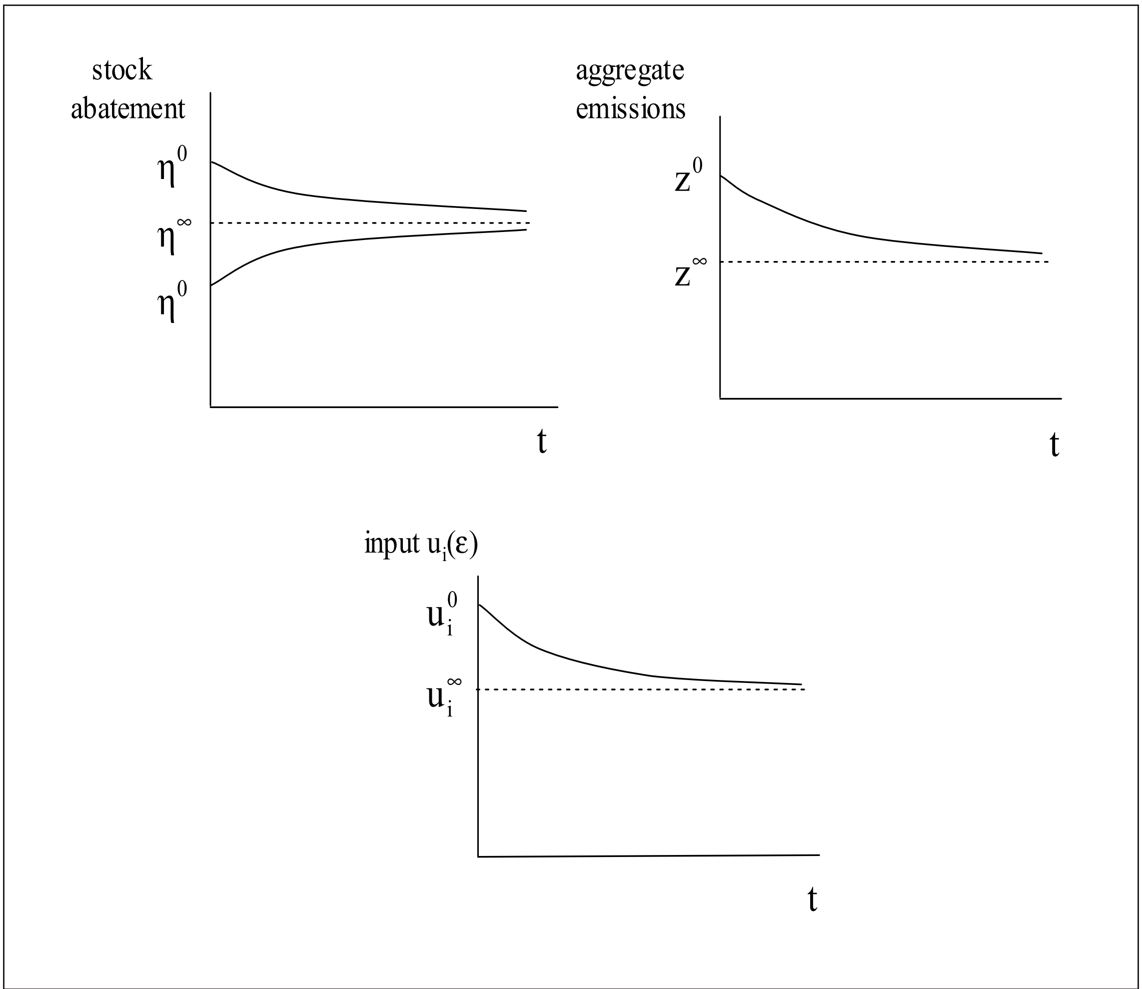
Suppose that the initial pollution stock is greater than its steady-state value, and the implementation of a pollution abatement policy is required. The inequality given in part 1 of Proposition 4 holds if the indirect effect of an increase in pollution on marginal abatement cost is of a minor order, either because the value of the cross derivative $k_{\eta s}$ is low or because the curvature of the convex abatement cost function in the level of abatement is highly pronounced (for $k_{\eta\eta}$ large, see equation 2.25). Additionally, it has to be the case that an increase in pollution has to be accompanied by a sharp increase in the marginal damage and marginal abatement cost. Under these conditions, initial stock abatement η^0 is not sufficient to reduce the stock of pollution, and it has to be reinforced with a high abatement effort at the source. Consequently, the initial level of input use has to be low. As the stock of pollution falls, the shadow price according to Proposition 3 decreases as well, which, in turn, leads to a decline in the stock abatement effort, and to an increase in aggregate emissions and input use. Therefore, an intertemporally and quality-differentiated optimal pollution abatement policy, for $s_0 > s^{\infty}$, can be characterized by choosing the levels of applied input initially below their steady-state values. As time passes, they increase until their steady-state values are reached. This case is depicted in Figure 2.6.

Figure 2.6: Optimal Intertemporal Restoration Policy, where the Direct Effect Dominates the Indirect Effect



However, according to part 2 of Proposition 4, it may be the case that the change in the marginal damage, and abatement costs with respect to s, as a result of an increase in the pollution stock is smaller than a change in the marginal abatement cost with respect to η. In this case, the optimal intertemporal abatement policy is characterized by a high initial stock abatement effort η^0, and aggregate emissions z, like the level of input u_i, $i = 1, 2$, decrease gradually towards their steady-state values. This case is illustrated in Figure 2.7.

Figure 2.7: Optimal Intertemporal Restoration Policy where the Indirect Effect Dominates the Direct Effect



Therefore, the curvature of the convex damage and abatement cost functions limits the possibility to abate pollution once it has been generated. In the case that the pollutant causes severe damages to human health or the environment (for example, substances that form part of the Toxic Release Inventory of the EPA such as Benzene, lead compounds, Methyl Ethyl Ketone, etc.) beyond low levels of concentration, the convexity of the damage function is highly pronounced, and the optimal policy will require high abatement effort at the source. The same optimal policy may be required if an increase in the pollution stock leads to a very high increase of the marginal abatement cost with respect to s, making stock abatement prohibitively expensive.

An especial case occurs if $k = k(s(t))\,\eta(t)$, that is, abatement costs are proportional to the amount of abated pollution stock. As abatement enters the model linearly, the restoration policy will be given either exclusively by source abatement or stock abatement depending on what policy has the lowest cost. During the early stages, when the pollution stock is high, source abatement is expected to be a dominant strategy, until pollution has decreased sufficiently and the unitary abatement cost has fallen below the marginal value of pollution, i.e., $k(s(t)) < V_z$. After this point in time, there will be no source abatement, and the pollution will be abated once it has been generated.

The pattern of adoption of the precision technology will also change over time as a decrease (increase) of shadow cost along the optimal path results in an increase (decrease) in the quasirent for both activities. In the case where the quasirents of different activities intersect, this increase will also lead to a different optimal technology adoption. In the case where quasirents of the different activities do not intersect along the entire optimal path, the optimal technology choice pattern does not change at all. However, the quasirent of the different technologies might intersect along some part of the optimal path, leading to a change in the optimal technology choice pattern during this time and constancy otherwise.

2.5 Optimal Quality Differentiated and Intertemporal Policies

The social optimum, characterized by the equations (2.9) - (2.13), however, is not equivalent to the private optimum since producers do not consider the externality. Their decision problem is simply given by

$$V(P) \equiv \max_{x_i,\, u_i} \int_{\epsilon_0}^{\epsilon_1} \Big(\sum_{i=1}^{2} (ph_i(\epsilon) f(\beta_i u_i) - cu_i - I_i) x_i \Big) \, d\epsilon \qquad (P1)$$

subject to

$$x_i(\epsilon) \geq 0,\ i = 1, 2, \quad x_1(\epsilon) + x_2(\epsilon) \leq X(\epsilon), \quad u_i(\epsilon) \geq 0,\ i = 1, 2.$$

The optimality conditions without pollution are similar to the ones in the optimal case. The only difference is that the shadow price of pollution stock, φ, is zero. Thus, the variable input is determined for each microunit, where the value of its marginal product is equal to its price. The model suggests some adoption of precision technologies even without pollution consideration. Precision technologies are adopted without pollution pricing if at the microunit level the gains from increased output and input cost savings exceed the extra cost of the technology. Taking explicitly into account the shadow cost of pollution stock will provide an extra incentive to adopt precision technology. The reality is that even without environmental considerations, there is significant adoption of precision technologies such as drip, modern irrigation technologies, insulation, soil carbon sequestration, etc.

In the case without intervention in the pollution stock, the assumption about fixed prices of output and input results in the same choices of input use and technology at all periods. Only the pollution stock evolves over time, and it may be growing very fast. Thus, the analysis of the social optimum suggests that explicit pricing of the environmental side effect is a triggering gradual adoption process over time and introduces dynamics in the behavior of individual firms that would not exist without it.

The private behavior will lead to aggregate emissions above the socially optimal level. A first-best policy calls for a tax on individual emissions. However, individual emissions cannot be observed due to high costs or technical infeasibility (Knopman and Smith, 1993); therefore, policymakers must resort to other policy measures where the key variables have to be observable and correlate as close as possible to the individual emissions (Braden and Segerson, 1993). These selection criteria are met by individually tailored input taxes supported by individually tailored technology taxes or subsidies. Since the pollution function is linear with respect to the fixed asset, the following proposition establishes policies that lead to optimal input use and technology adoption.

Proposition 5: *Provided that input use and technology choices can be observed at each unit with quality ϵ, an optimal policy can be obtained by*

- *a quality differentiated input tax $\tau_i,\ i = 1, 2$, given by $\tau_i(\epsilon) = \lambda^* \alpha_i g_{u_i}(u_i^*(\epsilon)),\ i = 1, 2$, together with*
- *a quality differentiated technology subsidy or tax per unit of asset $\sigma_i,\ i = 1, 2$, given by $\sigma_i(\epsilon) = -\tau_i(\epsilon) u_i^*(\epsilon) + \lambda^* \alpha_i g(u_i^*(\epsilon)) \gtreqless 0$.*

Proof: The private decision problem, in the presence of an input tax and a technology subsidy / tax, is given by: $\max_{x_i, u_i} \int_{\epsilon_0}^{\epsilon_1} \Big(\sum_{i=1}^{2} (p h_i(\epsilon) f(\beta_i u_i) - c u_i - I_i) x_i \Big) - \Big(\sum_{i=1}^{2} (\tau_i u_i x_i + \sigma_i x_i) \Big) d\epsilon$. Analyzing the necessary conditions of the problem, one can see that the private optimum coincides with the social optimum given by equations (2.9) to (2.13), thus the input tax $\tau_i,\ i = 1, 2$, together with the technology subsidy or tax $\sigma_i,\ i = 1, 2$, establishes the quality-differentiated optimal input use and technology adoption for every quality ϵ. ■

An input tax alone, however, is not sufficient to achieve the social optimum since it only establishes equations (2.9) and (2.10) but not equations (2.11) and (2.12), that is, the introduction of a tax on the intensive margin causes a distortion on the extensive margin, thus the social optimum is not realized. To establish the socially efficient allocation of technologies, the input tax needs to be complemented by a technology policy. The yet undetermined sign of σ_i, $i = 1, 2$, defines if one has a technology subsidy or tax. In the case where it is positive, one have a technology tax. If it is negative, one have in fact a subsidy. Substituting the value of the quality differentiated input tax τ_i, $i = 1, 2$, into σ_i, $i = 1, 2$, it is obtained:

$$\sigma_i = \lambda^* \alpha_i \big(g(u_i^*) - g_{u_i} u_i^* \big) \gtreqless 0. \tag{2.40}$$

Utilizing Theorem 2.17, page 258, de la Fuente (2000), allows to conclude that if $g(u_i)$ is strictly convex, i.e., the marginal contribution of applied input to pollution is increasing, σ_i, $i = 1, 2$, is negative. In other words σ_i, $i = 1, 2$, turns into a technology subsidy. However, if $g(u_i)$ is strictly concave, σ_i, $i = 1, 2$, turns into a technology tax, and if $g(u_i)$ is linear σ_i, $i = 1, 2$, is zero. The latter case implies that a quality-differentiated input tax alone is able to establish the social optimum and does not need to be complemented by a technology tax or subsidy. In this model, the emission function is assumed to be convex, thus, input taxes need to be complemented by technology subsidies. The introduction of an input tax leads to a different optimal intensity which, in turn, affects the technology adoption decision and, as a result of these two adjustments, the generated amount of pollution changes. Since the emission function is convex, the pollution expenditures per unit of asset ($\tau_i u_i^* = \lambda^* \alpha_i g_{u_i} u_i^*$) are higher than the shadow value of the emissions per unit of asset ($\lambda_i \alpha_i g(u_i^*)$). Therefore, it is necessary to apply a technology subsidy equal to the difference between the pollution expenditures and its shadow cost per unit of asset.

The specific design of policy instruments based on input and/or technology choices has to simultaneously take into account the varying quality of the asset and the aspect of time. In this way the policies can be adjusted according to the characteristics of the potential emissions of the production unit. Moreover, technology and input use are easy to monitor so that the policies can be enforced in practice as well.

These taxes are also adjusted over time in line with the changes of the shadow cost of the pollutant that varies according to the development of the stock of pollutant over time.

2.6 Summary and Conclusions

This chapter presents a modelling approach for the socially optimal management of an accumulating pollutant generated by heterogeneous producers. Source abatement is considered by reduction of input use, exit, and adoption of precision technologies, and stock abatement is also considered. The problem is analyzed in a general framework, that can be applied in different contexts, for instance in relation to carbon emissions, water logging, or accumulation of toxic chemicals. The solution procedure decomposes the optimization problem to a static optimization problem that determines the allocation of resources by heterogeneous production units subject to aggregate emission constraints.

The socially optimal intertemporal equilibrium is determined in the second stage where the optimal solution of the first stage is optimized over time. This sequential procedure to solve the problem within a quality-differentiated, intertemporal framework enhances the analytical tractability of the problem, and allows to obtain analytical results more easily. It turns out that the optimal intertemporal change in the shadow price towards the long-run equilibrium depends on the curvature of the convex abatement cost and damage functions. Since the shadow price of the first stage and the second stage are identical, the curvature of the abatement cost and damage functions also establishes the link between the short-run and long-run input demand and technology demand functions. Moreover, it determines the optimal mix of source abatement and stock abatement.

Due to the presence of an externality, the private net benefit-maximizing strategy of the producers does not produce the socially optimal outcome. Thus, environmental policies in the form of individually tailored input taxes (intensive margin) and individually tailored technology taxes or subsidies (extensive margin) are proposed to induce individual differentiated responses rather than uniform responses. The results show that regulations at the extensive margin should not be considered as a substitute for regulations at intensive margin but, rather, as complements. Moreover, regulations at the extensive margin may require the payment of subsidies to achieve the socially optimal outcome.

Considering the aspect of time and quality simultaneously permits us to formulate the necessary changes to transform an individually tailored optimal, yet static, environmental policy analysis to an intertemporally and individually tailored optimal policy. In particular, the temporal aspect of the regulation is of great importance, since it determines the optimal mix of degree of severity and the time schedule of the policy measures.

With the advent of geographic information systems, reduced computation cost, and improved monitoring technologies, the discriminatory policies presented here are becoming feasible. It is shown that optimality can be attained by incentives even without direct measurement of pollution at the microlevel. Good estimates of production and pollution generation functions and information on microlevel and input use at the microlevel are sufficient to yield optimal outcomes.

One of the main contributions of this chapter is to show that in the management of stock pollution, explored in chapters 2 and 3, introduction of pollution control policies will trigger a dynamic adoption. To simplify the analysis, other dynamic forces that may affect the adoption of new technologies have not been incorporated (see survey by Sunding and Zilberman (2001)). Some can be incorporated into the existing framework without altering the main results of this thesis. For example, learning by doing (reduction in cost of new technologies as manufacturers learn from experience) may be introduced by having $I_1(t)$ with $\partial I_1/\partial t < 0$, $\partial^2 I_1/\partial t^2 \geq 0$. Learning by using (improvement in the use of technology or users learning from their and others' experience) may be presented when the production function with the modern technology is $h_1(\epsilon, t)f(\cdot)$ when $\partial h_1/\partial t > 0$ or by $h_1(\epsilon, L_1, t)f(\cdot)$ when $L_1(t)$ is a second stock variable measuring aggregate acreage with modern technology over time, again with $\partial h_1/\partial t > 0$. The framework can be also modified to accommodate production risk and risk-averse behavior as long as the random elements do not affect pollution generation. It can also be expanded to industries that face negatively sloped demand. As demand is more inelastic, it serves to slow the adoption process. Therefore, the analytical techniques introduced in Proposition 1 can be extended and applied. However, two stage optimal control may have some problems in the situations where there is different accumulation of stocks at different locations, for instance soil erosion, or in situations where there is uncertainty in irreversibility, e.g. climate change, and one needs to apply the real option framework to understand the dynamics of adoption. The extension of the model for this situation is a challenge of future research.

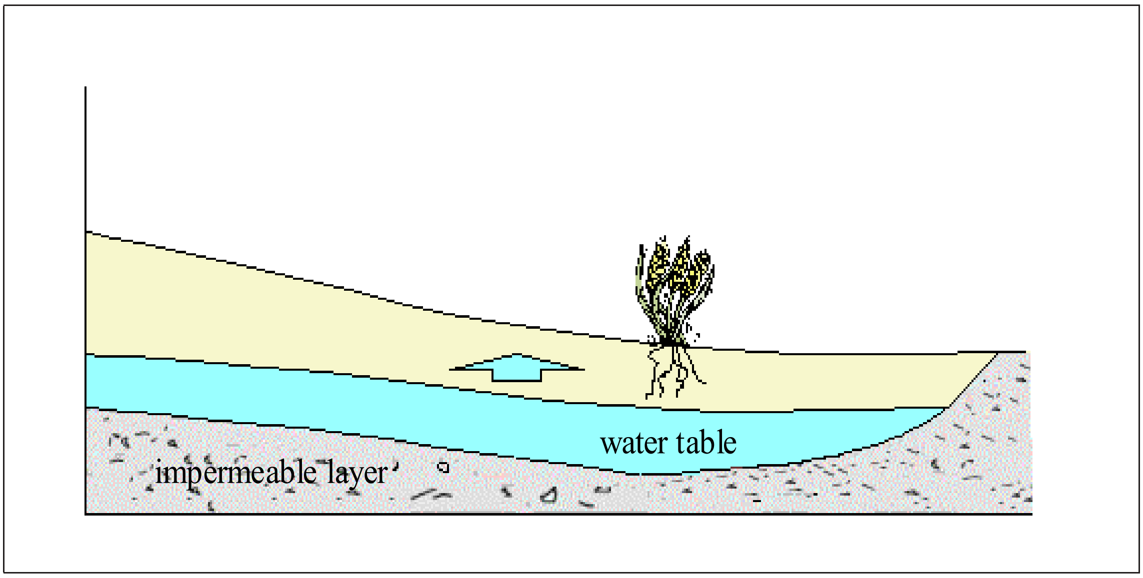
Chapter 3

Optimal Control of Waterlogging Caused by Irrigated Agriculture

3.1 Introduction

The problem of waterlogging occurs in impermeable or poorly drained soils when salt-laden waters accumulate at the subsurface. As a result, the water table may rise up to the crop-root zone where it leads to a reduction of crop yields. Figure 3.1 presents an scheme of the waterlogging problem. In the case of irrigated agriculture, individual farmers, however, do not take into account the extent to which their individual irrigation practices will lead to a rise in the water table of the entire irrigated land in the region. In this way, the water-storage capacity of the land can be considered as a shared good of all farmers located in the region, and optimal private depletion of the water-storage capacity does not coincide with the socially optimal depletion strategy (Hartwick and Olewiler, 1998). To establish the socially optimal outcome, corrective policies are required, assuring that privately optimal irrigation practices coincide with the socially optimal irrigation practices. These policies introduce policy for water management activities that recognize their impact

Figure 3.1: Scheme of the Waterlogging Process



on the waterlogging problem. Given the dynamic nature of water-storage, it is plausible that corrective policies should vary over time. Moreover, since land quality varies over space, the corrective policies should be tailored specifically to site, i.e. vary over space.

However, most of the waterlogging and drainage literature considers policies that either establish the socially optimal intertemporal solution — but assume that land quality is homogeneously distributed over space (Knapp et al., 1990; Shah, Zilberman, and Lichtenberg, 1995), — or neglect the intertemporal aspect of the waterlogging and take only the spatial heterogeneity of the land quality into account (Khanna, Isik, and Zilberman, 2002).

In this chapter the aspects of time and space are integrated in the economic analysis to find the optimal policies for the case of waterlogging problem in agriculture. To find the socially optimal allocation over space and time, it is utilized optimal control in two stages presented in the previous chapter. In the first, the optimal irrigation technology choice and the optimal level of irrigation for each location within the agricultural region are determined. Moreover, it is allowed for the possibility of restoring the water-storage capacity by lowering the water table of the entire irrigated area, for instance, by operating a drainwater disposal facility. The result of the spatial but static optimization is captured by a value function which, in turn, becomes the objective function for the intertemporal optimization, i.e., the second stage of the overall optimization procedure. The results of overall optimization allows to determine the socially optimal spatial and intertemporal adoption of irrigation technologies, the amount of water used for irrigation, and the disposal of drainwater. As a corrective measure to achieve the socially optimal outcome, the alteration of the price of water is proposed, by imposing a tax that is technologically, spatially, and temporally differentiated on the use of water. The two stages are linked by the common shadow cost (user costs) of the water-storage capacity. It allows to transform an optimal spatially targeted water-pricing policy, yet static, into an optimal, temporally, and spatially targeted water-pricing policy.

To demonstrate the applicability of the theoretical approach, two stages optimal control, the socially optimal water price in the presence of waterlogging is determined empirically for the case of cotton produced in the San Joaquin Valley in California. For this purpose, the mathematical model of the approach of optimal control is reformulated such that it can be solved numerically with commercial mathematical software programs on a desktop computer.

The results show that the welfare of the private optimum compared to the social optimum reduces by 1.4% to 3.8% for a water-storage capacity of 25 feet and by 9% to 37% for a water-storage capacity of 5 feet. The optimal tax imposed on applied water increases from the initial year to the final year of consideration by approximately 15. Likewise, the optimal tax is spatially and technologically differentiated. Depending on the two considered technologies, the optimal tax is approximately either 9 or 14 times higher for the lowest land quality compared to the highest land quality. As a result, one obtains an optimal water-pricing structure that is technologically, spatially, and temporally differentiated.

This chapter is organized as follows. Section 3.2 introduces the economic model, and section 3.3 presents the theoretical approach of optimal control in two stages together with the optimal outcome from a social point of view. Section 3.4 contrasts this result with the optimal outcome from a private point of view. The following section demonstrates the applicability of the theoretical model with a particular case study and discusses the empirically obtained results. The chapter ends with a summary and some conclusions.

3.2 The Economic Model

Consider an agricultural region where a single crop is produced. Production is based exclusively on the use of water, supplied by irrigation, and land. Each location within the region is classified according to its biophysical attributes, such that it can be characterized by a single number ϵ, $\epsilon \in [\epsilon_0, \epsilon_1]$ that presents its land quality. According to this concept of space, land quality stands for the capacity of the land to retain the applied water such that it is available for crop uptake and does not reach the impermeable layer where it leads to a depletion of the water-storage capacity. In this way, flat and heavy soils correspond to a high value of ϵ while steeper lands and sandy soils have a lower value of ϵ.

The number of acres with the same quality ϵ available at each location is denoted by $X(\epsilon)$. For simplicity, but without loss of generalization, it is assumed that there are two different irrigation technologies $i,\ i = 1, 2$ available. The subscript $i = 1$ represents a precision technology (drip irrigation), and $i = 2$ the traditional technology (furrow irrigation). The number of acres cultivated with technology $i,\ i = 1, 2$, at any moment of time t at location ϵ is denoted by $x_i(t, \epsilon)$.

The model assumes constant returns to scale with respect to the land. Thus, the production function per acre is presented by $f(\beta_i(\epsilon)u_i(t,\epsilon))$, $i = 1, 2$, where $u_i(t,\epsilon)$ denotes the amount of applied water per acre associated with technology i, and $\beta_i(\epsilon)$ the irrigation effectiveness, that is, the fraction of the applied water that is effectively utilized by the crop. The irrigation effectiveness $\beta_i(\epsilon)$ depends on the land quality ϵ and is specific for each technology i. It is assumed that the precision technology has a higher irrigation effectiveness for every ϵ, that is, $1 \geq \beta_1(\bar{\epsilon}) \geq \beta_2(\bar{\epsilon}) > 0$, $\forall \bar{\epsilon} \in (\epsilon_0, \epsilon_1)$. Additionally, it is also assumed that the soil quality increases the effectiveness of water use for every technology i, i.e., $d\beta_i/d\epsilon > 0$, $i = 1, 2$. The production function has regular neoclassical properties, that is, $f_{u_i} > 0$ and $f_{u_i u_i} < 0$.

The product and water prices, denoted by p and c, respectively, are exogenously determined. It is assumed that the annualized fixed costs per acre I are larger for technology 1 than for technology 2, i.e., $I_1 > I_2$, as specialized equipment is needed.

The part of the applied water that is not utilized by the crop can percolate below the crop-root zone. The percolation per acre, specific for each particular irrigation technology i, $i = 1, 2$, is given by $\alpha_i(\epsilon)\ u_i(t,\epsilon)$, where $\alpha_i(\epsilon)$ is the drainage coefficient per unit of applied water with technology i given land quality ϵ. It is assumed, irrespective of the technology, that the drainage coefficient is zero when land quality is at its maximum, i.e., $\alpha_1(\epsilon_1) = \alpha_2(\epsilon_1) = 0$; and that $d\alpha_i/d\epsilon < 0$, $i = 1, 2$. The later assumption implies that the drainwater generated per unit of applied water increases as land quality declines. Since the modern technology increases the input-use efficiency, it is also assumed that it decreases the drainage coefficient, that is, $\alpha_2(\bar{\epsilon}) > \alpha_1(\bar{\epsilon})$, for $\forall \bar{\epsilon}$, $(\epsilon_0 < \bar{\epsilon} < \epsilon_1)$.

The irrigated area is characterized by an impermeable soil layer that impedes the percolation of the irrigated water (drainwater) below a certain depth. Thus, the drainwater will accumulate above this soil layer leading to a rise in the water table. It is assumed that the production function is independent of the stock of drainwater $s(t)$, while the top level of the stock is below the crop-root zone. Above this level, where $s > \bar{s}$, the soil is not productive anymore and $f(\cdot) = 0$ for $s(t) > \bar{s}$. In order to mitigate the negative effects of the accumulation of the drainwater, there exists the possibility to employ a disposal technique of the drainwater that lowers the water level. The disposal costs are given by $k\,\eta(t)$, where k is a constant and denotes the disposal costs per acre-foot for the entire irrigated area. The variable $\eta(t)$ denotes the quantity of acre-feet (AF) of water removed from the irrigated area. As it refers to the entire area of irrigated land, it does not depend on land quality ϵ.

3.3 The Socially Optimal Outcome

It is assumed that a social planner exists, for example, a local government body or an irrigation district body. Furthermore, it is assumed that the social planner maximizes the present discounted net benefits from agricultural production over time while taking into account the social economic losses due to the accumulation of the drainwater stock, that is, the foregone profits of the decrease in future production. Given the regional focus of the analysis, it is assumed that the product and input prices are not influenced by regional production decisions and, therefore, they are taken as given.

The social planner's decision problem reads as

$$\begin{aligned} \max_{u_i(t,\epsilon),\, x_i(t,\epsilon),\, \eta(t)} \quad & \int_0^\infty e^{-\delta t}\Big(\int_{\epsilon_0}^{\epsilon_1} \Big(\sum_{i=1}^{2} \big(pf(\beta_i(\epsilon)u_i(t,\epsilon)) \\ & -cu_i(t,\epsilon) - I_i\big)x_i(t,\epsilon)\Big)d\epsilon - k\eta(t)\Big)\, dt, \end{aligned} \tag{S}$$

subject to

$$\begin{aligned} \dot{s}(t) \;\; = \;\; & \int_{\epsilon_0}^{\epsilon_1} \Big(\sum_{i=1}^{2} \alpha_i(\epsilon)u_i(t,\epsilon)x_i(t,\epsilon)\Big)d\epsilon - \eta(t), \\ & s(0) = s_0, \quad s(t) \le \bar{s}, \quad 0 \le \eta(t) \le s(t), \quad u_i(t,\epsilon) \ge 0,\ i = 1,2, \\ & x_i(t,\epsilon) \ge 0,\ i = 1,2, \quad x_1(t,\epsilon) + x_2(t,\epsilon) \le X(\epsilon), \end{aligned}$$

where s_0 denotes the stock of drainwater at the initial point of calendar time, and $\delta > 0$ the social discount rate.

Utilizing Pontryagin's Maximum Principle, the current Hamiltonian of problem (S) is given by

$$\begin{aligned}\mathcal{H} \;\equiv\; & \int_{\epsilon_0}^{\epsilon_1} \Big(\sum_{i=1}^{2} \big(pf(\beta_i(\epsilon)u_i(t,\epsilon)) - cu_i(t,\epsilon) - I_i\big)x_i(t,\epsilon)\Big)\, d\epsilon - k\eta(t) \\ & -\mu(t)\Big(\int_{\epsilon_0}^{\epsilon_1} \Big(\sum_{i=1}^{2} \alpha_i(\epsilon)u_i(t,\epsilon)x_i(t,\epsilon)\Big)\, d\epsilon - \eta(t)\Big).\end{aligned}$$

To facilitate the interpretations of the costate variable μ, it has been multiplied by minus one. In this way μ has a positive value. As in the previous chapter, the arguments ϵ and t of the variables and of the Lagrange multipliers $\omega_1, ..., \omega_8$, to be introduced later, will be suppressed to simplify the notation unless it is required for an unambiguous notation. Taking account of the constraints on the control variables leads to the Lagrangian: $\mathcal{L} \equiv \mathcal{H} + \omega_1 u_1 + \omega_2 u_2 + \omega_3 x_1 + \omega_4 x_2 + \omega_5 (X - x_1 - x_2) + \omega_6 \eta + \omega_7 (s - \eta) - \omega_8 s$. The solution of problem (S) has to satisfy the following necessary conditions stated in accordance with Theorem 1, page 276, Seierstad and Sydsæter (1987):

$$\begin{aligned}
\mathcal{L}_{u_1} &\equiv (p\beta_1 f_{u_1} - c)x_1 - \mu\alpha_1 u_1 x_1 + \omega_1 = 0, && (3.1)\\
\mathcal{L}_{u_2} &\equiv (p\beta_2 f_{u_2} - c)x_2 - \mu\alpha_2 u_2 x_2 + \omega_2 = 0, && (3.2)\\
\mathcal{L}_{x_1} &\equiv py_1 - cu_1 - I_1 - \mu\alpha_1 u_1 + \omega_3 - \omega_5 = 0, && (3.3)\\
\mathcal{L}_{x_2} &\equiv py_2 - cu_2 - I_2 - \mu\alpha_2 u_2 + \omega_4 - \omega_5 = 0, && (3.4)\\
\mathcal{L}_{\eta} &\equiv k + \mu + \omega_6 - \omega_7 = 0, && (3.5)\\
\dot{\mu}(t) &= \delta\mu + \mathcal{L}_s = \delta\mu + \omega_7 - \omega_8, && (3.6)\\
\dot{s}(t) &= \int_{\epsilon_0}^{\epsilon_1} \Big(\sum_{i=1}^{2} \alpha_i u_i x_i \Big)\, d\epsilon - \eta(t), \quad s(0) = s_0. && (3.7)
\end{aligned}$$

Modifying the framework described in chapter 2, problem (S) is solved in two stages. In the first stage it is analyzed the optimal spatial solution given by the optimal level of applied water and the optimal technology choice is analyzed. More precisely, it is determined, at every location ϵ, the optimal irrigation technology, the optimal amount of applied water, and the optimal size of the idle land given by $X(\epsilon) - \big(x_1(\epsilon) + x_2(\epsilon)\big)$. In the second stage the optimal intertemporal solution of the previously obtained optimal spatial solution is derived.

3.3.1 The Optimal Spatial Solution

In the first stage the solution of the spatial social planner's decision problem is given by the value function $V(z)$ defined as:

$$V(z) \equiv \max_{u_i(\epsilon),\, x_i(\epsilon)} \int_{\epsilon_0}^{\epsilon_1} \Big(\sum_{i=1}^{2} \big(pf(\beta_i(\epsilon)u_i(\epsilon)) - cu_i(\epsilon) - I_i\big)x_i(\epsilon)\Big)\, d\epsilon \tag{S1}$$

subject to

$$z = \int_{\epsilon_0}^{\epsilon_1} \Big(\sum_{i=1}^{2} \alpha_i(\epsilon)u_i(\epsilon)x_i(\epsilon)\Big)\, d\epsilon,$$

$$u_i(\epsilon) \geq 0,\ i = 1,2, \quad x_i(\epsilon) \geq 0,\ i = 1,2, \quad x_1(\epsilon) + x_2(\epsilon) \leq X(\epsilon),$$

where z denotes the flow of drainwater, generated over the entire range of ϵ, from ϵ_0 to ϵ_1, that accumulates above an impermeable soil layer.

Taking account of the constraints on the control variables, the Lagrangian of the spatial maximization problem (S1) is given by

$$\begin{aligned}\mathcal{L}1 \equiv\ & \int_{\epsilon_0}^{\epsilon_1} \Big(\sum_{i=1}^{2} \big(pf(\beta_i u_i) - cu_i - I_i\big)x_i\Big)\, d\epsilon + \lambda\Big(z - \int_{\epsilon_0}^{\epsilon_1} \big(\sum_{i=1}^{2} \alpha_i u_i x_i\big)\, d\epsilon\Big) \\ & + \upsilon_1 u_1 + \upsilon_2 u_2 + \upsilon_3 x_1 + \upsilon_4 x_2 + \upsilon_5\big(X - x_1 - x_2\big).\end{aligned}$$

A solution of the problem has to satisfy the following necessary conditions:

$$
\begin{aligned}
\mathcal{L}1_{u_1} &\equiv (pf_{u_1} - c - \lambda\alpha_1 u_1)x_1 + \upsilon_1 = 0, && (3.8)\\
\mathcal{L}1_{u_2} &\equiv (pf_{u_2} - c - \lambda\alpha_2 u_2)x_2 + \upsilon_2 = 0, && (3.9)\\
\mathcal{L}1_{x_1} &\equiv pf(\beta_1 u_1) - cu_1 - I_1 - \lambda\alpha_1 u_1 + \upsilon_3 - \upsilon_5 = 0, && (3.10)\\
\mathcal{L}1_{x_2} &\equiv pf(\beta_2 u_2) - cu_2 - I_2 - \lambda\alpha_2 u_2 + \upsilon_4 - \upsilon_5 = 0, && (3.11)\\
\mathcal{L}1_{\lambda} &\equiv z - \int_{\epsilon_0}^{\epsilon_1} \big(\sum_{i=1}^{2} \alpha_i u_i x_i\big)\, d\epsilon = 0. && (3.12)
\end{aligned}
$$

The Lagrange multiplier λ is interpreted as the shadow costs of the prespecified level of the flow of drainwater, z. Please note that z does not depend on ϵ. Thus, λ is constant over space. Given the interpretation of λ, the necessary conditions (3.8) and (3.9) indicate for $u_i > 0$ $i = 1, 2$, that at every location and for each technology, water should be applied up to the point where the value of the marginal product per acre equals the sum of the marginal cost of water and of the marginal cost of generated drainwater per acre. Equations (3.10) and (3.11) govern the optimal choice of technology at every location ϵ. Since the production function and drainwater generation function are linear in land, the irrigation technology that leads to a higher quasirent per acre, say, $\mathcal{L}1_{x_i} - \upsilon_{i+2} + \upsilon_5 \equiv pf(\beta_i u_i) - cu_i - I_i - \lambda\alpha_i u_i$, will be completely preferred to the technology with the lower quasirent. Hence, the technology that yields the highest quasirent should be adopted on the entire land available at location ϵ. Consequently, for every location ϵ, boundary solutions are exclusively obtained, given either by $x_1(\epsilon) = X(\epsilon)$, $x_2(\epsilon) = X(\epsilon)$, or $x_1(\epsilon) = x_2(\epsilon) = 0$. In this case, the Lagrange multipliers of the binding constraint reflect the difference between the value of the marginal net benefits and the marginal drainwater generation cost. The adoption of the conservation technology will take place when the quasirent of the precision technology is positive and larger than the traditional technology.

3.3.2 The Optimal Allocation over Space and Time

In the first stage, the socially optimal spatial technology choice and input use from a static point of view were derived. To analyze how this optimal allocation is affected over time, the value function V, obtained in the first stage, is maximized over time. Hence, the social planner's decision problem is given by:

$$W^S(z^*(t), \eta^*(t); \bar{s}) \equiv \max_{z(t),\, \eta(t)} \int_0^\infty e^{-\delta t}\Big(V(z(t)) - k\eta(t)\Big)\, dt \tag{S2}$$

subject to

$$\dot{s}(t) = z(t) - \eta(t), \qquad s(0) = s_0, \qquad s(t) \leq \bar{s}, \qquad 0 \leq \eta(t) \leq s(t).$$

The total amount of generated drainwater of the entire irrigated area, presented by the parameter z in the first stage, becomes a decision variable in the second stage. Thus, it now depends on t. Another decision variable is given by the amount of disposed drainwater $\eta(t)$. These two decision variables allow to analyze the optimal mix of on-farm and off-farm measures to control waterlogging.

The current value Hamiltonian of the second stage is given by: $\mathcal{H}2 = V(z(t)) - k\eta(t) - \varphi(t)\,(z(t) - \eta(t))$, where φ denotes the shadow cost of the drainwater stock. Taking account of the constraints on the control variables leads to the Lagrangian: $\mathcal{L}2 \equiv \mathcal{H}2 + \upsilon_6\eta + \upsilon_7(s - \eta) - \upsilon_8 s$. The first-order conditions read as

$$\mathcal{L}2_z \equiv V_z - \varphi = 0 \qquad \Rightarrow \qquad \lambda(t) = \varphi(t), \tag{3.13}$$

$$\mathcal{L}2_\eta \equiv \varphi(t) - k + \upsilon_6 - \upsilon_7 = 0, \tag{3.14}$$

$$\dot{\varphi} = \delta\varphi + \mathcal{L}2_s = \delta\varphi + \upsilon_7 - \upsilon_8, \tag{3.15}$$

$$\dot{s} = z - \eta, \qquad s(0) = s_0. \tag{3.16}$$

Equation (3.13) indicates that the marginal value of the generated drainwater in the irrigated area should equal its marginal shadow cost, φ, which, in turn, is equal to the shadow cost of the spatial allocation problem, λ. Equation (3.15) states that, as long as $s < \bar{s}$ and $\eta < \bar{s}$, the shadow cost φ will increase at the rate of discount, as a consequence of the depletion of the subsurface storage capacity. Given that disposal $\eta(t)$ enters linearly in the model, the evaluation of equation (3.14) allows to determine the optimal choice of η as a function of k, υ_6, and υ_7. Thus, equation (3.14) yields

$$\begin{aligned} \varphi < k \quad &\Rightarrow \quad \eta(t) = 0, \\ \varphi > k \quad &\Rightarrow \quad \eta(t) = s(t), \\ \varphi = k \quad &\Rightarrow \quad 0 \leq \eta(t) \leq s(t). \end{aligned} \tag{3.17}$$

At the maximum there are two possible candidates for the most rapid approach path towards the steady state. If disposal of the drainwater is not available or it is very costly, production will end when $s = \bar{s}$. However, in the case where disposal cost is reasonably low, one may distinguish two periods. In the first period where $t < t_1$, the shadow cost of the stock of drainwater φ is lower than the drainwater disposal costs k. Thus, no disposal takes place and the water accumulates in the subsurface until it reaches the crop-root zone. Equation (3.13) is solved globally and uniquely for $z = z^*(\phi)$ provided that V_{zz} does not vanish over its entire domain. Hence, the optimal evolution of the generated drainwater over time can be determined. The substitution of $z = z^*(\phi)$ into equation (3.16) describes the change of the stock of drainwater $\dot{s}$, given that $\eta = 0$, $\forall t < t_1$. After a certain point in time, t_1, where the stock of drainwater has reached $\bar{s}$, the disposal of drainwater exceeding $\bar{s}$ will take place in order to maintain agricultural production at the previous level. This outcome leads to a steady state where $z(t) = \eta(t)$, that is, for $t > t_1$, the amount of generated drainwaater is removed form the agricultural region.

3.4 The Common Access Outcome

Let now assume that there are many competitive farmers in the area, and the stock of drainwater is a result of the flow of drainwater of every individual farmer. Thus, the stock of drainwater is a bad with common access to all farmers. As established in the literature (Hartwick and Olewiler, 1998), the private optimum is not equivalent to the social optimum, characterized by equations (3.8) - (3.12) since the individual farmer will not take user costs into account. Therefore, with private costs lower than social costs, the private depletion of the water-storage capacity will be higher compared to the optimal depletion from a social point of view. Consequently, the water-storage capacity will be depleted optimally earlier from a private, rather than social, viewpoint. These considerations suggest that a government intervention or a concerted action of the farmers is necessary to achieve efficiency.

A first-best policy would call for a tax on the individual generation of drainwater. However, the individual contribution to the common stock of drainwater cannot be observed and policymakers have to rely on other policy measures that need to be observable and need to correlate as closely as possible to the individual contribution to the stock of drainwater (Braden and Segerson, 1993). These criteria are met by technologically, spatially, and temporally-differentiated input taxes, provided they are based on site-specific information. The following proposition defines a policy that assures the optimal amount of applied water and the optimal choice of technology at every location ϵ.

Proposition 6: *For a given amount of generated drainwater z, and provided that the amount of applied water and technology choices can be observed at each location ϵ, an optimal policy can be obtained by a technologically and spatially-differentiated input tax $\tau_i^*(\epsilon),\ i = 1, 2$, given by $\tau_i^*(\epsilon) = \lambda^*(z)\alpha_i(\epsilon),\ i = 1, 2$.*

Proof: In the case of an input tax on applied water, the private decision problem

(T1) is given by:

$$\begin{aligned} FNM(\tau_i(\epsilon)) \;\equiv\; & \max_{x_i(\epsilon),\, u_i(\epsilon)} \int_{\epsilon_0}^{\epsilon_1} \Big(\sum_{i=1}^{2} \big(pf(\beta_i(\epsilon)u_i(\epsilon)) - cu_i(\epsilon) - I_i\big)x_i(\epsilon) \\ & - \sum_{i=1}^{2} \tau_i(\epsilon)u_i(\epsilon)x_i(\epsilon)\Big)\, d\epsilon, \end{aligned} \tag{T1}$$

subject to

$$u_i(\epsilon) \geq 0,\ i = 1,2, \quad x_i(\epsilon) \geq 0,\ i = 1,2, \quad x_1(\epsilon) + x_2(\epsilon) \leq X(\epsilon).$$

The first-order conditions read as

$$(pf_{u_i} - c - \tau_i u_i)x_i + \upsilon_i = 0, \quad i = 1,2, \tag{3.18}$$

$$pf(\beta_i u_i) - cu_i - I_i - \tau_i u_i + \upsilon_{i+2} - \upsilon_5 = 0, \quad i = 1,2. \tag{3.19}$$

Analyzing the first-order conditions of the private problem (T1) and of the social problem (S1) shows that the optimal private choice of the technology employed $x_i(\epsilon)$ and of the amount of applied water $u_i(\epsilon)$ coincide with the socially optimal value of $x_i^*(\epsilon)$ and $u_i^*(\epsilon)$, provided that the tax on applied water $\tau_i(\epsilon)$ is set equal to $\lambda^*(z)\,\alpha_i(\epsilon),\ i = 1,2.$ ■

The optimal input taxes are specific for each employed technology and depend on land quality ϵ. Since the optimal shadow cost of drainwater λ^* changes over time, according to the stock of drainwater, the input taxes also need to be temporally differentiated. The following proposition establishes how the tax τ_i^* evolves optimally over time.

Proposition 7: *Provided that the amount of applied water and technology choices can be observed at any point in time t, at each location ϵ, the optimal technologically, spatially and temporally differentiated tax on applied water $\tau_i^*(t,\epsilon),\ i = 1,2,$ is given by $\tau_i^*(t,\epsilon) = \phi^*(t)\,\alpha_i(\epsilon),\ i = 1,2.$*

Proof: Define $T(\tau_i(t,\epsilon))$ as the amount of collected taxes, that is given by

$$T(\tau_i(t,\epsilon)) = \int_{\epsilon_0}^{\epsilon_1} \Big(\sum_{i=1}^{2} \tau_i(t,\epsilon)\alpha_i(\epsilon)u_i(t,\epsilon)x_i(t,\epsilon)\Big)\, d\epsilon.$$

Moreover, social welfare where a tax on applied water is introduced $W^T(\tau_i^*(t,\epsilon),\eta^*(t))$, is defined as the sum of the present value of farm net margin, $FNM(\tau_i(t,\epsilon))$, and of the collected taxes, $T(\tau_i(t,\epsilon))$, minus the present value of the disposal costs of drainwater $k\,\eta(t)$.[1] Thus, the private problem in the presence of taxes is given in the second stage by

$$\begin{aligned} W^T(\tau_i^*(t,\epsilon),\eta^*(t)) \;&\equiv\; \max_{\tau_i(t,\epsilon),\eta(t)} \int_0^\infty exp^{-\delta\, t}\Big(FNM(\tau_i(t,\epsilon)) \\ &\quad + T(\tau_i(t,\epsilon)) - k\,\eta(t)\Big)\, dt \end{aligned} \tag{T2}$$

subject to

$$\dot{s}(t) \;=\; z(\tau_i(t,\epsilon)) - \eta(t), \qquad s(0) = s_0, \qquad s(t) \leq \bar{s}, \quad 0 \leq \eta(t) \leq s(t).$$

Since the shadow cost of the spatial allocation problem $\lambda(t)$ is equal to the shadow cost of the stock of drainwater in the intertemporal allocation problem $\phi(t)$ by equation (3.13), the input tax $\tau_i^*(t,\epsilon) = \varphi^*(t)\alpha_i(\epsilon),\ i = 1,2$, assures the optimal choice of technology and the amount of applied water at every location ϵ from a social point of view. A comparison of the first-order conditions, corresponding to the problems (T2) and (S2), shows that the optimal intertemporal solution are identical. Thus, the level of welfare $W^T(\tau^*(t,\epsilon),\eta^*(t))$ obtained by imposing a technologically, spatially, and temporally differentiated input tax on applied water $\tau_i^*(t,\epsilon)$ is identical to the level of welfare of the socially optimal solution $W^S(z^*(t),\eta^*(t);\bar{s})$. ∎

The input taxes depend on the employed technology, on space, and on time. The use of information on land quality allows one to target specific locations so that policies based on the applied water and a chosen irrigation technology can be adjusted to the

[1] Consumer surplus is taken to be zero because the study focuses on a small agricultural region, and, therefore, it is assumed that product prices are not influenced by changes in output. It is also assumed that consumer surplus is not affected by a rise in the production externality.

site-specific potential with respect to the generation of drainwater. These spatially optimal taxes are adjusted over time according to the evolution of the shadow cost of the stock of drainwater. Since the employed irrigation technology and the actual use of water are easy to monitor, the policies can be implemented in practice as well.

3.5 Empirical Study

The empirical part of the chapter demonstrates on the one hand how the theoretical model presented above can be applied in empirical work. For his purpose, the theoretical model has to be reformulated such that it can be solved numerically with standard mathematical software. On the other hand, the empirical part of the analysis demonstrates the magnitude of the inefficiency of the private versus the social outcome. In this way it clarifies whether the inefficiency, derived in economic literature (Hartwick and Olewiler, 1998), has importance in a real-world study. Additionally, the empirical study shows to which extent the proposed policy is technology specific and to which extent it changes over time and space.

The empirical study is based on the cotton produced on 400,000 irrigated acres in the San Joaquin Valley in California. In order to simplify the study, it is assumed that there are only two irrigation technologies available, furrow and drip irrigation.[2] The specification of the production function is given by $y_i(\epsilon) = Max[-1589 + 2311(\beta_i(\epsilon)\, u_i(\epsilon)) - 462(\beta_i(\epsilon)\, u_i(\epsilon))^2, 0]$, where $i = 1$ denotes drip irrigation, and $i = 2$ accounts for furrow irrigation (Caswell, Lichtenberg, and Zilberman, 1990). The value of the parameters of the production function are obtained from a study by Hanemann et al. (1987). The land quality index is calibrated such that it coincides with the irrigation effectiveness of the traditional technology, i.e., $\beta_2(\epsilon)$ is the identity function $\beta_2(\epsilon) = \epsilon$. In this way, the quality of the land of the

[2] An extension of the study by including more than two technologies is straightforward within the context of this numerical analysis.

irrigated area under consideration ranges from 0.2 (steep and sandy soils) to 0.8 (flat and heavy soils). It is assumed that the quality of the land is distributed uniformly, with an average land quality of 0.5. Hanemann et al. (1987) show that the efficiency of drip irrigation reaches 0.95 in soils where the efficiency of furrow is 0.6. This information is used to calibrate the efficiency function of drip irrigation with constant elasticity. It is given by $\beta_1(\epsilon) = \epsilon^{0.1}$. The information in Hanemann et al. (1987) is also utilized to calibrate the drainage functions for the different technologies, resulting in $\alpha_1(\epsilon) = (1 - \beta_1)^{1.074}$ and $\alpha_2(\epsilon) = (1 - \beta_2)^{1.092}$. Cotton price is assumed to be \$0.60 (U.S.) per pound and water price is \$55 (U.S.) per acre-foot. The fixed cost of adoption of furrow is taken to be \$500 (U.S.) per acre while the fixed cost of drip is \$633 (U.S.) per acre (Khanna, Isik, and Zilberman, 2002). The social discount rate is set equal to 0.04.

As described in the theoretical part of this chapter, the social optimization problem is solved in two successive stages. In the first stage, the optimal spatial allocation is determined by solving problem (S1) numerically. As noted in the proof of Proposition 6, the solutions of the first-order conditions of problem (S1) and of problem (T1) are identical if the socially optimal tax $\tau_i^*(\epsilon)$ is set equal to $\lambda^* \alpha_i(\epsilon)$. To obtain the numerical solutions of the first-order conditions of equations (3.18) and (3.19), these two equations are formulated in Mathematica®, with $\tau_i^*(\epsilon)$ replaced by $\lambda^* \alpha_i(\epsilon)$. The programming code is available upon request. For a preestablished λ^*, the solution provides the optimal choices of irrigation technologies (x_i^*), the amount of applied water (u_i^*), and the amount of produced cotton ($f(\beta_i u_i^*)$) at each location ϵ. The generated drainwater in the entire irrigated area, z, associated with the predetermined λ^* is obtained based on equation (3.12). Thereafter, this procedure is repeated for different values of λ^*, which was systematically incremented starting with zero.

Thus, it was able to obtain a series of u_i^*, x_i^*, $f(\beta_i u_i^*)$ and λ^* that goes together with a particular z. Moreover, it allowed to estimate the function $z(\varphi(t)) = z(\lambda)$, with the software package SPSS, that is required in problem (S2) to determine the optimal evolution of the stock of drainwater. The function $z(\varphi(t))$ was plugged into equation (3.16) of problem (S2). Furthermore, it is known from equation (3.17) that the disposal of drainwater, η, is equal to zero until the water-storage capacity is completely depleted. Thus, equations (3.15) and (3.16) simplify to a system of two ordinary differential equations in φ and s that can be solved easily with Mathematica®. The solution of the two differential equations provides the optimal time paths of the shadow cost and of the stock of drainwater, together with the point in time t_1 where the resource should be depleted. Knowing the value of the shadow cost at each point in time allows to retrieve the optimal values of the variables of the spatial problem and examine their evolution over time.

The empirical analysis is started by calculating the socially optimal outcome given a water-storage capacity of 10 feet. The results are summarized in Tables 3.1 and 3.2. Table 3.1 presents the case where disposal of drainwater is available at a cost of $100/AF, and Table 3.2 presents the case where the disposal of drainwater is not available.

The results show that with disposal costs of $100/AF, the shadow cost starts out with $13.23 and increases by 4% until it reaches $100 in the 50$^{\text{th}}$ year. That is, in the 50$^{\text{th}}$ year a further depletion of the water-storage capacity will lead to foregone profits of $100. At this point in time, the shadow costs are equal to the disposal cost of $100/AF. Thereafter, as the shadow cost raises annually by 4%, depletion would be more expensive than disposal. For this particular reason the shadow cost increases up to $100 exactly in the moment where the water storage is completely depleted. The increase in the shadow cost continuously decreases the share of land where a particular technology is employed, for instance drip irrigation turns unprofitable after the 24$^{\text{th}}$ year. The disposal of drainwater allows to maintain 31.6% of the agricultural land in production; however, this is only 56% of the land that was cultivated initially.

Table 3.1: Results of the Social Optimization for the Entire Irrigated Area (where the water-storage capacity is 10 feet and the disposal cost is $100/AF)

Year	Shadow Cost	Technology adoption[(a)] Drip	Technology adoption[(a)] Furrow	Technology adoption[(a)] Idle Land	Applied water (10^3 AF)[(b)]	Drainwater flow (10^3 AF)[(b)]	Drainwater stock (10^3 AF)[(b)]	Yield (in 10^6 lb)	Social welfare (current value, 10^3 $)	Social welfare (discounted value, 10^3 $)
0	13.23	7.11 %	49.61 %	43.28 %	798	113	0	293	14431	14431
10	19.73	4.71 %	47.63 %	47.66 %	737	102	1031	270	14251	9553
20	29.43	1.57 %	44.99 %	53.44 %	657	88	1960	240	13924	6256
30	43.91	-	41.69 %	58.31 %	580	74	2769	214	13394	4034
40	65.51	-	37.01 %	62.99 %	506	59	3444	191	12576	2539
50	97.72	-	31.80 %	68.20 %	424	44	3974	164	11389	1541
50.6+	100.00	-	31.61 %	68.39 %	421	43[(c)]	4000	163	7043[(c)]	$7043\, exp^{(-0.04t)}$

[(a)] Indicates the share of land where a particular technology is adopted.

[(b)] To compute the mean water use per acre, drainwater flow and drainwater stock per acre, we must divide those values by 400, since the region under analysis comprises 400,000 irrigated acres.

[(c)] From year 50.6 onwards, drainwater disposal comes into operation and, therefore, $\eta(t) = z(t) = 43,\ \forall t \geq 50.6$. The current value of the drainwater disposal costs, $4.3 million, leads to substantial loss in social welfare. For this reason, V drops from 16,389 thousand dollars to 7,043 thousand dollars.

Table 3.2: Results of the Social Optimization for the Entire Irrigated Area (where the water-storage capacity is 10 feet and disposal is not available)

Year	Shadow Cost	Technology adoption			Applied water (10^3 AF)	Drainwater flow (10^3 AF)	Drainwater stock (10^3 AF)	Yield (in 10^6 lb)	Social welfare (current value, 10^3 \$)	Social welfare (discounted value, 10^3 \$)
		Drip	Furrow	Idle Land						
0	18.42	5.17 %	48.02 %	46.81 %	749	104	0	274	14290	14920
10	27.49	2.16 %	45.49 %	52.35 %	672	91	948	246	13994	9380
20	41.01	-	42.14 %	57.86 %	591	76	1778	217	13501	6066
30	61.18	-	37.83 %	62.17 %	519	61	2477	195	12741	3838
40	91.27	-	32.74 %	67.26 %	438	46	3032	169	11620	2346
50	136.16	-	27.02 %	72.98 %	352	33	3442	139	10102	1367
60	203.13	-	20.91 %	79.09 %	264	21	3717	108	8208	745
70	303.04	-	14.67 %	85.33 %	180	12	3879	75	6016	366
80	452.08	-	8.56 %	91.44 %	102	6	3961	44	3641	148
90	674.42	-	2.77 %	97.23 %	32	2	3993	14	1215	33
95.1	826	-	-	100.00 %	-	-	4000	-	-	-

In contrast, when the disposal of drainwater is not feasible (Table 3.2), the initial shadow costs ($18.42) are higher, and complete depletion of the water-storage capacity is delayed until year 95. Surprisingly, the share of land where drip irrigation is employed is lower than in Table 3.1, and it already vanishes completely after the 16^{th} year. These land-use changes are due to the high fixed cost per acre of drip irrigation, making it not viable if water prices are relatively high. However, these land-use changes do not imply an expansion of the land cultivated with furrow irrigation instead. The nonavailability of the disposal of the stock of drainwater increases the initial share of idle land from 43.28% (Table 3.1) to 46.81% (Table 3.2). The output and input prices and the fixed cost per acre for each technology shows that on-farm control of waterlogging is mainly achieved by leaving the land fallow. This result is consistent with reality where some land of the San Joaquin Valley is not utilized, due to a poor water-storage capacity.

The following section presents a sensitivity analysis with respect to the parameters of the model, i.e. with respect to the price of cotton, p, the water price, c, the disposal costs, k, and the heterogeneity of the land.

3.5.1 Effect of Parameter Changes on the Optimal Technology Allocation

The sensitivity analysis evaluates the changes in the share of land where a particular irrigation technology is adopted, as a result of a change in a parameter of the model. To measure the heterogeneity of the land quality the beta distribution is chosen, since it allows a wide variety of different shapes of the distribution.

Table 3.3 presents the different distributions used in the numerical analysis. Besides 1 uniform distribution of the land quality, where the two parameters of the beta distribution, denoted by γ and ϕ are identical and are equal to 1, 3 n-shaped distributions ($\gamma = \phi > 1$), and 3 u-shaped distributions ($\gamma = \phi < 1$) were used. In order to attribute the resulting changes in land-use exclusively to a change in the heterogeneity of the land quality the mean of the different distributions is maintained constant. Thus the maximum possible variance of the land quality is given by 0.09, that is where half of the land has the lowest land quality and the other half has the highest land quality. To facilitate the graphical presentation of the results, an index of the heterogeneity of the land quality has been constructed, and it is denoted by ξ. It is constructed by dividing the variance of each distribution by the highest variance (0.09). In this way, the most heterogeneous distribution of the land quality is associated with the index 1, and the heterogeneity of the land quality declines as the index number decreases.[3]

Table 3.3: Properties of the Beta Distributions

Parameters	mean	variance	index of land heterogeneity ξ
(0.1,0.1)	0.5	0.075	0.83
(0.5,0.5)	0.5	0.045	0.5
(0.8,0.8)	0.5	0.035	0.38
(1,1)	0.5	0.03	0.33
(2,2)	0.5	0.018	0.20
(3,3)	0.5	0.013	0.14
(5,5)	0.5	0.008	0.09

Figure 3.2 a-c) shows the effect of a successive increase in the price of cotton from \$0.60/lb to \$0.70/lb on the share of land where a particular irrigation

[3]The socially optimal solution of the agricultural allocation problem with an index of land heterogeneity of 1 is not calculated because the distribution is not continuous and therefore its computation presents some numerical difficulties. The more heterogeneous considered land is a beta distribution with parameters $\gamma = \phi = 0.1$, which has an index of heterogeneity of 0.83.

technology is adopted. The calculations are based on the case where the initial water-storage capacity of 10 feet, and the disposal of draiwater is not available. Figure 3.2 a-b) shows that an increase of the price of cotton from \$0.60/lb to \$0.70/lb in the first year leads to a decrease in the share of land with furrow irrigation from 42% to 32%. Yet, the share of land with drip irrigation increases from 5% to 67%. This pattern of increase, however, is maintained only until the 31st year. Thereafter the continuous increase in the shadow cost of the drainwater leads also to a decrease of the share of land with drip irrigation. In the 31st year the share of land with drip irrigation reaches 84%, with furrow irrigation 16%, and with no agricultural activity 0%. The decrease of the share of land with drip irrigation, however, does no lead to an increase of the share of land with furrow irrigation, but to an increase of the land that is taken out of production (Figure 3.2 c). The increase of the share of idle land is the only way that some part on the land can be maintained in agricultural production.

Figure 3.3 a-c) shows how the share of land, where a particular technology is adopted, changes as a response to a change in the district water price, given $\bar{s} = 10$ feet and p =\$0.60/lb. Figure 3.3 shows that drip irrigation is more sensitive to changes in the water price than furrow irrigation (Figures 3.3 a-b). For example, a decrease in the district water price from \$60/AF to \$45/AF leads in the first year to an increase in the share of land with drip irrigation from 0% to 54%. The share of land with drip irrigation declines faster over time if the district water price is intermediate than if it is low. However, the share of land with furrow irrigation declines slightly faster over time if the district water price is low. A decrease in the district water price from \$60/AF to \$45/AF advances the cease of agricultural production by 24 years (see Figure 3.3 c).

Figure 3.4 a-c) shows the effect of an increase in the disposal costs on the share of land where a particular technology is adopted, given $\bar{s} = 10$ feet and p =\$0.65/lb. For a cotton price of \$0.65/lb, disposal costs of \$100/AF maintain the entire irrigated

Figure 3.2: The Share of Cultivated Land as a Result of a Variation in Cotton Price for a Given Technology

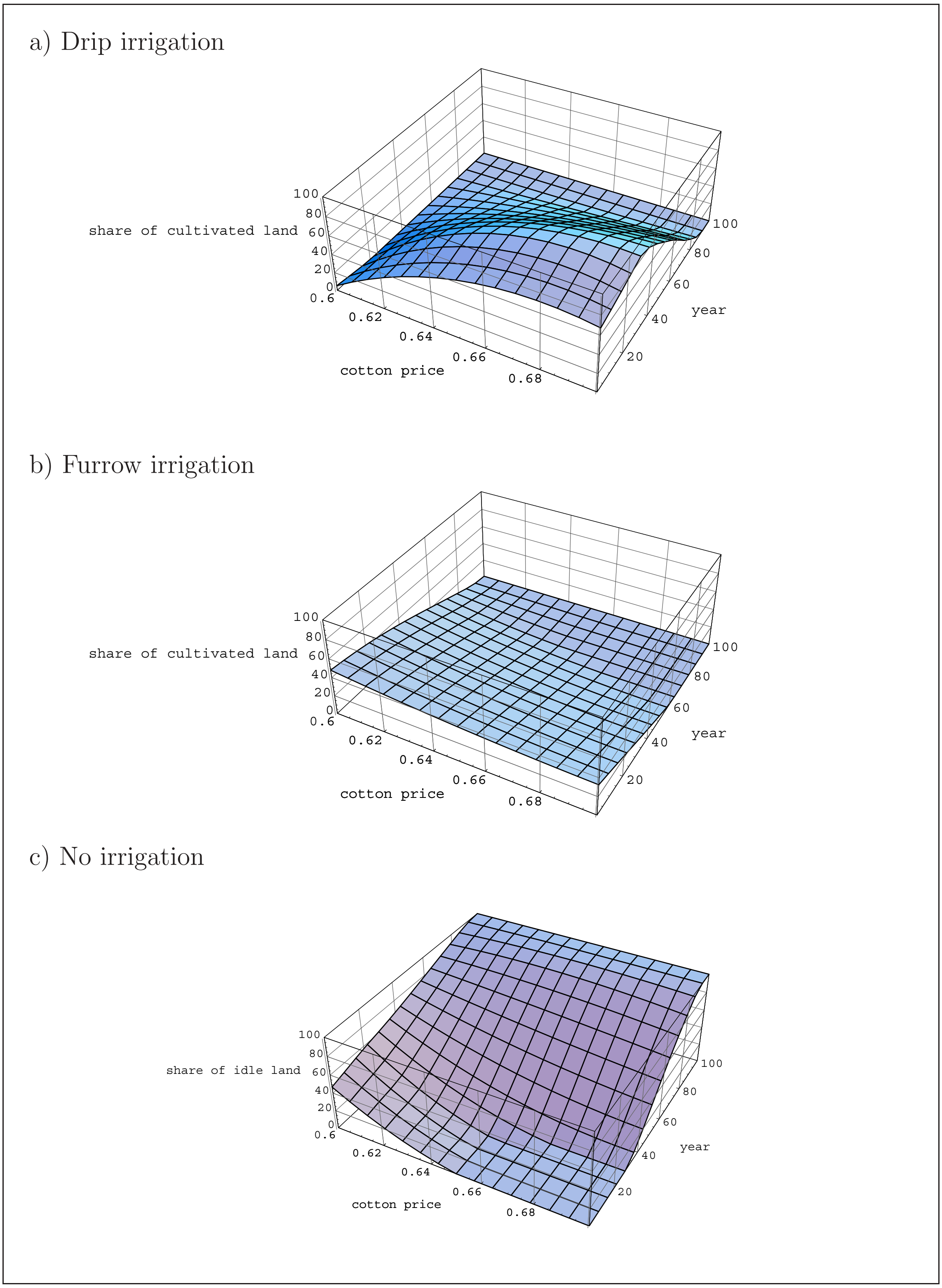


Figure 3.3: The Share of Cultivated Land as a Result of a Variation in District Water Price for a Given Technology

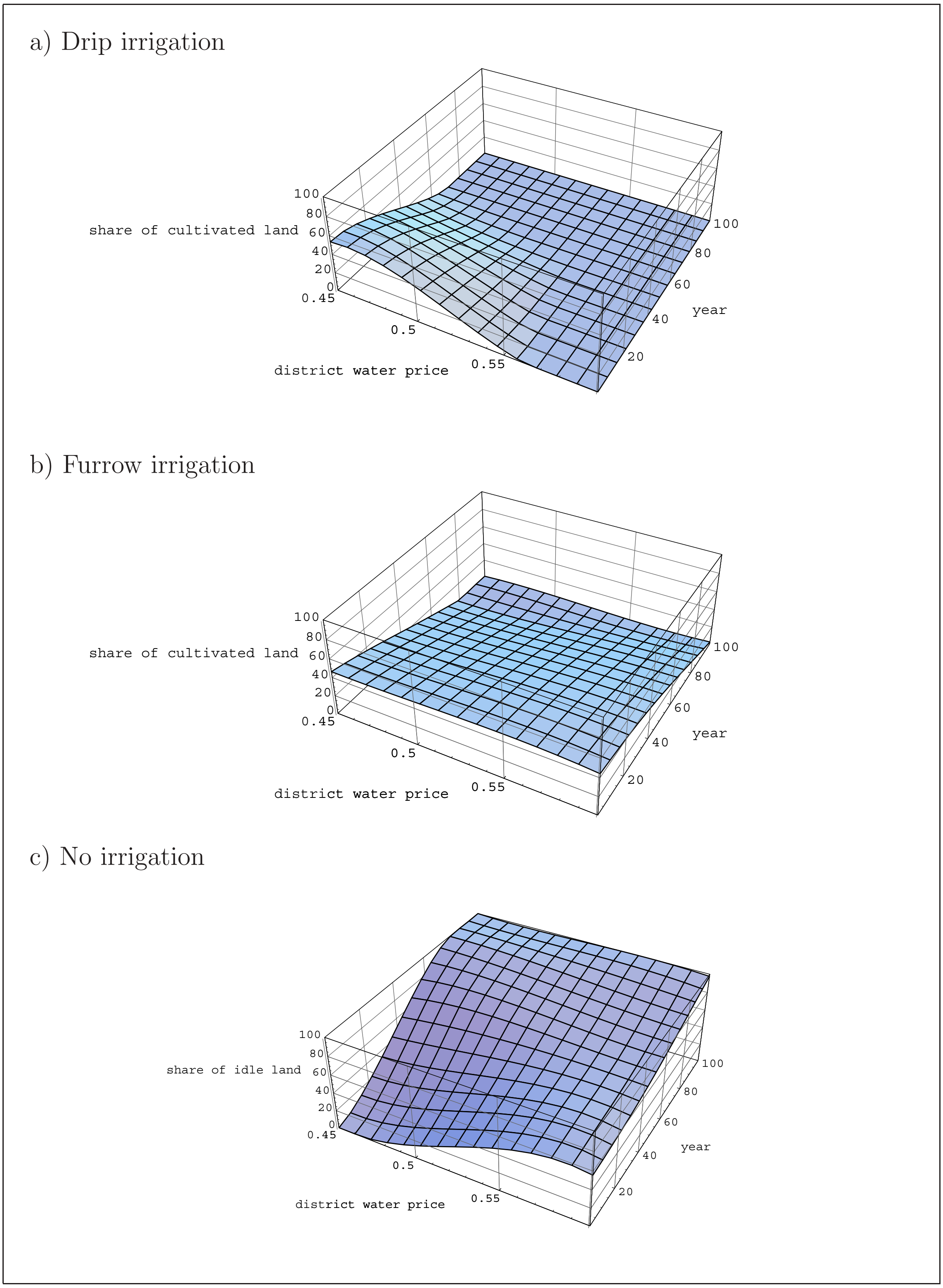
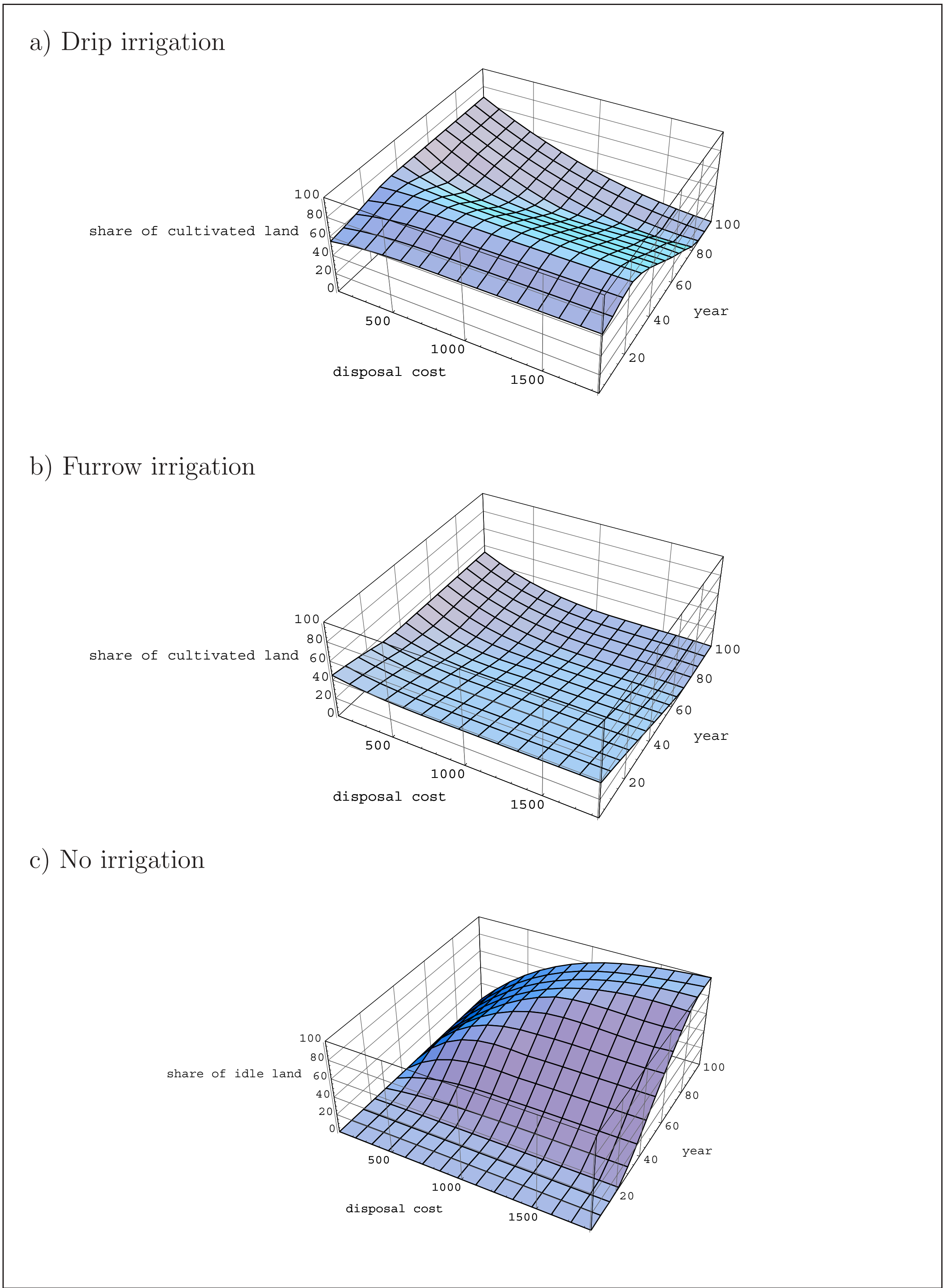


Figure 3.4: The Share of Cultivated Land as a Result of a Variation in Disposal Costs for a Given Technology



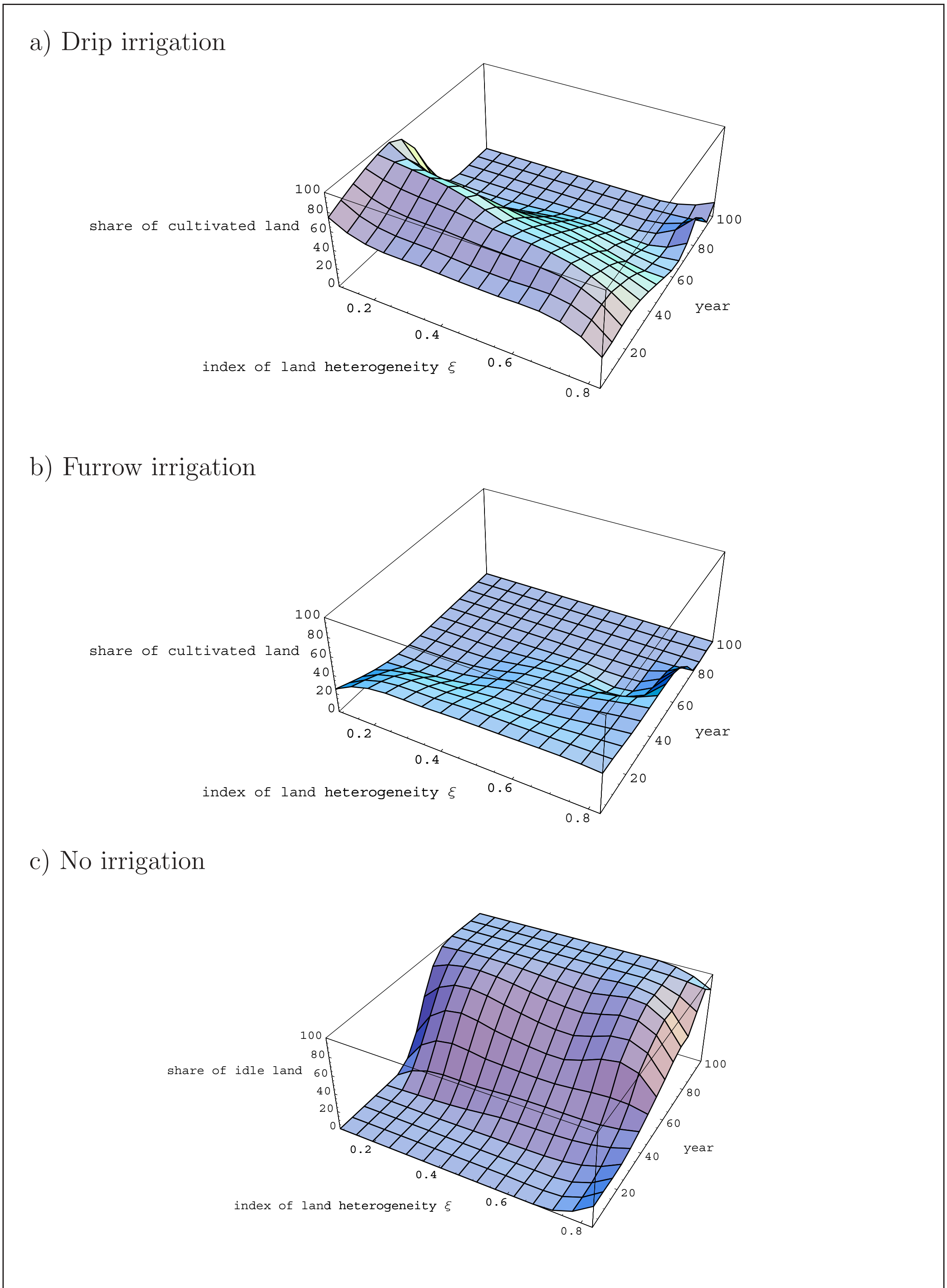
area in agricultural production for most of the time horizon. The share of cultivated land drops to 48% if disposal costs are 500$/AF, and to 20% if disposal costs are $1000/AF. The share of land with drip or furrow irrigation basically declines slightly over time for low disposal costs and strongly over time for high disposal costs. The share of land with drip irrigation even increases over time if the disposal costs are sufficiently low. Provided that disposal costs are not very high Figure 3.4 a-c) shows that some part of the land can be maintained in agricultural production.

Figure 3.5 a-c) shows the effect of an increase in the heterogeneity of the land quality on the share of land where a particular technology is adopted, given $\bar{s} = 10$ feet and $p =$\$0.65/lb. In the first year, the share of land with drip or furrow irrigation does not vary with the heterogeneity ξ. However, for more heterogeneously distributed land quality, furrow irrigation declines over time slower than for less heterogeneously distributed land quality. The stronger decline of furrow irrigation for a low ξ is compensated for a certain time period by an increase in the share of land with drip irrigation. However, after approximately 80 years the share of land with drip and furrow irrigation has declined to zero, and agricultural production does not take place anymore.

3.5.2 Comparison Between Social and Common Access Outcomes

Since the water-storage capacity is a good with common access to all farmers located within the irrigated area, the optimal private behavior will not coincide with the optimal social behavior. More precisely, economic theory has established that goods with common access are privately overutilized in comparison with their optimal utilization from a social point of view. The numerical analysis helps to determine whether this theoretically established inefficiency in terms of social welfare loss is of significant magnitude, or whether it can be neglected.

Figure 3.5: The Share of Cultivated Land as a Result of a Variation in Land Heterogeneity for a Given Technology



To determine the social welfare loss, the private maximization problem (T1) and (T2) is solved, where $\tau_i(t,\epsilon)$ is set to zero, and this outcome is compared with the optimal outcome of the social maximization problem (S). That is, it is calculated the difference between $W^S(z^*(t),\eta^*(t);\bar{s}) - W^T(0,\tilde{\eta}(t))$.

Tables 3.4 and 3.5 present the results of the social and private optimizations, respectively. These tables show that the depletion of the resource occurs earlier in the case of private-maximizing behavior since the farmers do not take into account the scarcity rent associated with the resource use. The difference between the social and the private outcome is even more pronounced if the possibility of disposal of the drainwater does not exist and the water-storage capacity is small. For instance, if $\bar{s}=5$, the depletion will occur five times faster than the socially optimal depletion. In this case, the welfare losses are very accentuated and signify nearly 40% of the social welfare, as shown in Figure 3.6.

Figure 3.6: Welfare Losses of the Private Equilibrium in Percentage over the Social Welfare

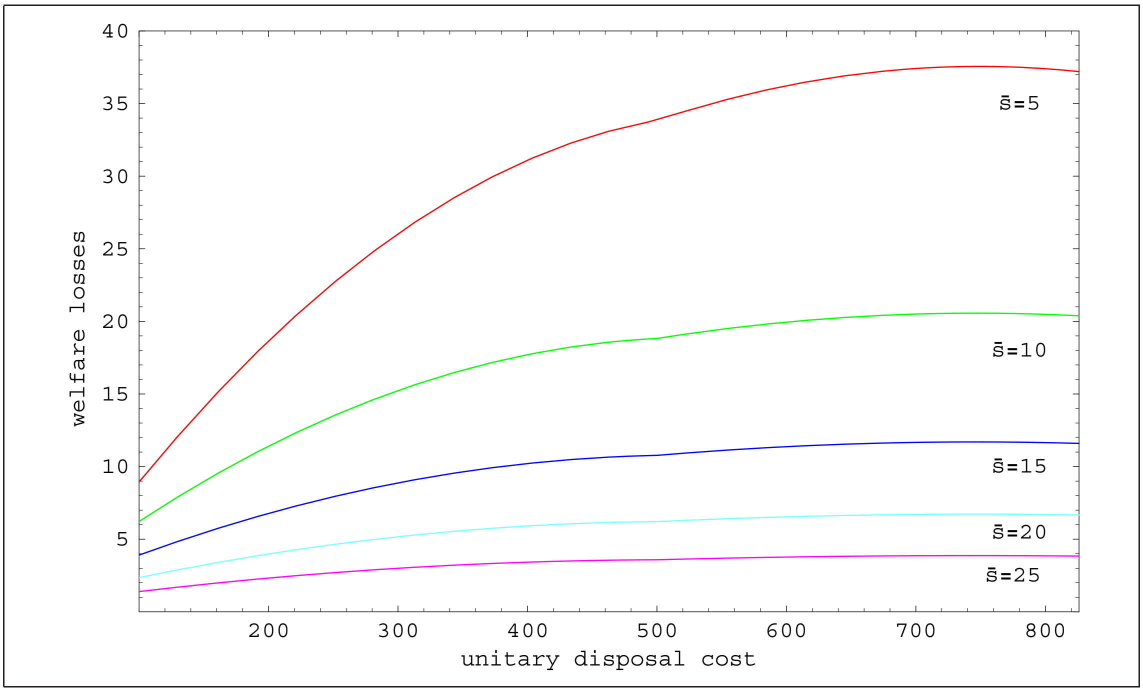


Table 3.4: Results of the Social Optimization.*

Disposal cost		Initial water-storage capacity $\bar{s}$				
		5 feet	10 feet	15 feet	20 feet	25 feet
$100/AF	Time (years)	30.1	50.6	68.1	84.2	99.5
	Farm net margin$^{(a)}$	222.759	270.033	303.020	325.069	339.568
	Collected taxes$^{(a)}$	91.408	67.678	47.424	32.130	21.286
	Disposal costs$^{(a)}$	-32.294	-14.247	-7.072	-3.713	-2.011
	Welfare$^{(a)}$	281.911	323.464	343.384	353.488	358.843
$250/AF	Time (years)	44.7	67	85.3	101.8	117.3
	Farm net margin	176.309	244.511	287.855	315.805	333.838
	Collected taxes	98.682	75.400	53.643	36.713	24.494
	Disposal costs	-17.175	-7.037	-3.392	-1.754	-0.942
	Welfare	257.816	312.887	338.106	350.765	357.390
$500/AF	Time (years)	59.9	82.8	101.2	117.8	133.4
	Farm net margin	161.460	236.991	283.551	313.225	332.261
	Collected taxes	94.912	75.092	54.175	37.346	25.029
	Disposal costs	-5.422	-2.173	-1.039	-535	-287
	Welfare	250.950	309.911	336.688	350.036	357.009
Disposal not available or $k > 826$	Time (years)	72.1	95.1	113.6	130.2	145.8
	Farm net margin	158.839	235.680	282.800	312.774	331.984
	Collected taxes	90.856	73.704	53.632	37.131	24.949
	Disposal costs	0	0	0	0	0
	Welfare	249.695	309.384	336.432	349.905	356.933

* based on the uniform land distribution, and a cotton price of $0.6/lb.

$^{(a)}$ Indicates the present value of those variables, in thousand dollars.

Table 3.5: Results of the Private Optimization*

Disposal cost		Initial water-storage capacity $\bar{s}$ 5 feet	10 feet	15 feet	20 feet	25 feet
$100/AF	Time (years)	14	28	42	56.1	70.1
	Farm net margin	256.704	303.346	329.971	345.171	353.848
	Welfare	256.704	303.346	329.972	345.171	353.848
	Welfare losses	8.94%	6.22%	3.91%	2.35%	1.39%
$250/AF	Time (years)	14	28	42	56.1	70.1
	Farm net margin	199.462	270.669	311.318	334.523	347.769
	Welfare	199.462	270.669	311.318	334.523	347.769
	Welfare losses	22.63%	13.49%	7.92%	4.63%	2.69%
$500/AF	Time (years)	14	28	42	56.1	70.1
	Farm net margin	166.023	251.581	300.421	328.302	344.218
	Welfare	166.023	251.581	300.421	328.302	344.218
	Welfare losses	33.84%	18.82%	10.77%	6.21%	3.58%
Disposal not available or $k > 826$	Time (years)	14	28	42	56.1	70.1
	Farm net margin	156.806	246.318	297.418	326.588	343.239
	Welfare	156.806	246.318	297.418	326.588	343.239
	Welfare losses	37.20%	20.38%	11.60%	6.66%	3.84%

* based on the uniform land distribution, and a cotton price of $0.6/lb.

Table 3.6: Optimal Taxes on Applied Water (in \$/AF)

Drip irrigation							
Year	Land quality ϵ						
	0.2	0.3	0.4	0.5	0.6	0.7	0.8
0	5.96	4.45	3.37	2.53	1.84	1.26	0.77
10	8.88	6.64	5.03	3.77	2.74	1.88	1.14
20	13.25	9.91	7.50	5.63	4.09	2.81	1.71
30	19.77	14.79	11.19	8.39	6.11	4.18	2.55
40	29.50	22.06	16.70	12.52	9.11	6.24	3.80
50	44.00	32.91	24.91	18.68	13.59	9.31	5.67
60	65.65	49.09	37.17	27.87	20.27	13.89	8.45
70	97.93	73.23	55.45	41.57	30.24	20.73	12.61
Furrow irrigation							
Year	Land quality ϵ						
	0.2	0.3	0.4	0.5	0.6	0.7	0.8
0	30.20	23.43	17.47	12.35	8.08	4.67	2.16
10	45.05	34.95	26.07	18.43	12.05	6.97	3.22
20	67.21	52.14	38.89	27.49	17.98	10.40	4.81
30	100.27	77.78	58.01	41.01	26.83	15.52	7.18
40	149.59	116.03	86.54	61.18	40.02	23.15	10.71
50	223.15	173.10	129.11	91.27	59.70	34.54	15.97
60	332.91	258.23	192.60	136.16	89.06	51.53	23.83
70	496.64	385.24	287.33	203.13	132.87	76.87	35.55

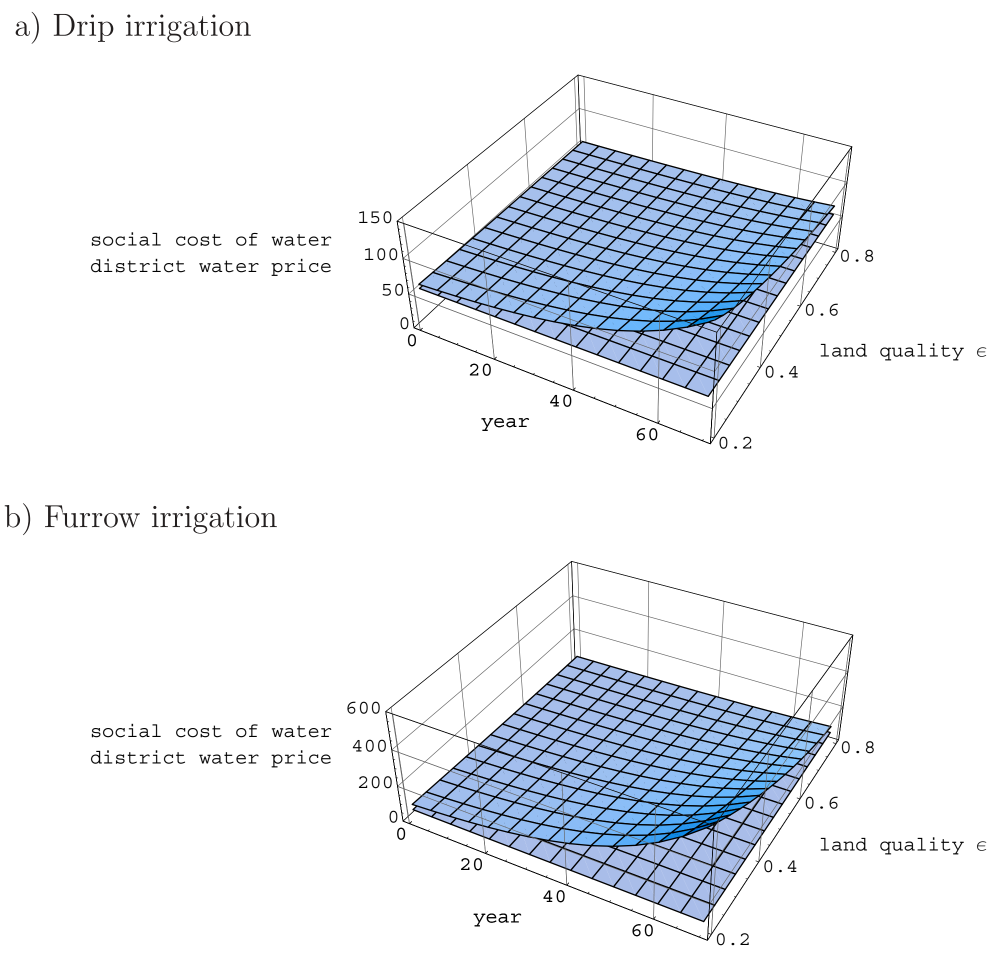
To induce farmers to behave optimally from a social point of view, the social planner could impose a spatially and temporally differentiated tax on the applied water. It is given by $\lambda(t)\,(1-\epsilon^{0.1})^{1.07448}$ and $\lambda(t)\,(1-\epsilon)^{1.9022}$ for drip and furrow irrigation respectively, where the optimal value of the shadow cost at each point in time is known for a predetermined set of parameters. For example, when the water-storage capacity is 25 feet, the optimal shadow cost is $\lambda(t) = 2.42\,exp^{0.04t}$ and, for a water-storage capacity of 5 feet, it is $\lambda(t) = 46.17\,exp^{0.04t}$. For the case of $\bar{s} = 5$, $p =$\$60/lb, a uniform distribution of the land quality, and no availability of drainwater disposal, Table 3.6 presents the socially optimal intertemporal tax scheme as a function of land quality ϵ. It shows that the optimal taxes on water applied with furrow irrigation are more sensitive to a change in land quality than the taxes on water applied with drip irrigation. In this way, the taxes on water applied with furrow irrigation are about 14 times higher at locations with a poor land quality in comparison with the taxes at locations with a high land quality. Moreover, the optimal taxes increase over time by the rate of discount, which exacerbates the difference between taxes imposed on low- and high-quality lands.

Figure 3.7 shows how the social cost of water changes as a function of time and land quality for drip (Figure 3.7 a) and furrow irrigation (Figure 3.7 b). The social cost of water consists of the water price that is charged by the water district[4] plus the optimal technologically, spatially, and temporally differentiated taxes on the applied water. Figure 3.7 shows for both available technologies that the tax for high-quality lands is never as high as the water price. However, the share of the tax in social water price increases as land quality declines. Moreover, the socially optimal tax increases exponentially over time, so that the proportion of the social water price that corresponds to the tax increases as time goes by. For instance, in land with average quality, the tax on water, applied with furrow irrigation, exceeds the water price from the 38$^{\text{th}}$ year onwards and, in land with low quality, from the

[4]Figure 3.7 is based on a water price of \$55/AF.

15th year (furrow irrigation) and from the 55th (drip irrigation) year onwards. At the end of the planning horizon, the social cost of water applied with furrow irrigation reaches \$600/AF on the lowest quality land.

Figure 3.7: Evolution of the District Water Price and the Social Cost of Water (Water price plus the socially optimal tax) over Time and Land Quality ϵ



To see how a change in the price of water, charged by the water district, affects the optimal tax the social optimal outcome based on district prices of \$45/AF,

\$50/AF and \$60/AF was calculated. Additionally, it was analyzed to which extent the change in the optimal tax, as a result of a change in the district water price, varies over time and over land quality. For example, given a water-storage capacity of 5 feet, a decrease in the district water price from \$55/AF to \$50/AF leads, in the first year, to an increase in the socially optimal taxes from \$2.53/AF to \$3.69/AF (drip irrigation) and from \$12.35/AF to \$18.06/AF (furrow irrigation) for a site with average land quality ($\epsilon = 0.5$).[5]

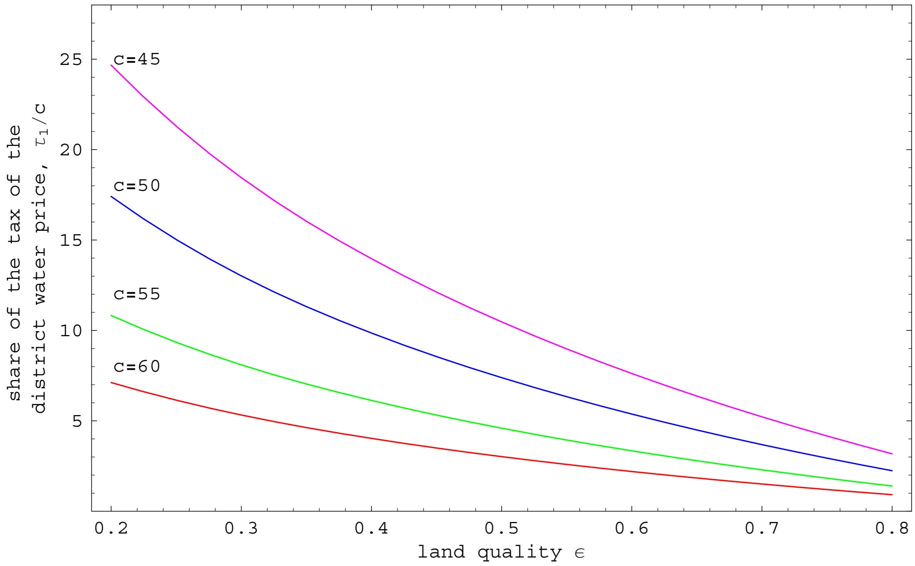
Figure 3.8 shows how the share of the tax of the district water price changes in the first year of the planning horizon as a function of land quality for different district water prices, for drip irrigation (Figure 3.8 a), and furrow irrigation (Figure 3.8 b). Likewise Figure 3.9 shows the evolution of the share of the tax of the district water price over time for different district water prices, for drip irrigation (Figure 3.9 a), and furrow irrigation (Figure 3.9 b), on land with average land quality. The results presented in Figure 3.8 and 3.9 are based on calculations with a water-storage capacity of 5 feet, a cotton price of \$0.60/lb and no availability of disposal of the drainwater.

Figure 3.8 a-b) shows that the share of the tax of the district water price increases with a rise in the district water price. This effect exacerbates as the quality of the land declines. For instance, in the case of drip irrigation a reduction of the district water price from \$60/AF to \$45/AF, causes the share of the tax of the district water price to increase from 0.9 to 3.1% on high-quality lands, and from 7.1 to 24.67% on low-quality lands. The same effect is even more pronounced in the case of furrow irrigation since more drainwater is generated. The share of the tax of the district water price, on water applied with furrow irrigation, on low quality land reaches up 125% when the district water price is \$45/AF.

[5]For a decrease in the district water price from \$55/AF to \$50/AF the change of the socially optimal taxes as a function of ϵ are given in general by $46.17\, exp^{0.04t}\ \alpha_i(\epsilon)$, and $67.48\, exp^{0.04t}\ \alpha_i(\epsilon)$ respectively.

Figure 3.8: Variation in the Share of the Optimal Tax of the District Water Price as a Function of Land Quality ϵ for Different District Water Prices

a) Drip irrigation



b) Furrow irrigation

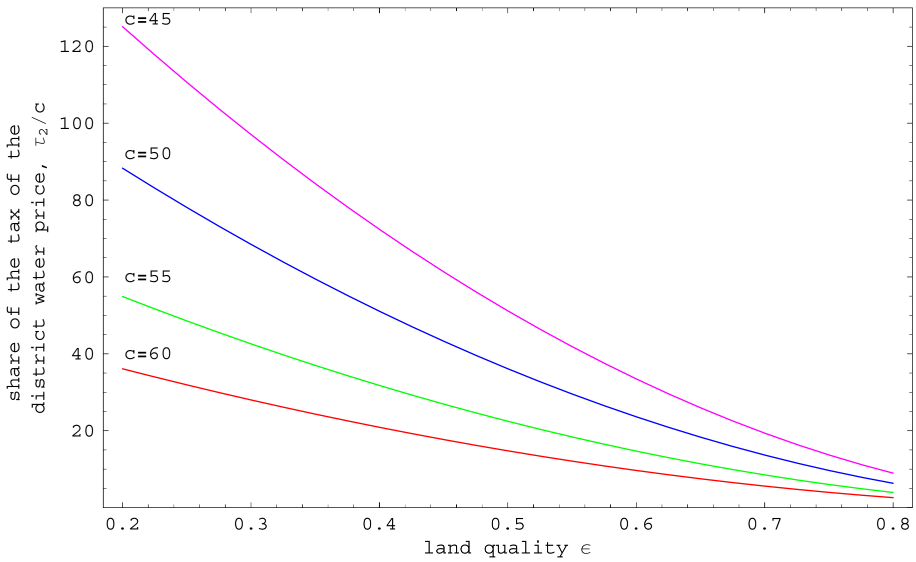
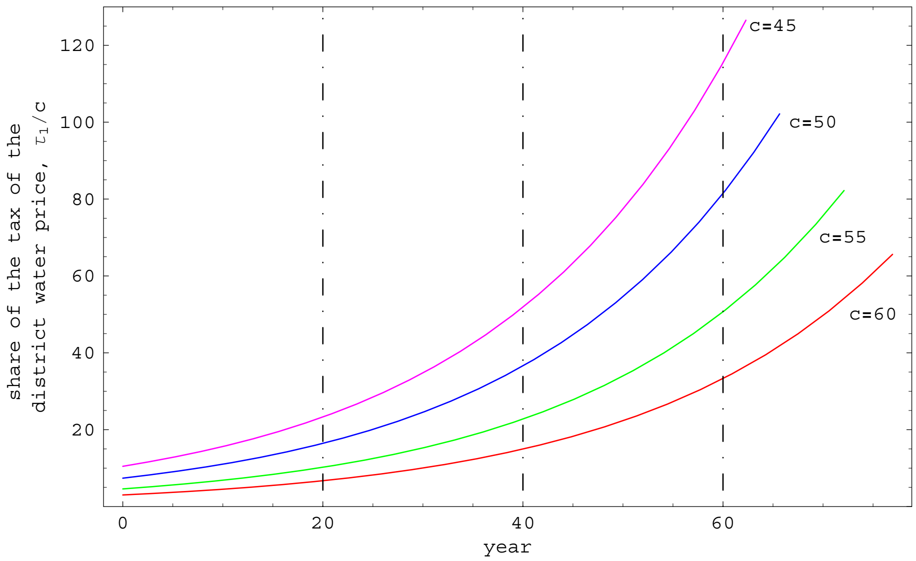


Figure 3.9: Variation in the Share of the Optimal Tax of the District Water Price as a Function of Time for Different District Water Prices

a) Drip irrigation



b) Furrow irrigation

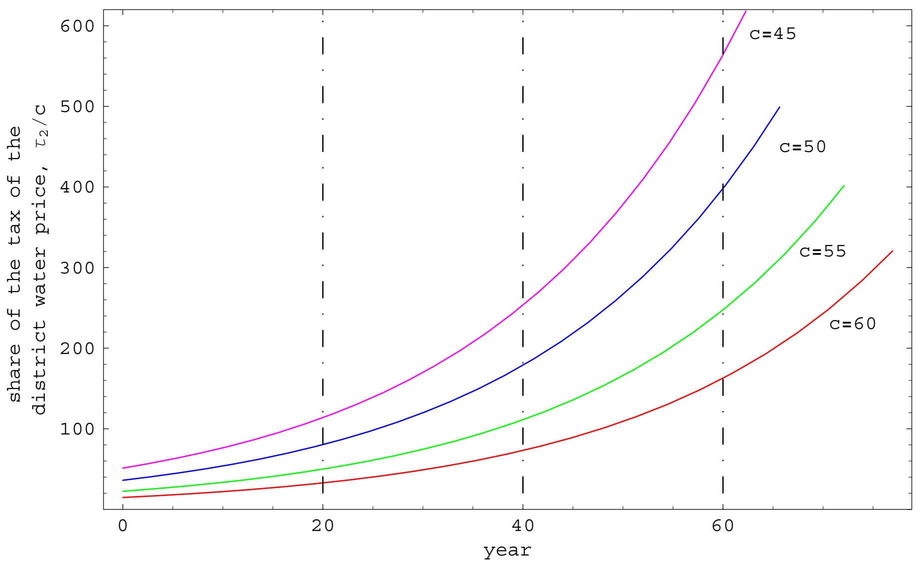


Figure 3.9 shows that the share of the optimal tax of the district water price increases over time. The increase is even stronger the lower is the district water price. Thus, for land with average quality, the tax on water applied with drip irrigation constitutes the principal share of the district water price if it is below \$50/AF. In the case of furrow irrigation the share of the tax of the district water price increase even more sharply over time. At the end of the planning horizon the tax corresponds to 300% of the district water price if the district water price is \$60/AF and to 600% of the district water price if the price is \$45/AF.

The elasticities of the socially optimal tax with respect to the district water are -1.41, -3.23, -4.53 and -1.91 given a district water price of \$45/AF, \$50/AF, \$55/AF, \$60/AF respectively. These elasticities are identical for drip irrigation and furrow irrigation. For medium water prices (50 and \$55/AF) a decrease in the district water price produces a strong increase in the socially optimal tax. This result is due to the fact that a decrease in the district water prices increases the private profitability of the land, and therefore, land that had been idle before, comes into production leading to an overall increase in drainwater generation. However, if the district water prices are either low or high a decrease in the price produces a less stronger increase in the socially optimal tax (see also Figure 3.3). In the case of low prices this reduced effect is a consequence of the exhausted possibility to bring in more land into agricultural production. In the case of high district water prices one observes that no additional land will be put into production, thereby limiting the generation of drainwater.

3.6 Implementation of Uniform Policies

Considering the heterogeneity of land implies that one should take into account both the spatial allocation and the temporal allocation in designing the correct policies to lead to the social optimum. Ignoring any of these aspects will lead to inefficient outcomes. This section explores the magnitude of the inefficiencies resulting from a deviation of the socially optimal solution, when imposing a second-best uniform policy under heterogeneous conditions. With this purpose, the land allocation problem is modelled in the case that the regulator does not posses all the necessary information to design a spatially and temporally differentiated policy, or even having it, the administrative costs of a differentiated policy are considered too high. Firstly, it is considered the case where the social planner does not know the whole distribution of the land quality over the region, making necessary to implement a uniform policy over space. Afterwards it is considered the case in which the policy maker does not know the optimal timing of the problem or she is not able to change the policy over time due to unaffordable administration costs.

3.6.1 Uniform Policies over Space

Assume the farmers know the physical attributes of their own land in making their decisions, but the social planner does not know the whole distribution of the land quality in the region, she only knows the mean land quality $\bar{\epsilon}$. Therefore, the social planner does not possess all the necessary information to implement a targeted policy over space and must implement a uniform policy instead. In that case it is not possible to regulate the private behavior to lead to the social optimum, producing as a consequence, a loss in social welfare.
When a constant tax over space $\hat{\tau}_i(t,\bar{\epsilon}) = \tau(t)\,\alpha_i(\bar{\epsilon})$ is levied on the applied water, each farmer will choose the application rate that maximizes the farm net margin after the tax, given by:

$$FNM(\hat{\tau}_i(t,\bar{\epsilon})) \;\equiv\; \max_{x_i,\,u_i} \int_{\epsilon_0}^{\epsilon_1} \Big(\sum_{i=1}^{2} \big(pf(\beta_i(\epsilon)u_i) - cu_i - I_i - \tau\alpha_i(\bar{\epsilon})u_i\big)x_i \Big)\, d\epsilon \quad (\mathrm{U}\bar{\epsilon})$$

subject to

$$x_i(\epsilon) \geq 0,\ i = 1, 2, \quad x_1(\epsilon) + x_2(\epsilon) \leq X(\epsilon), \quad u_i(\epsilon) \geq 0,\ i = 1, 2$$

The objective of the social planner is then, to find the constant tax rate over space that maximizes the social welfare over time. The theory suggest that the discounted social welfare obtained with a uniform policy over space, $W(\hat{\tau}_i(t, \bar{\epsilon}), \hat{\eta}(t))$, will be inferior than the social welfare obtained with the spatially and temporally-differentiated optimal policy, $W(\tau_i^*(t, \epsilon), \eta^*(t))$, as the allocation of technologies and water use differs from those of the socially optimal outcome. To investigate the extent of these inefficiencies, the solution of the uniform policy over space is computed for the case of the cotton produced in the San Joaquin Valley in California.[6]

To examine empirically the inefficiency resulting from applying a spatially-uniform policy over the agricultural region, it is assumed that the social planner only knows that the mean land quality is 0.5, but she does not know the whole distribution of the land quality. As a consequence, she is not able to impose the optimal differentiated tax, and she fixes instead a uniform tax on the applied water with drip irrigation equal to $\tau(t)\,(1 - 0.5^{0.1})^{1.074}$, and a tax $\tau(t)\,(1 - 0.5)^{1.092}$ on the applied water with furrow irrigation, selecting $\tau(t)$ in each period to maximize the social welfare. This policy favors the use of low quality and more polluting lands against the lands with higher quality, since the tax is fixed on the base of the drainwater generated on the mean-quality lands. The optimization was carried out for the land quality distributions defined in Table 3.3.

Table 3.7 summarizes the results of the uniform optimization over space, and Figure 3.10 depicts the welfare losses due to the implementation of the uniform tax over space as a function of the land heterogeneity ξ. One can observe from Table 3.7 that heterogeneity of land has a considerable influence on the magnitude of the

[6] To keep the analysis simple, the solutions were only computed for the exhaustible resource scenario, but the inefficiencies could also be evaluated for the case in which drainwater disposal is feasible.

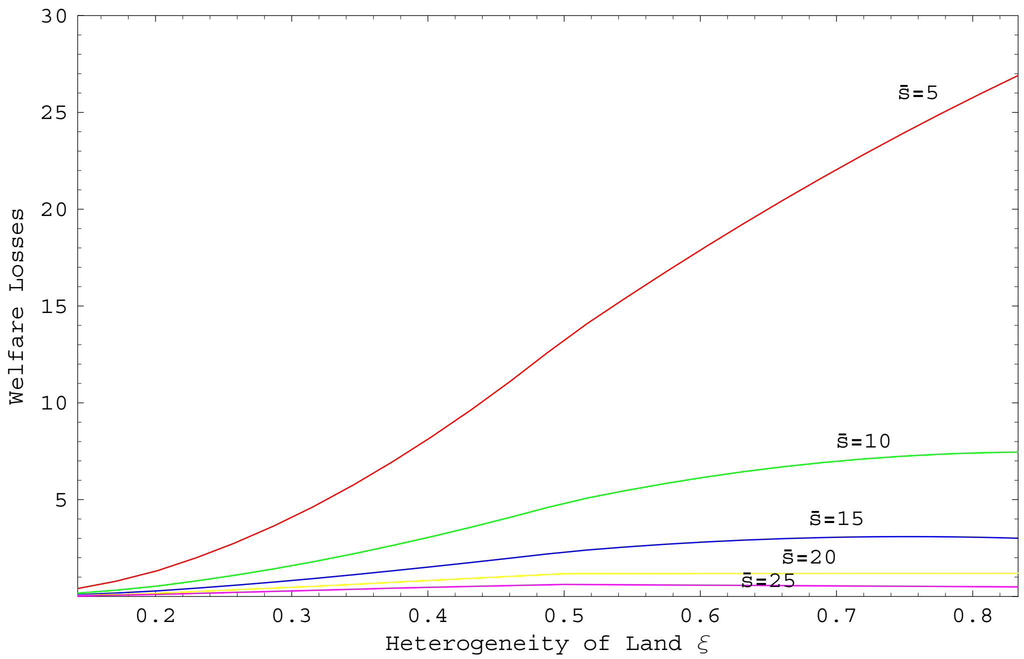
Table 3.7: Results of the Optimization with Constant Taxes over Space*

Index of land heterogeneity ξ		Initial water-storage capacity $\bar{s}$				
		5	10	15	20	25
0.83	Farm net margin	194.737	495.909	708.233	821.283	878.373
	Collected taxes	312.547	274.279	159.069	84.209	42.756
	Welfare	507.281	770.187	867.302	905.492	921.129
	Welfare losses	26.91%	7.45%	3.01%	1.20%	0.49%
0.50	Farm net margin	239.741	526.529	708.133	825.929	902.369
	Collected taxes	327.228	275.547	203.760	143.449	98.761
	Welfare	566.969	802.075	911.893	969.378	1000.000
	Welfare losses	13.28%	4.83%	2.30%	1.18%	0.62%
0.38	Farm net margin	265.605	527.125	687.641	792.049	863.106
	Collected taxes	301.134	249.264	189.603	140.150	100.826
	Welfare	566.738	776.388	877.244	932.199	963.932
	Welfare losses	7.42%	2.80%	1.40%	0.77%	0.45%
0.33	Farm net margin	276.832	523.360	675.135	774.721	843.746
	Collected taxes	285.719	238.719	184.327	138.777	101.553
	Welfare	562.551	762.079	859.462	913.497	945.299
	Welfare losses	5.29%	2.03%	1.04%	0.59%	0.35%
0.20	Farm net margin	290.685	506.758	635.707	723.144	783.930
	Collected taxes	249.449	208.190	168.063	132.372	103.717
	Welfare	540.134	714.948	803.770	855.517	887.647
	Welfare losses	1.30%	0.53%	0.28%	0.17%	0.10%
0.14	Farm net margin	298.844	496.847	614.324	695.020	752.739
	Collected taxes	225.827	191.475	158.744	128.657	103.157
	Welfare	524.671	688.323	773.069	823.677	855.896
	Welfare losses	0.41%	0.17%	0.09%	0.06%	0.03%

*For a cotton price of $0.65/lb, and where drainwater disposal is not available.

deadweight losses from a uniform policy over space. The divergence between the outcome of the uniform policy and the social optimum becomes more pronounced when the water logging capacity falls to less than 10 feet. When the land is rather homogeneous, the implementation of a uniform tax on the applied water can be considered as a substitute of an heterogeneous policy, for instance, if the variance of the land quality is lower than 0.018 ($\xi = 0.20$), the deadweight loss is less than 1% of social welfare. However, as heterogeneity of land increases and the water logging capacity gets smaller, one can not utilize uniform policies to achieve the social optimum.

Figure 3.10: Welfare Losses of a Uniform Tax over Space as a Percentage over the Social Welfare



Contrary to the results by Helfand and House (1995), it is find that there are cases in where the welfare loses of a uniform policy measure can be very significative.

For example, when the subsurface storage capacity is 5 feet and the soil is uniformly distributed ($\xi = 0.33$), there will be a loss in social welfare of 5%, and the welfare loss reaches 27% with an index of land heterogeneity of 0.83.

3.6.2 Uniform Policies over Time

The maximization of the social welfare requires to impose an increasing tax over time, as the water-storage capacity declines. However, in many occasions, the government is not able to change the policy continuously. Instead, she implements a static policy, imposing a specific tax schedule over space and maintaining it in the overall time period, or changing the tax after some time (almost static). The welfare obtained in this case should be clearly inferior (Perman et al., 1999). The median shadow price of the drainwater flow, that is, the shadow price at half the temporal horizon of the optimization problem, is denoted by $\tilde{\varphi}$, and it is assumed that the authority imposes a tax $\tilde{\tau}_i(\bar{t}, \epsilon) = \tilde{\varphi}\, \alpha_i(\epsilon)$. The farmer will choose the technology and the level of applied water to maximize

$$FNM(\tilde{\tau}_i(\bar{t}, \epsilon)) \equiv \max_{x_i,\, u_i} \int_{\epsilon_0}^{\epsilon_1} \Big(\sum_{i=1}^{2} \big(pf(\beta_i(\epsilon)u_i) - cu_i - I_i - \tilde{\varphi}\alpha_i(\epsilon)u_i\big)x_i \Big)\, d\epsilon \quad (\mathrm{U}\bar{t})$$

subject to

$$x_i(\epsilon) \geq 0,\ i = 1, 2, \quad x_1(\epsilon) + x_2(\epsilon) \leq X(\epsilon), \quad u_i(\epsilon) \geq 0,\ i = 1, 2$$

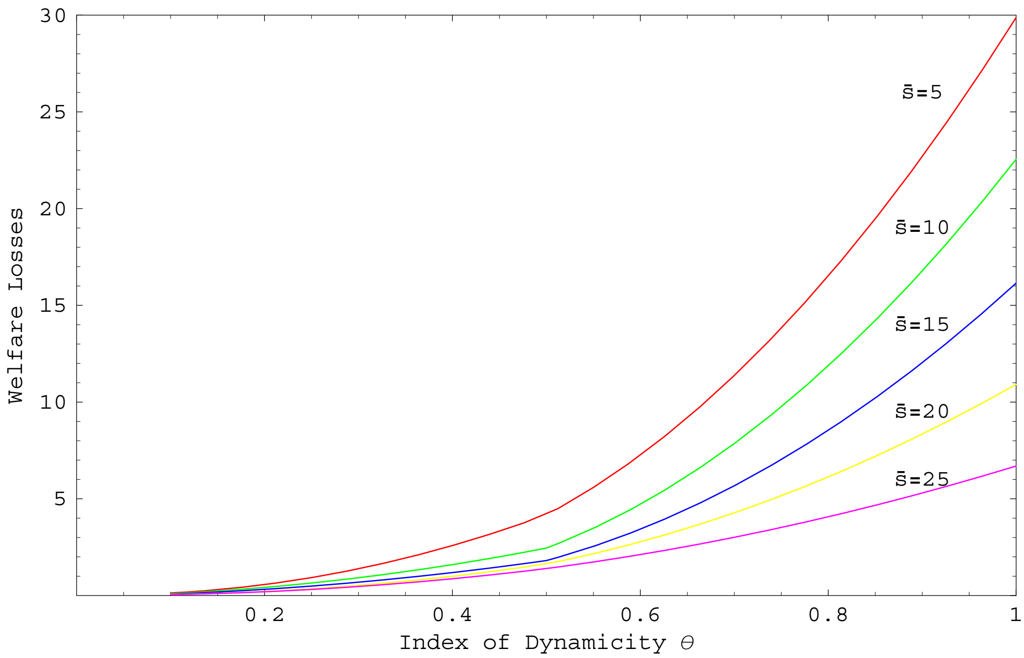
Given that Proposition 7 states that the optimal input taxes should be fixed according to $\tau_i^*(t, \epsilon) = \varphi(t)\, \alpha_i(\epsilon)$, $i = 1, 2$, and knowing that the optimal shadow price of the drainwater stock increases over time at the discount rate, one obtains that these taxes must increase at the same rate. Thus, the imposition of static or almost static taxes should result in a loss of the social welfare, given by $W(\tau_i^*(t, \epsilon), \eta^*(t)) - W(\tilde{\tau}_i(\bar{t}, \epsilon), \tilde{\eta}(t))$.

To analyze empirically the inefficiencies resulting from applying a static policy over time, 4 different optimizations were computed.[7] In the fist one, the median shadow price was taken to calculate the water taxes, and those taxes were maintained constant in the overall period. In the second, the temporal horizon was divided in two periods and the median shadow price of each period was taken to compute the corresponding taxes, applying each tax half of the time frame. The third and fourth optimizations were performed by dividing the time frame in 4 and 10 periods respectively. To facilitate the representation of results, an index of dynamicity is constructed, defined as $\theta \equiv 1 - 1/n$, where n denotes the number of periods considered. In this way, an index $\theta = 0$ corresponds to a totally static policy, while an index $\theta = 1$ stands for a totally dynamic policy. The last one is equivalent to the socially optimal solution. Table 3.8 presents the results of these optimization problems, and Figure 3.11 depicts the welfare losses of the static policy as a function of the index of dynamicity. One can see from Table 3.8 that the welfare losses increase as a more static policy is applied, reaching a deadweight loss of 27% of the social welfare if a constant tax over time is imposed, when the water storage capacity is only 5 feet. However, the welfare losses decrease significatively when the planning horizon is divided in more than 1 period (index of dinamicity $\theta = 0.5$).

Comparing the uniform policy over space with the static policy over time, one also observes that not taking into account heterogeneity of land and thus implementing uniform taxes over space can lead, if the land is rather heterogeneous, to a greater deadweight loss than an almost static policy (considering only two different periods). Surprisingly, it could also be the case that a second-best uniform policy is in some situations worse than the private solution, given that it leads to a lower social welfare.

[7] The result of the static policy is also computed only for the exhaustible resource scenario.

Figure 3.11: Welfare Losses of a Static Tax



3.7 Summary and Conclusions

This chapter presents a theoretical model that allows to determine the intertemporally optimal amount of water use and intertemporally optimal technology choice in the presence of spatial heterogeneity of land. To determine the optimal spatial and temporal decision two stage optimal control is utilized, where the first stage serves to determine the optimal spatial allocation of the resources, and the second stage to determine the optimal intertemporal development of the already optimal spatial decision problem. The method is presented in the context of the problem of waterlogging. Due to the common property character of the available water-storage capacity of the entire irrigated agricultural area, the optimal private outcome does not coincide with the optimal social outcome. Therefore, a corrective price policy is required that takes account of the spatial heterogeneity of the land, of the different

Table 3.8: Results of the Optimization with Static Taxes*

Index of Dynamicity θ		Initial water-storage capacity $\bar{s}$				
		5	10	15	20	25
0	Farm net margin	158.871	285.358	390.314	478.663	556.192
	Collected taxes	257.630	317.108	337.901	339.926	328.905
	Welfare	416.501	602.465	728.216	818.590	885.096
	Welfare losses	29.88%	22.55%	16.15%	10.91%	6.69%
0.5	Farm net margin	299.241	477.780	677.805	779.259	843.410
	Collected taxes	269.947	281.010	174.953	124.588	91.976
	Welfare	569.188	758.790	852.757	903.847	935.386
	Welfare losses	4.17%	2.45%	1.81%	1.63%	1.39%
0.75	Farm net margin	320.934	556.676	691.017	797.031	860.895
	Collected taxes	267.509	216.180	173.205	118.669	84.713
	Welfare	588.443	772.856	864.221	915.700	945.608
	Welfare losses	0.93%	0.65%	0.49%	0.35%	0.32%
0.9	Farm net margin	328.994	559.739	707.469	801.533	865.512
	Taxes	264.165	217.391	160.352	116.763	82.625
	Welfare	593.159	777.130	867.821	918.296	948.136
	Welfare losses	0.13%	0.10%	0.08%	0.06%	0.05%
1	Farm net margin	332.320	562.569	709.233	803.732	866.320
	Collected taxes	261.633	215.321	159.240	115.142	82.282
	Welfare	593.952	777.891	868.473	918.874	948.602
	Welfare losses	-	-	-	-	-

*For a cotton price of $0.65/lb and where drainwater disposal is not available.

technologies, and of the dynamic aspects of the depletion of the water-storage capacity. Consequently optimal water prices are spatially, technologically, and temporally differentiated.

To demonstrate the applicability of this approach an empirical illustration is presented for the case of cotton produced in the San Joaquin Valley in California, USA. The theoretical model is reformulated such that it can be solved numerically with standard mathematical software. The empirical part of the chapter shows to which extent the inefficiency of the private versus the social outcome is relevant in an empirical context. With a water-storage capacity of 5 feet, for instance, the depletion will occur five times faster than the socially optimal depletion leading to an overall welfare loss of 40%. The simultaneous consideration of technology, time and space permits to show to which extent the changes in water prices are attributed to a change in the technology, in time or in land quality. Moreover, the empirical study demonstrates the sensitivity of the optimal water pricing policy with respect to changes in land quality given a certain irrigation technology. For example, the optimal tax on water applied with drip irrigation is approximately 9 times higher for the lowest land quality compared to the highest land quality, while the optimal tax on water applied with furrow irrigation is 14 times higher.

The analysis suggests that site and technology specific information is needed in every moment of time to design optimal pricing policies. With the development of new technologies, such as remote sensing and geographic information systems, and the enhancing of monitoring practices, these differentiated policies are becoming feasible. The analysis is expanded to cases where policymakers are not able to distinguish between land qualities, and therefore, the pricing policy can only be technologically and temporally differentiated, and to cases were policymakers are not able to change the policies continously over time. In this cases, the model serves to determine to which extent uniform policies over space or static taxes can be a substitute for the optimal policies.

Chapter 4

Optimal Control of Size-Distributed Forests

4.1 Introduction

Interest in forests has grown significatively in many countries over the last years. The United Nations Economic and Social Council, in its decision 1995/226, endorsed the recommendation of the third session of the Commission on Sustainable Development to establish the Intergovernmental Panel on Forests (IPF). Its objective was to pursue consensus and coordinated proposals for action to support conservation, management, and sustainable development of forests. The implementation of the proposals for action has been approved by the United Nations, and the European Union. Therefore, it becomes necessary to design forest policies that support the sustainable management of forests.

Forests provide a large variety of services such as: timber production; recreation and landscape; natural habitat for numerous species; protection of watersheds; protection of villages from avalanches and landslides; buffering and carbon sequestration (Rojas, 1996). Thus, from a social point of view the optimal management of forests must take these multiple services into account. The variety of services is especially pronounced in Catalan forests, where principal benefits from forestry are in form of by-products (mushrooms, truffles, hunting, ...). These benefits account for 65% of the commercial value of forest products, and only 35% are attributed to timber and firewood sale (Raddi, 1997). One of the most important instruments to support and reinforce the multiple services provided by the forest is the selective-logging regime. However, previous economic literature has either ignored completely the possibility of a selective-logging regime, or this logging regime has been treated without an explicit recognition of the complex growth process driven by time and size of the tree.

Optimal tree rotation of a forest where all trees have the same age was solved by Faustmann (1849), Pressler (1860), and Ohlin (1921) (Conrad and Clark, 1987). However, if the multiple services that forest provides are taken into account, the Faustmann solution is not optimal, as is originated from focusing exclusively on timber production. Moreover, the Faustmann approach is limited as it does not allow to analyze the optimal management of a size-distributed forest. To determine the optimal management of such forest it is necessary to consider, in addition to time, the size of the trees.

This chapter presents a theoretical model to find the optimal selective-logging regime of a size-distributed forest. The law of motion of the economic model is governed by a partial differential equation that describes the evolution of the forest stock over time. To find the solution of the resulting distributed optimal control problem a numerical solution technique known as "Escalator Boxcar Train" is proposed. The empirical part of the chapter determines the optimal selective-logging regime of a forest that consists of *pinus sylvestris* from a private and social perspective. Additionally, the optimal logging regime that converts an even-sized forest into an uneven-sized or size-distributed forest is analyzed. The analysis allows to compare the optimal selective-logging regime with the optimal clear-cutting of the

Faustmann model. The results show that the clear-cutting regime leads to lower benefits than the selective-logging regime. This is due to the rigidity of the Faustmann model with respect to thinning and planting. Finally, the effect of incorporating a monetary value of diversity on the optimal management regime of the forest is analyzed.

The chapter is organized as follows. The following section describes the basic features of the model. Section 4.3 defines the distributed optimal control problem. In section 4.4, a numerical method for solving the distributed optimal control problem is presented. Section 4.5 determines the selective-logging regime of a size-distributed forest. Finally, section 4.6 presents some conclusions.

4.2 The Economic Model

In order to specify the economic model, one has to determine initially how the size of a tree is measured. In forestry, the size of a tree, and consequently the size of a forest, is usually measured by the diameter at breast height, that is, the diameter of the trunk at a height of 1.30 m above the ground. Adopting this measurement, distributed optimal control emerges as the most suitable tool to determine the optimal selective-logging regime of a privately owned size-distributed forest. Apart from defining size and time as the relevant exogenous determinants for the growth process, it is also taken into account that timber prices per m^3 increase with the diameter of the tree. More specifically, time, denoted by t, and diameter, denoted by $l \in \Omega,\ \Omega \equiv [l_0, l_m)$, are incorporated as the domain of the control and state variables. The upper boundary of the diameter domain, l_m, can be interpreted as the maximum diameter that a tree can reach under perfect environmental conditions. It is assumed that a diameter-distributed forest can be fully characterized by its number of trees and by the distribution of the diameter of the trees. In other words, the spatial distribution or particular location of the trees is not accounted for. It is assumed that all individuals have the same environmental conditions and the same amount of space. Moreover, given that the diameter of a tree lies in the interval $[l_0, l_m)$, and that the number of trees is large by assumption, the forest can be represented by a density function. It is denoted by $x(t,l)$ and indicates the population distribution with respect to the structuring variable, l, at time t. Given this definition, the number of trees in the forest at time t is given by

$$X(t) = \int_{l_0}^{l_m} x(t,l)\, dl. \tag{4.1}$$

In order to model the dynamics of the forest, the four processes: growth, reproduction, mortality and the influence of the individual tree on in the vital functions of other individuals have to be determined. Let define $g(E,l)$ as the rate of change in the diameter of a tree as a function of its current diameter, and of a collection of environmental characteristics, E, that affect individual growth. These environmental characteristics can be given, for example, by the total number of trees, or the basal area[1] of the forest. A large basal area decreases the rate of growth of a single tree, since, for instance, less light is available for photosynthesis. Thus, the change in the diameter over time of a single tree is given by

$$\frac{dl}{dt} = g(E,l). \tag{4.2}$$

The instantaneous mortality rate, that is, the rate at which the probability of survival of a tree with diameter l, given the environment E decreases with time, is defined by $d(E,l)$. This chapter concentrates on the case where the forest is managed in form of a plantation, i.e., all young trees are planted with diameter l_0 and no biological reproduction takes place. Thus, the control variables of the forest owner are given by $u_1(t,l)$ and $u_2(t;l_0)$ and denote cutting density at time t with diameter l, and planting density at time t with diameter l_0 respectively. These features

[1]Basal area is the area of the cross section of a tree at a height of 1.30 m above the ground. Basal area is often used to measure and describe the density of trees in the forest, using the sum of the basal area of all trees expressed per unit of land area (e.g., square meters per hectare).

are typical for time and diameter dependent optimal control problems where a distributed control u_1 (distributed over l) is complemented by a boundary control u_2. Following Tuljapurkar and Caswell (1997), the partial differential equation (PDE) that governs the dynamics of the forest, complemented by the variable $u_1(t,l)$ can be written as:

$$\frac{\partial x(t,l)}{\partial t} + \frac{\partial g(E,l)\, x(t,l)}{\partial l} = -d(E,l)x(t,l) - u_1(t,l). \tag{4.3}$$

4.3 The Distributed Optimal Control Problem

The optimal management of a diameter-distributed forest can be formulated as a distributed optimal control problem (Calvo and Goetz, 2001). Using the definitions given in the preceding section, the formal decision problem of the forest owner can be stated as:

$$\begin{aligned} \max_{u_1(t,l),u_2(t;l_0)} \quad & \int_0^\infty \int_{l_0}^{l_m} V_1(x(t,l), u_1(t,l), l)e^{-rt}\, dl\, dt \\ & - \int_0^\infty V_2(x(t,l_0), u_2(t;l_0), l_0)e^{-rt}\, dt, \end{aligned} \tag{P}$$

subject to the constraints

$$\begin{aligned} &\frac{\partial x(t,l)}{\partial t} = f(E,t,l) = -\frac{\partial g(E,l)\, x(t,l)}{\partial l} - d(E,l)x(t,l) - u_1(t,l), \\ &x(t_0,l) = x_0(l), \quad x(t,l_0) = u_2(t;l_0), \quad u_1 \in U_1, \quad u_2 \in U_2, \end{aligned}$$

where r denotes the discount rate. The function $V_1(\cdot)e^{-rt}$ presents the discounted net margin of the timber, i.e. the revenue of the timber sale minus cutting and maintenance costs. The function $V_2(\cdot)e^{-rt}$ captures the discounted cost of planting trees with diameter l_0. The term $x_0(l)$ denotes the initial diameter distribution of the trees, and the restriction $x(t,l_0) = u_2(t;l_0)$ requires that the planted density coincides with the density of the stock variable with diameter l_0.

The expressions U_1, U_2 denote some bounded control sets, such as bio-physical or non-negativity constraints. Using Pontryagins Maximum Principle the present value Hamiltonian is given by

$$\begin{aligned}\mathcal{H} &\equiv \int_{l_0}^{l_m} \big[V_1(\cdot)e^{-rt} + \lambda(t,l)f(\cdot)\big]dl - V_2(\cdot)e^{-rt} + \lambda_0(t)\big[u_2(t;l_0) - x(t,l_0)\big] \\ &\equiv \int_{l_0}^{l_m} \mathcal{H}_1\,dl + \mathcal{H}_2,\end{aligned}$$

where $\mathcal{H}_1$ stands for $V_1(\cdot)e^{-rt}+\lambda(t,l)f(E,t,l)$, and $\mathcal{H}_2$ for $-V_2(\cdot)e^{-rt}+\lambda_0(t)\big[u_2(t;l_0)-x(t,l_0)\big]$. The variables $\lambda(t,l)$ and $\lambda_0(t)$ denote the costate variable and a Lagrange multiplier respectively. The term $\mathcal{H}_1$ is associated with the distributed optimal control part of the problem, and the term $\mathcal{H}_2$ with the boundary optimal control part (Feichtinger and Hartl, 1986), and (Muzicant, 1980). That is why $\mathcal{H}_1$ is integrated over the range of l but not $\mathcal{H}_2$. In other words $\mathcal{H}_2$ is similar to a standard optimal control problem (lumped optimal control) since it is valid for all moments of time but only for a single value of l, i.e. l_0 (lumped). However, it is not a proper optimal control problem as the constraint $u_2(t;l_0) - x(t,l_0) = 0$ is constant over time. As a result, the first order conditions associated with this part of the problem do not involve a system of canonical differential equations. Taking into account the constraints of the control sets U_1 and U_2, leads to the Langrangian $\mathcal{L}$ given by

$$\mathcal{L} \equiv \int_{l_0}^{l_m} \mathcal{H}_1\,dl + \mathcal{H}_2 + \mu_1 U_1 + \mu_2 U_2,$$

where μ_1 and μ_2 are Langrange multipliers.

For an interior solution the following necessary conditions (Sage, 1968) are obtained, and it is assumed that they are also sufficient.[2]

[2]In the literature very little is said about sufficient conditions. Robson (1985) shows that the necessary conditions for a specific quasi-linear distributed control problem are also sufficient if the maximized Hamiltonian is concave in the state variable. As the thesis is primarily concerned with the presentation of a numerical solution technique, sufficient conditions are of minor importance.

$$
\begin{aligned}
\frac{\partial \mathcal{H}_1}{\partial u_1} &\equiv V_{1_{u_1}} e^{-rt} + \lambda(t,l) = 0, \;\; \forall t, \;\; \forall l && (4.4)\\
\frac{\partial \mathcal{H}_2}{\partial u_2} &\equiv -V_{2_{u_2}} e^{-rt} + \lambda_0(t) = 0, \;\; \forall t && (4.5)\\
\frac{\partial \mathcal{H}_2}{\partial \lambda_0} &\equiv u_2(t; l_0) - x(t, l_0) = 0, \;\; \forall t && (4.6)\\
-\frac{\partial \mathcal{H}_1}{\partial x} &= \frac{\partial \lambda(t,l)}{\partial t} + \frac{\partial\big(g(E,l)\lambda(t,l)\big)}{\partial l} && (4.7)\\
\frac{\partial x(t,l)}{\partial t} &= -\frac{\partial\big(g(E,l)\,x(t,l)\big)}{\partial l} - d(E,l)x(t,l) - u_1(t,l). && (4.8)
\end{aligned}
$$

The first necessary condition, equation (4.4), states that along the optimal path the discounted marginal net margin of the timber should equal the shadow price of the forest stock for every t and l, since the thinning variable is equivalent to a reduction of the forest density. In contrast to lumped optimal control, distributed optimal control requires that this equation holds along the optimal path not only with respect to time, but also with respect to diameter. Thus, the private owner maximizes his/her benefits not only over time but also over diameter at every instant of time. In other words, the private owner practices selective logging. Equation (4.5) states that the discounted marginal cost of planting trees with diameter l_0 should equal at very moment of time the discounted marginal benefits of planting this tree, e.g. the discounted marginal net benefits that accrue from time t to t_1. Hence, in correspondence with the first necessary condition the private owner also practices to some extent selective planting by choosing the time and the number of trees to be planted, however not their diameter. Equation (4.6) reproduces the constraint associated with $\lambda_0(t)$ and reflects the fact that the number of planted trees has to coincide with the stock variable at diameter l_0. Necessary condition (4.7) shows that the marginal change in the overall discounted net benefits of selective logging due to a decrease in the stock, captured by $-\frac{\partial \mathcal{H}_1}{\partial x}$, has to equal the marginal change in the shadow price with respect to time plus the marginal change of the product of the growth rate by the shadow price with respect to diameter. The last necessary condition is just a restatement of the law of motion, and therefore, it will not be discussed here.

4.4 The Numerical Approach

In practice, the necessary conditions, including a system of partial differential equations, can only be solved analytically under severe restrictions with respect to the specification of the mathematical problem, basically linearity of the state and control variables (Muzicant, 1980). Therefore, numerical methods have to be used to solve most distributed optimal control problems encountered in economics. The different available numerical techniques can be distinguished by their approximation approach.

The first type of approximation is based on Finite Differences. This method discretizes the diameter-domain[3] of the variables, Ω, into a number m of cells. In this way, the PDE governing the dynamics of the model results in a set of difference equations. In the optimal control problem, these difference equations will contain the control functions evaluated at the grid points, and the problem is to adjust these values so as to minimize the given criterion function while satisfying all the difference equations. This problem can be solved by linear or non-linear programming techniques. However, different discretization schemes produce distinct temporal and diameter-distributed pattern formation yielding different regions of stability (Gang and Zhaojun, 1994).

Another type of approximation, known as the method of Galerkin, consists of proposing specific functional forms for $x^*(t,l)$ and $\lambda^*(t,l)$ that approximate the unknown true functions. The parameters of the approximation function $x^*(t,l)$ and

[3]The discretization must be done in the domain of the distributed parameter, in this case, the diameter. In other problems it could be, for instance, age or space.

$\lambda^*(t,l)$ are the unknowns to be determined. The substitution of the approximation functions in equations (4.7) and (4.8) in comparison with the true functions, leads to an error term. Thus, the unknowns are chosen to minimize the weighted error over the entire domain of the state equations. Introducing the approximation functions into equations (4.7) and (4.8) and integrating each with respect to l eliminates this variable and allows to obtain a system of ordinary differential equations in t (Tzafestas, 1980). This method assumes implicitly that each approximation function is defined by a single expression which is valid throughout the entire domain Ω. A further development of the method of Garlekin, known as the Finite Elements Method divides the region Ω into a number of non-overlapping subdomains or elements Ω^e, and approximates the underlying function in a piecewise manner over each subdomain.

Alternatively, in order to reduce the numerical complexity, one can transform the distributed optimal control into a classic optimal control problem by transforming the independent variable, l, into a state variable of the system $l(t)$. The numerical approach is based on the work by de Roos (1988), who developed a special numerical method called the "Escalator Boxcar Train" for solving physiologically structured population models. The principal idea of this method is to group individuals into cohorts with a low heterogeneity between the trees of each cohort. In this way, each cohort can be characterized by the number of trees and their average diameter. By assumption, cohorts of trees stay together as isolated groups. The number of trees of a cohort diminishes only if trees die or are cut. The range $\Omega = [l_0, l_m)$ is divided into m small intervals $\Omega_i = [l_i, l_{i+1}), i = 1...m$, such that all trees with a diameter within the interval Ω_i can be considered approximately identical, and are presented by a single cohort with index i.

The total number of trees with a diameter within the interval Ω_i is given by

$$X_i(t) = \int_{l_i}^{l_{i+1}} x(t,l)\,dl, \tag{4.9}$$

and the average diameter of these individuals by

$$L_i(t) = \frac{1}{X_i} \int_{l_i}^{l_{i+1}} l\, x(t,l)\, dl. \tag{4.10}$$

The cohort with index i is assumed to be fully characterized by its number of trees, X_i, and average diameter of the trees, L_i. This assumption allows to approximate the density function $x(t,l)$ by a collection of cohorts with X_i trees and diameter L_i. Following de Roos, (1988), integrals over the entire range $[l_0, l_m)$ can be approximated up to second-order precision by the following sum:

$$\int_{l_0}^{l_m} \varphi(l)\, x(t,l)\, dl = \sum_{i=1}^{m} \varphi(L_i(t))\, X_i(t), \tag{4.11}$$

where $\varphi(l)$ is a measurable characteristic of the stand. Therefore, all stand characteristics of interest, such as the stand basal area or stand volume, can be computed as a weighted sum over the cohort characteristics. For example, the stand basal area can be approximated by $\sum_{i=1}^{m} \pi \left(\frac{L_i(t)}{2}\right)^2 X_i(t)$.

The variable $U_{1i}(t)$ is defined as the number of thinned trees in the interval $[l_i(t), l_{i+1}(t))$ at time t, that is

$$U_{1i}(t) = \int_{l_i(t)}^{l_{i+1}(t)} u_1(t,l)\, dl. \tag{4.12}$$

Once the diameter domain is divided, the dynamics of the population determines the evolution of the diameter distribution. Since the trees do not switch from one cohort to another, the expression $\partial g(E,l)\, x(t,l)/\partial l$ of equation (4.3) is zero over time and thus, the change of x with respect to time is exclusively given by $-d(E,l)x(t,l) - u_1(t,l)$. Hence, for a cohort i one has

$$\frac{dX_i}{dt} \;=\; -d(E,L_i)X_i - U_{1i}, \qquad\qquad i = 1, \cdots, m. \tag{4.13}$$

Although the trees of cohort i, i.e. where $\Omega_i \equiv [l_i, l_{i+1})$, do not switch over time to another cohort, the diameter of the trees is growing with time. Additionally, since the trees of cohort i are considered approximately identical, the diameter of each tree is given by the average diameter L_i of the cohort. Hence, to capture the growth effect of the diameter over time equation (4.2) has to be considered additionally. In terms of a cohort, this equation is given by

$$\frac{dL_i}{dt} = g(E, L_i), \qquad i = 1, \cdots, m. \tag{4.14}$$

As a result, the PDE governing the evolution of the forest can be decoupled into a system of $2m$ ordinary differential equations in $X_i(t)$ and $L_i(t)$.

It is only left to define the planting variable within this context. Planted density over time, $u_2(t; l_0)$, cannot be added only to the first cohort, since it would cause the range of Ω_1 to increase continuously over time. Consequently the average diameter of the cohort, L_1, would grow continuously over time and it would become a poor approximation of the diameter of the individual trees of this cohort. Hence, the approximation would break down. Therefore, new cohorts of individuals must be created at regular time intervals or time periods $\triangle t$, that consist of the planted trees within the range $[t, t + \triangle t)$. It is considered that planted trees within the range $[t, t + \triangle t)$, for a sufficiently small $\triangle t$, can be considered homogeneous and thus, belong to the same cohort. In this way, U_{2j} is defined as the number of planted trees from time $(j-1)\triangle t$ to $j\triangle t$, that is

$$U_{2j} = \int_{(j-1)\triangle t}^{j\triangle t} u_2(t; l_0)\, dl. \tag{4.15}$$

Thus, every time period, $\triangle t$, a new cohort j is set up. It evolves over time exactly like cohorts i that were established at the beginning of the planning horizon. Additionally, cohorts j have to comply with the boundary condition $X_{i+j}(t) = U_{2j}$.

Since the optimization problem presented in section 4.3 has an infinite time horizon, the numerical technique "Escalator Boxcar Train" would require an infinite number of variables, making its solution unfeasible. However, Getz and Haight

(1989) state that is possible to approximate an infinite time horizon problem by repeatedly solving a finite time horizon problem. Let t_1 denote the terminal point in time of a finite time horizon problem. The resulting value function of the resource at time t_1 corresponds to the net margin of the timber of the whole stand. Hence, for a sufficiently large planning horizon and a positive discount rate, the net margin of timber at the end of the planning horizon, defined by $\sum_{i=1}^{m} V_3(X_i(t_1),\, L_i(t_1))e^{-rt_1}$, does not affect the optimal solution of the key variables for the first periods, since the discounted value of the terminal stock is insignificant (Haight, Brodie, and Adams, 1985). Thus, the infinite time horizon problem can be broken apart into a sequence of overlapping finite horizon problems. The first finite horizon problem, for example where $t_1 = 200$, is solved and the optimal values, U_{1i} and U_2, in the initial time period are accepted as the optimal values of the infinite horizon problem of the first period $\triangle t$. Thereafter, the value of the state variables X_i and L_i for each cohort at time $\triangle t$ are taken as the starting value of the numerical solution of the second finite horizon problem, from $\triangle t$ to $t_1 + \triangle t$. The optimal values of the control variables, U_{1i} and U_2, in the initial time period $\triangle t$ of the second finite horizon problem are now taken as the optimal values of the infinite horizon problem of the period $2\triangle t$. This procedure is repeated $(t_1 + 100)/\triangle t$ times to obtain the optimal values of the control variables, U_{1i} and U_2, and of the state variables, X_i and L_i, over 300 years. Following this procedure, each finite time horizon problem has m initial cohorts and $n \equiv \frac{t_1}{\triangle t}$ additional or planted cohorts.

Given these transformations, the formal decision problem can be restated as:

$$\begin{aligned}\max_{U_{1i}(t),U_{2j}} \quad & \int_0^{t_1} V_1(\bar{X}(t), \bar{U}_1(t), \bar{L}(t))e^{-rt}\, dt \\ & - \sum_{j=1}^{n} V_2(U_{2j})\, e^{-rj\triangle t}\, dt \,+\, V_3(\bar{X}(t_1),\, \bar{L}(t_1))e^{-rt_1},\end{aligned} \tag{P$'$}$$

subject to the constraints

$$
\begin{aligned}
\frac{dX_i(t)}{dt} &= -d(E, L_i)X_i(t) - U_{1i}(t) && i = 1, \cdots, m+n, \\
& X_i(t_0) = X_{0i} && i = 1, \cdots, m, \\
& X_{i+j}(t) = U_{2j}(t) && i + j = m+1, \cdots, m+n, \\
\frac{dL_i(t)}{dt} &= g(E, L_i) && i = 1, \cdots, m+n, \\
& L_i(t_0) = L_{0i} && i = 1, \cdots, m, \\
& L_{i+j}(t) = L_{0j}(t) && i + j = m+1, \cdots, m+n, \\
& U_{1i} \in U_1, \quad U_{2j} \in U_2,
\end{aligned}
$$

where $\bar{X}$, $\bar{U}_1$, and $\bar{L}$ denote the vectors $\bar{X} = (X_1, \cdots, X_{m+n})$, $\bar{U}_1 = (U_{11}, \cdots, U_{1\,m+n})$, and $\bar{L} = (L_1, \cdots, L_{m+n})$ respectively.

4.5 Empirical Study

The purpose of the empirical analysis is to determine the optimal selective-logging regime of a diameter-distributed forest, that is, the selective logging that maximizes the discounted net benefits from timber production of a stand of *pinus sylvestris* (Scots pine), over an infinite time horizon, and compare the selective logging with the clear-cutting regime.

4.5.1 Data and Specification of Functions

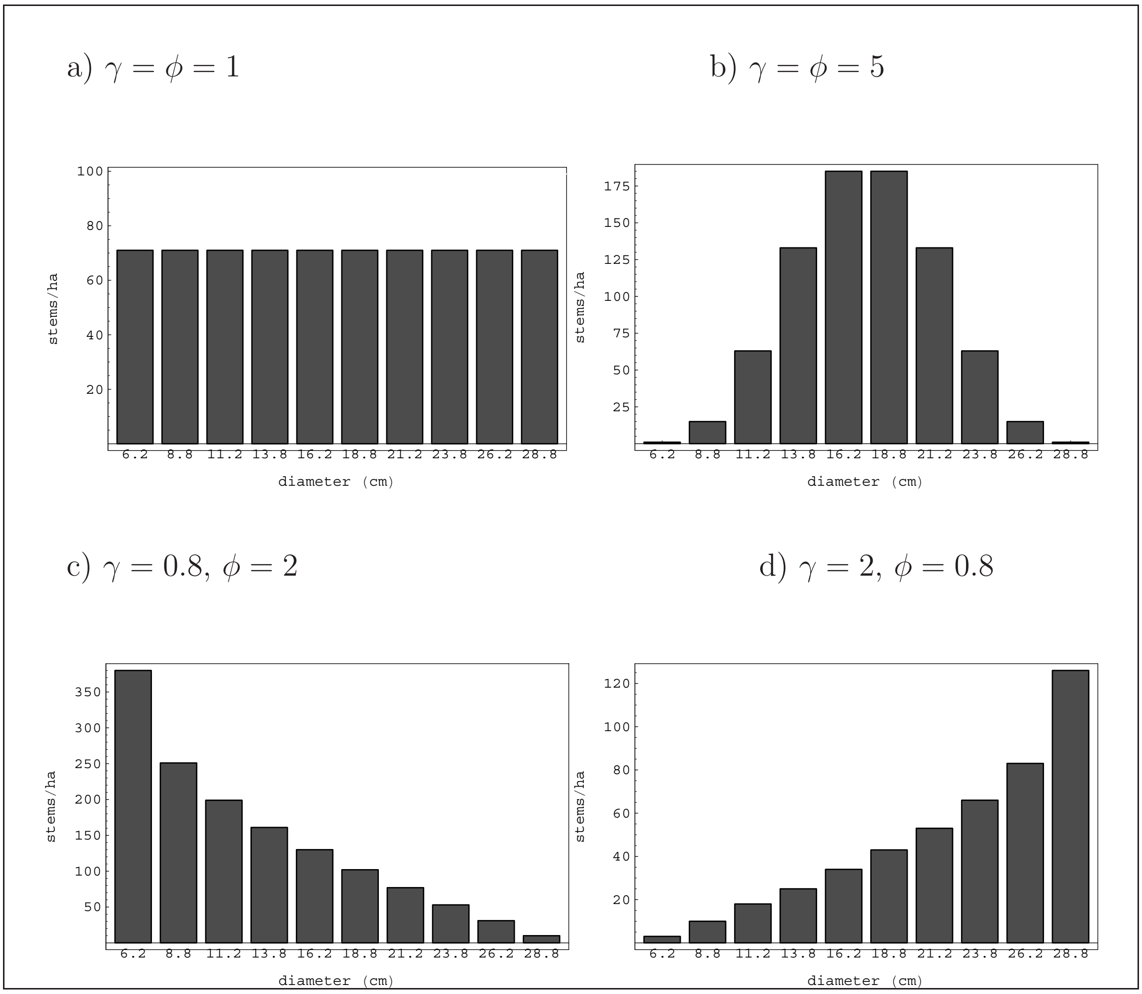
For given specifications of the economic and biophysical functions of the model, and a given initial diameter distribution of the trees, X_{0i}, a numerical solution of the decision problem (P′) can be obtained. To proceed with the empirical study, different initial diameter distributions of a forest have been chosen, specified on the base of a transformed beta density function $h(l)$ with shape parameters γ and ϕ (Mendenhall, Wackerly and Scheaffer, 1990). To be more concrete, the initial forest consists of a population of trees over the interval 5 cm $\leq l \leq$ 30 cm. The distribution of the diameter of the trees is given by:

$$h(l;\,\gamma,\,\phi) = \begin{cases} \dfrac{1}{25}\dfrac{\Gamma(\gamma+\phi)}{\Gamma(\gamma)\Gamma(\phi)}\left(\dfrac{l}{25}\right)^{\gamma-1}\left(1-\dfrac{l}{25}\right)^{\phi-1}, & \gamma,\phi > 0;\ 5 \leq l \leq 30, \\ 0, & \text{elsewhere}, \end{cases} \tag{4.16}$$

where $h(l;\,\gamma,\,\phi)$ denotes the density function of the diameter of trees. Thus, the integral $\int_{l_i}^{l_{i+1}} h(l;\gamma,\,\phi)\,dl$ gives the proportion of trees lying within the range $[l_i, l_{i+1})$. The beta density function is utilized because it is defined over a closed interval and allows to define a great variety of distinct shapes of the initial distributions of the diameter of the trees. The interval [5, 30] is divided into 10 initial subintervals of identical length. In this way, each cohort comprises trees that differ in their diameter by a maximum of 2.5 cm, and thus, they can be considered homogeneous. The initial number of trees in each cohort, X_{i0}, $i = 1, \cdots, m$, is calculated in such a way that the basal area of the stand is constant (20 m^2/ha) in order to allow for comparisons between the results of the different optimization outcomes. Figure 4.1, cases a-d), presents four different initial distributions obtained by varying the parameters γ and ϕ of the beta density function.

The function $V_1(\bar{X}(t), \bar{U}_1(t), \bar{L}(t))$ accounts for the net margin of the timber at time t, and is defined as: $\left[\sum_{i=0}^{m+n}\left(p(L_i)\,V(L_i)\,WP(L_i) - C_1\right)U_{1i}(t)\right] - [C_2(X(t))]$. The first term in square brackets denotes the sum of the revenue of the timber sale minus the cutting costs of each cohort i, and second term $C_2(X(t))$, accounts for the maintenance costs. The parameter $p(L_i)$ denotes timber price per cubic meter of wood as a function of the diameter, $VT(L_i)$ the total volume of a tree as a function of its diameter, $VM(L_i)$ the part of the total volume of the tree that is marketable, and C_1 cutting costs.

Figure 4.1: Types of Initial Distributions of the Diameter of the Trees



Data about costs and prices was provided by Tecnosylva.[4] This data shows that timber price per cubic meter is an increasing function of the diameter of the tree, but its second derivative is negative. Thus, a quadratic price function was estimated, given by $p(L) = -23.02 + 4.35L - 0.049L^2$. Thinning costs comprise logging and pruning, and the costs of collecting and removing the residues. These costs are set equal to €0.60/stem. It is assumed that maintenance cost is an increasing function

[4]Tecnolylva is a private firm, and it elaborates forest management plans in many places of the national territory.

of the number of stems per hectare, and it is given by $C_2(X) = 0.07X + 1.18\,10^{-5}X^2$. Finally, planting costs, $V_2(U_{2j})$ are assumed to be linear in the amount of planted trees. The planting cost per tree is denoted by C_3, and it is set equal to €0.60.

The value of the parameters of tree volume, $VT(L_i)$, and the marketable part of the tree volume, $VM(L_i)$, are estimated using the information in the study by Cañellas, Martinez, and Montero (2000). The tree volume is assumed to follow the allometric relation $VT(L) = 0.0002949L^{2.167563}$, and it is assumed that the marketable part of the volume of timber in each tree is an increasing function of the diameter, given by $VM(L) = 0.699 + 0.000411L$. As a tree specie *pinus sylvestris* has been chosen since it occupies most of the catalan forest territory. The thinning and planting period, $\triangle t$, is set equal to 10 years, which is a common practice for a *pinus sylvestris* forest (Cañellas, Martinez and Montero, 2000).

To determine the dynamics of the forest, the growth of a diameter-distributed stand of *pinus sylvestris* without thinning was simulated with the bio-physical simulation model GOTILWA (Growth Of Trees Is Limited by Water).[5] The model simulates the biophysical growth processes and allows to explore how these processes are influenced by the climate, the trees itself and the composition of the stand. The model is defined by 11 input files specifying more than 90 parameters related to site, soil composition, tree species, photosynthesis, stomatal conductance, composition of the forest, canopy hydrology and to climate. To generate a wide variety of possible initial distributions, 103 pairs of γ and ϕ were used, where the values of γ and ϕ are taken from the set M, M=$\{$ 0, 0.2, 0.4, 0.6, 0.8, 1, 2, 4, 6, 8, 10$\}$.[6] Fourteen of the 103 simulations were rejected because their stand density (stems per ha) was too high.

[5]This program has been developed by C. Gracia and S. Sabaté at the Department of Ecology, University of Barcelona, and CREAF (Centre de Recerca Ecolgica i Aplicacions Forestals), Autonomous University of Barcelona respectively.

[6]The set M allows to generate 121 possible pairs of γ and ϕ, but some of them give rise to equivalent initial distributions.

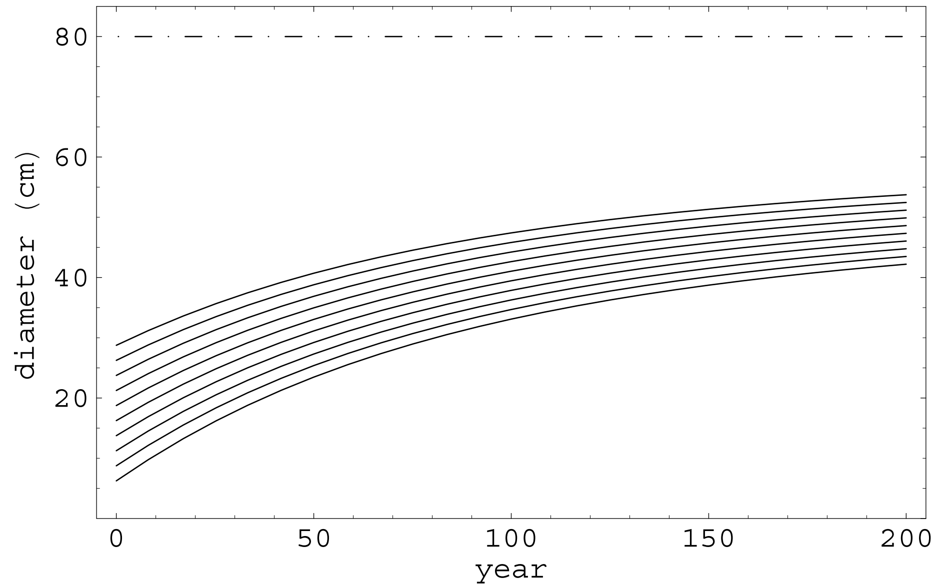
The results of the simulation were utilized to estimate the function $g(E, L_i)$, which describes the rate of diameter change. It was specified a von Bertalanffy growth curve (von Bertalanffy, 1957), generalized by Millar and Myers (1990) by allowing the rate of diameter growth to vary with some environmental variables. Thus, the function $g(E, L_i) = (l_m - L_i)(\beta_0 - \beta_1\, BA)$ is estimated by OLS, where β_0 and β_1 are proportionality constants, and BA is the stand basal area. The estimation yielded the following growth function: $g(E, L) = (80 - L)(0.00702 - 0.000043\, BA)$. Other functional forms of $g(E, L)$ were evaluated as well, but explained the observed variables to a lesser degree. Figure 4.2 shows the evolution of the diameter, L_i of each cohort i, $i = 1, \cdots, m$, over time for the initial distribution of the diameter of the trees as shown in Figure 4.1, case a), in the case of a non-managed forest. Although the potential for the maximum diameter is 80 cm, the simulated diameter seems to reach a maximum at about 50 cm. This is due to the fact that the increase of the stand basal area causes a decline in the rate of diameter change, slowing the growth process. In this model, the maximum basal area that the simulated forest can support is 163.25 m^2/ha.

As Gotilwa only simulates the survival or death of an entire cohort but not of an individual tree, it was not possible to obtain an adequate estimation of the function $d(E, L_i)$ describing the mortality of the forest. Nevertheless, the information obtained from Tecnosylva suggests that in a managed forest, the mortality rate can be considered almost constant over time and independent of the diameter. Thus, according to the data supplied by Tecnosylva, $d(E, L_i)$ was chosen to be constant over time and equal to 0.01 for each cohort.

4.5.2 Optimization Results

The optimizations were carried out with the Conopt2 solver that is available in the optimization package GAMS (General Algebraic Modelling System) (Brooke,

Figure 4.2: Evolution of the Diameter Distribution of Each Cohort over Time in a Non-Managed Forest



Kendrick and Meeraus, 1992). The programming code can be obtained from the author upon request.

For a given initial distribution, the numerical solution of the decision problem determines the optimal thinning, U_{1i}, and planting, U_{2j}, at every 10 year period, the optimal values of the state variables, X_i and L_i, and in consequence of the economic variables, such as the revenue from timber sale, cutting costs, planting costs, and maintenance costs, in each period. Various optimizations with different random initializations of the control variables were carried to assure that the numerical method provides robust solutions, i.e., solutions that are independent of the initially chosen values to start the numerical optimization.

All the optimizations were carried out on a per-hectare basis. Given the initial

Table 4.1: Optimal Selective-Logging Regime (where the initial diameter distribution is determined by $\gamma = \phi = 1$)

Year	Number of trees[(a)]	Planted trees	Thinning					Maintenance costs (€/ha)	Planting costs (€/ha)	Net benefit (€/ha)	Discounted net benefit (€/ha)
			Thinned trees	BA (m^2/ha)	Volume (m^3/ha)	Timber (m^3/ha)	Revenue - thinning costs (€/ha)				
0	710	0	0	0	0	0	0	566.32	0	-566.32	-566.32
10	1089	386	141	10.44	69.64	49.61	3129.43	783.31	232.17	2113.95	1730.76
20	1079	140	123	8.42	55.78	39.71	2432.37	790.33	83.90	1558.13	1044.45
30	1080	133	95	6.55	43.40	30.89	1893.86	817.27	80.08	996.51	546.90
40	1130	155	127	8.67	57.40	40.86	2499.19	834.75	93.13	1571.31	706.03
50	1155	161	136	8.96	59.15	42.08	2532.93	849.00	97.03	1586.90	583.79
60	1183	175	177	10.35	67.71	48.13	2764.79	837.60	105.06	1822.14	548.82
70	1207	211	254	16.98	112.21	79.84	4839.31	787.79	126.82	3924.70	967.82
80	1093	149	131	8.77	57.91	41.21	2493.57	795.60	89.72	1608.25	324.70
90	1103	151	125	8.33	55.01	39.14	2364.86	810.35	90.57	1463.94	241.99
100	1131	163	146	9.67	63.85	45.43	2742.45	817.73	98.25	1826.48	247.19
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
200	1124	161	148	9.80	64.69	46.03	2780.72	809.90	97.05	1873.76	34.32
300	1138	168	159	10.56	69.75	49.63	2998.18	812.44	101.01	2084.74	5.17

[(a)] The number of trees in the forest is calculated just after the trees are planted, and before the thinning takes place.

diameter distribution of the trees in Figure 4.1, case a), Table 4.1 summarizes the results of the optimization, that are based on calculations with a rate of discount of 2%. The discount rate corresponds to the loan rate minus the inflation rate. It is shown that, with a discount rate of 2%, the first thinning is delayed until the end of the first 10 year period, since the initial amount of trees is smaller than the number of trees in the long-run distribution. Consequently, the forest owner has to wait 10 years to obtain the first benefits from the forest, supporting a total of maintenance cost of €566 per hectare in the first 10 year period. The optimal management is also characterized by a large plantation at the beginning of the time horizon.[7] It can be observed that all economic, as well as biophysical variables, show a cyclical pattern over time. However, this cyclical pattern decreases as time goes by. In the long-run, the forest consists of approximately 1138 trees, and approximately 159 of these trees are cut each 10 year period. The thinned volume is 70 m^3, corresponding to a timber volume of 50 m^3. This amount of timber generates a current revenue from timber sale net of thinning costs, of about €3000 per hectare. The current net benefits of the forest in the long-run are over about €2000. The total sum of discounted net benefits of the forest over 300 years are €7634 per hectare.

Figure 4.3 a-f), depicts the number of trees in each cohort together with their corresponding average diameter, at different years of the planning horizon, to illustrate the optimal evolution of the forest over time.[8] Each bar account for one cohort, the grey bars present the optimal thinning in each 10 year period, while the dark bars stand for the number of trees that remain in the stand after thinning. Figure 4.3 shows that it takes more than 100 years to reach a diameter distribution of the trees which is relatively stable over time.

[7]The process of plantation is assumed to start not before the end of the first 10 year period.

[8]In order to simplify the analysis, the initial average diameter of each cohort is set to the mid-value of class instead of the weighted average diameter. Since the intervals have a wide of 2.5 cm, the simulated data shows that the difference between the simple and weighted average is not significative.

Figure 4.3: Evolution of the Optimal Diameter Distribution where $\delta = 0.02$

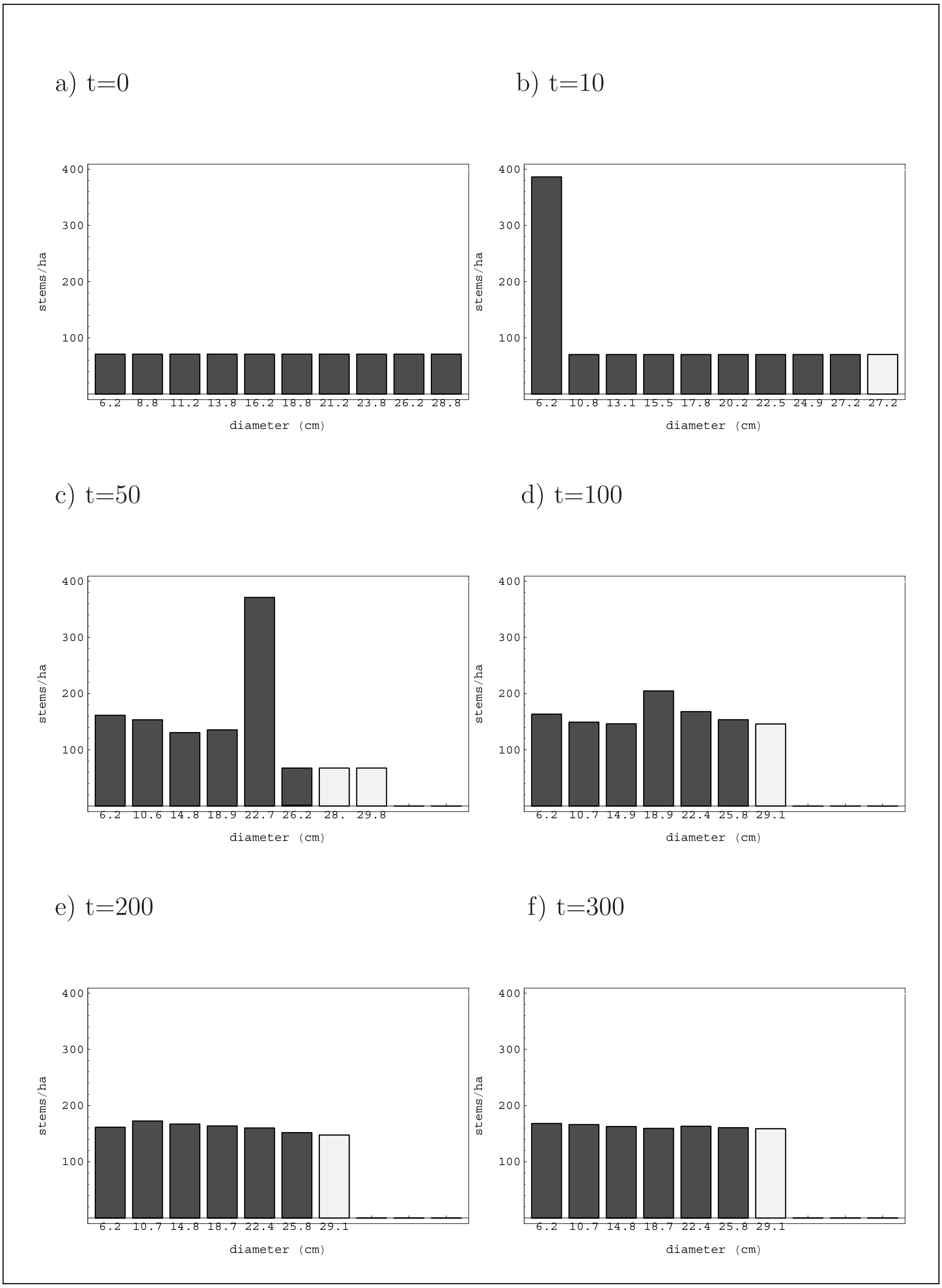


Table 4.2: Conversion From an Even-Diameter Forest into an Uneven-Diameter Forest

Year	Number of trees$^{(a)}$	Planted trees	Thinning: Thinned trees	Thinning: BA (m^2/ha)	Thinning: Volume (m^3/ha)	Thinning: Timber (m^3/ha)	Thinning: Revenue - thinning costs (€/ha)	Maintenance costs (€/ha)	Planting costs (€/ha)	Net benefit (€/ha)	Discounted net benefit (€/ha)
0	1137	0	0	0	0	0	0	964.11	0	-964.11	-964.11
10	1126	0	0	0	0	0	0	952.96	0	-952.96	-780.22
20	1114	0	0	0	0	0	0	941.95	0	-941.95	-631.41
30	1103	0	0	0	0	0	0	931.08	0	-931.08	-510.99
40	1280	188	432	18.05	114.66	81.29	3920.36	689.99	112.87	3117.50	1400.78
50	1144	304	395	21.57	140.12	99.54	5521.35	600.62	182.99	4737.74	1742.92
60	1039	297	256	17.77	117.80	83.86	5156.91	631.18	178.74	4346.99	1309.29
70	775	0	0	0	0	0	0	624.15	0	-624.15	-153.91
80	811	44	0	0	0	0	0	656.80	26.44	-683.24	-137.94
90	997	194	179	9.94	64.69	45.96	2572.42	663.21	116.47	1792.73	296.34
100	1053	243	290	16.29	106.13	75.41	4242.30	613.80	145.95	3482.55	471.31
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
200	995	186	196	11.03	71.82	51.03	2873.46	645.78	111.55	2116.13	38.76
300	970	170	166	9.37	61.04	43.37	2441.02	650.01	101.90	1689.10	4.19

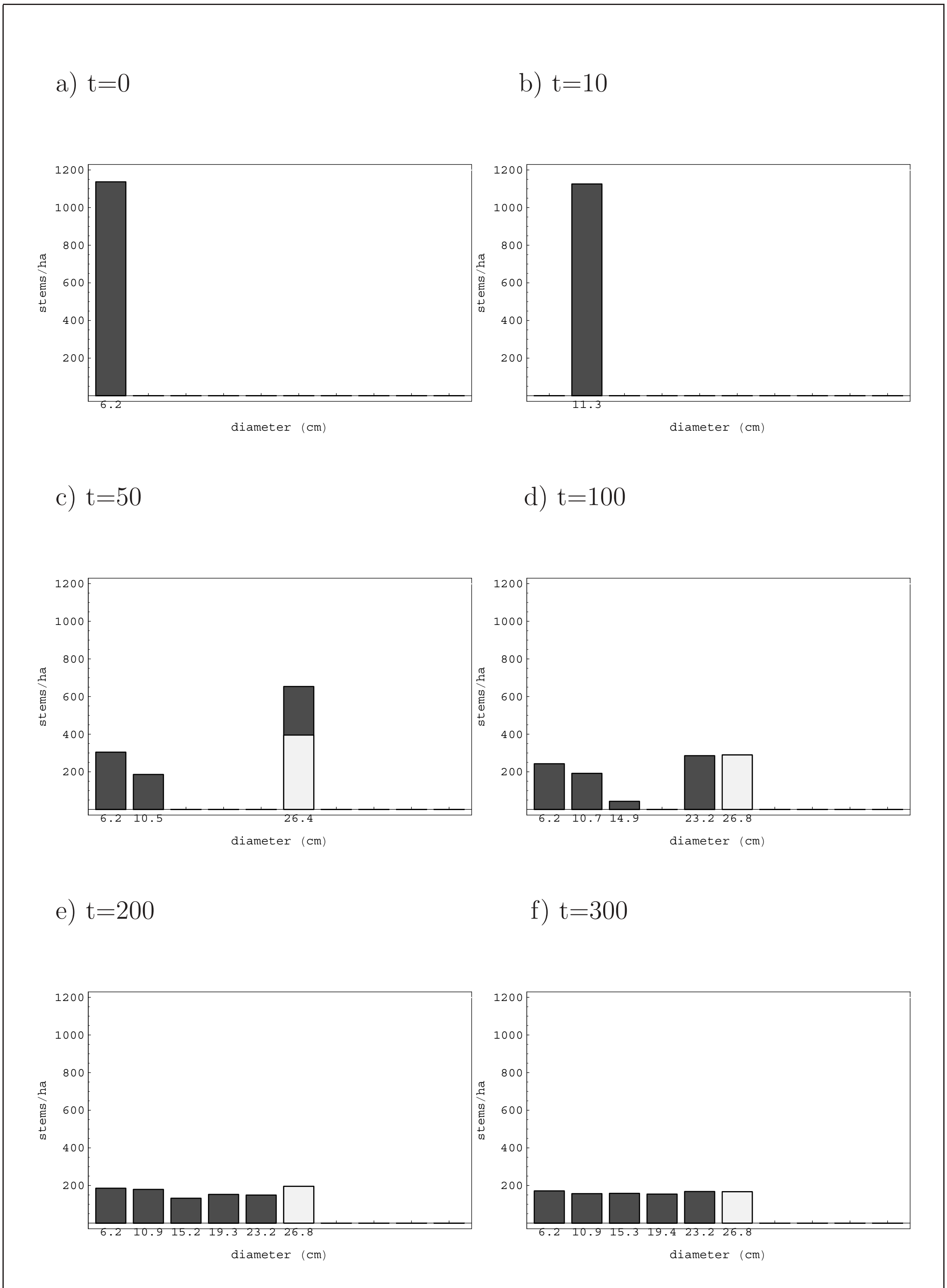
In the cases where forests were managed following a clear-cutting regime, it is necessary to transform an even-diameter forest (where all trees have the same diameter) into an uneven-diameter forest. Table 4.2 presents the results of the optimal management regime of the conversion of an even-diameter forest into an uneven-diameter forest, where the entire initial forest consists of 1138 trees with a diameter of 6.25 cm. Table 4.2 shows that it is optimal to let the forest grow without any intervention until the 40$^{\text{th}}$ year. At this point in time, 432 trees are thinned, and 188 new trees are planted. In the long-run, the number of trees decreases slightly compared to the initial forest density. The total sum of discounted net benefits of the forest is €2941/ha. The evolution of the diameter distribution of the trees is depicted in Figure 4.4, where it can be observed that 200 years are necessary to achieve a diameter distribution of the trees which is stable over time.

Alternatively to adopting a selective-logging management, the forest owner could continue with the clear-cutting regime, that is, thinning the entire stand at regular time periods. In this case, the optimal rotation period can be calculated directly via the Faustmann formula. However, in order to allow comparing the different outcomes, the Faustmann model needs to be adapted to the formulation given in this chapter. Additionally, it is necessary to assume that, contrary to the Faustmann model, no trees are planted at time 0 and the initial diameter distribution of the trees is valid for the Faustmann model and for the selective-logging model, i.e., 1138 trees, with a diameter of 6.25 cm. The Faustmann model maximizes the net present value of the perpetual returns from the forest. Its solution is given by the solution of the following optimization problem:

$$\max_{T} \quad \frac{F(T)}{1 - e^{(-\delta T)}}, \tag{F}$$

where $F(T)$ denotes the discounted net benefits obtained from cutting the entire stand at time T, i.e., the rotation period. In this model, $F(T)$ accounts for the

Figure 4.4: Transformation from an Even-Diameter Forest into an Uneven-Diameter Forest



revenue from the timber sale minus thinning, maintenance[9] and planting costs, that is:

$$
\begin{aligned}
F(T) \;=\; & \Big(p(L(T))\, VT(L(T))\, VM(L(T)) - C_1\Big)\, X\, e^{-\delta\, T} \\
& - \frac{C_2(X) e^{-\delta T} - C_2(X)}{e^{-10\delta} - 1} - C_3 X\, e^{-\delta\, T}
\end{aligned}
$$

The resulting optimal rotation length is 62 years. In must be noted that trees are planted with a diameter of 6.25 cm and, thus, they are approximately 15 years old. The total sum of the discounted net benefits obtained from a clear-cutting regime are about €1800/ha. Therefore, using the same parameter values for the clear-cutting and the selective-logging regime, the clear-cutting regime generates lower discounted net benefits than the selective-logging regime. That is, the rigidity of clear felling, given by the requirement of cutting the whole stand instead of only a proportion of it at different time periods, together with the unfeasibility of choosing the plantation levels, causes a loss of the clear-cutting regime of approximately 38% compared to the selective-logging regime.

The benefits of clear felling could only be superior to selective logging if timber prices increase with the amount of timber offered. A clear-cutting regime allows to offer a large amount of timber planks of a particular size. Consequently, the obtained timber prices per m^3 may be higher than with selective logging. While this is true for the case of a small forest area, it may not be true for the case of a large forest area, where the volume obtained from selective logging may be sufficiently large to achieve high timber prices per m^3.

[9]In selective logging, forest owner incurs in maintenance costs every 10 year period. In order to account for these costs adequately in the Faustmann model, the maintenance costs cannot be added simply at time T since they incur every 10 years. Thus, the correct maintenance cost that is added as a single payment to the Faustmann model is calculated as the sum of discounted periodic payments of the maintenance costs.

4.5.3 Effects of a Change in the Rate of Discount on the Optimal Selective-Logging Regime

In this section a sensitivity analysis is presented in order to evaluate how the optimal management of the forest changes as a result of a variation in the discount rate. The results of the optimizations with discount rates of 1.5, 2.5 and 3% are presented in Figures 4.5, 4.6, and 4.7 respectively. Figures 4.5, 4.6, and 4.7 show that the discount rate has a significant influence on the chosen optimal selective-logging regime. When the discount rate increases from 1.5% to 3%, it is optimal to decrease the amount of trees that the forest sustains in the long-run, from 1424 to 669, and the average diameter at which the trees are cut decreases from 28.5 cm to 27 cm. Therefore, with a discount rate of 3% the trees are cut earlier than with a rate of discount of 1.5%. In a parallel manner, the amount of planted trees in the initial period is also influenced by the rate of discount. Thus, an increase in the discount rate from 1.5% to 3% causes the optimal number of planted trees to decrease by 84% in the first 10 year period.

4.5.4 Effects of a Change in the Initial Diameter Distribution of the Trees on the Selective-Logging Regime

To illustrate how the initial diameter distribution of the trees alter the choice of the optimal selective-logging regime, problem (P') is solved additionally for the initial distributions of the diameter of the trees shown in Figure 4.1, cases b-d). The results of the optimizations with a rate of discount of 3% are presented in Figures 4.8, 4.9, and 4.10. The underlying initial distribution in Figure 4.8 corresponds to the pattern of the initial distribution presented in Figure 4.1, case b). Likewise, the underlying initial distributions in Figures 4.9, and 4.10 correspond to the patterns shown in 4.1, cases c) and d) respectively.

Figure 4.5: Evolution of the Optimal Diameter Distribution where $\delta = 0.015$

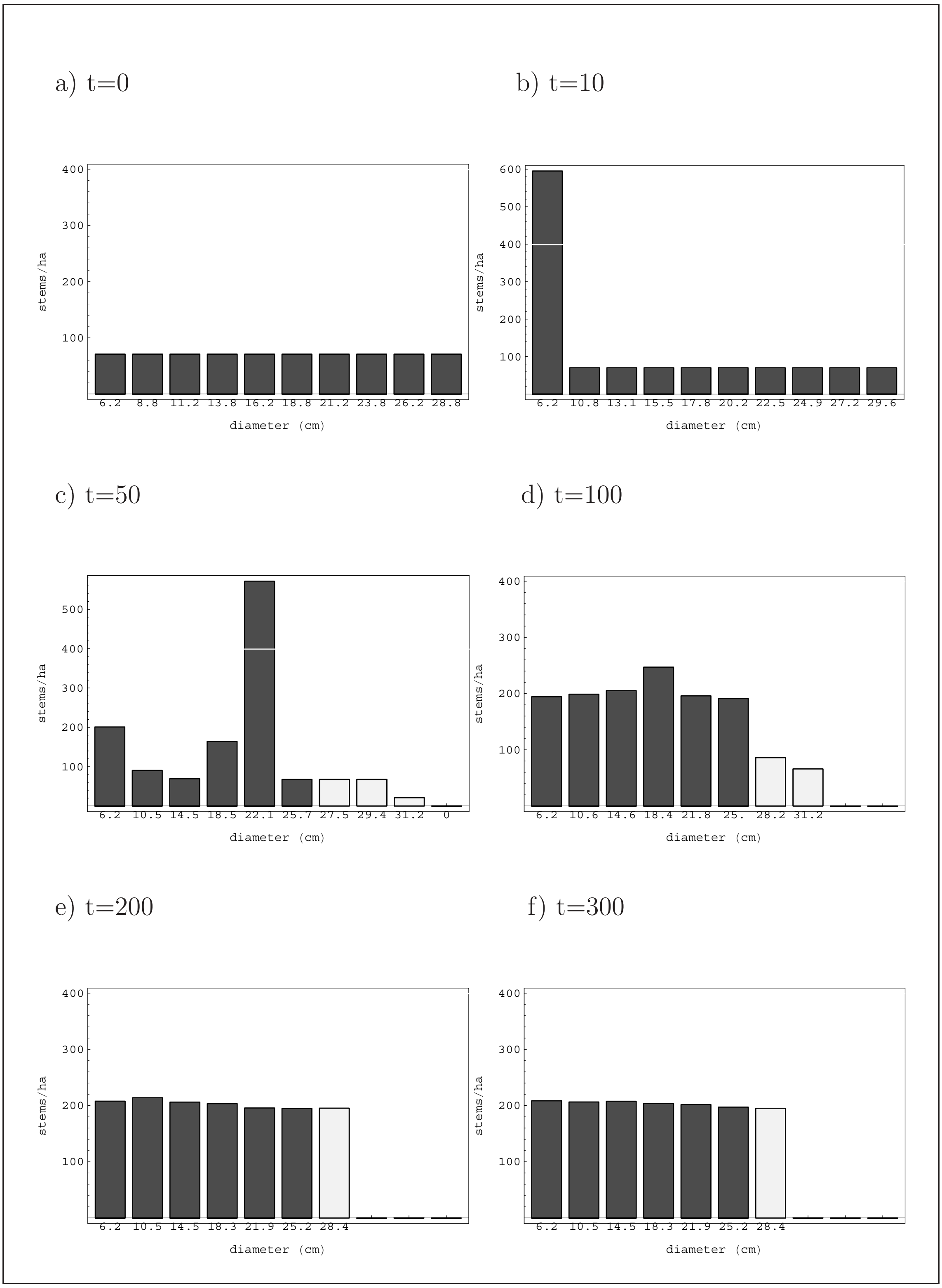


Figure 4.6: Evolution of the Optimal Diameter Distribution where $\delta = 0.025$

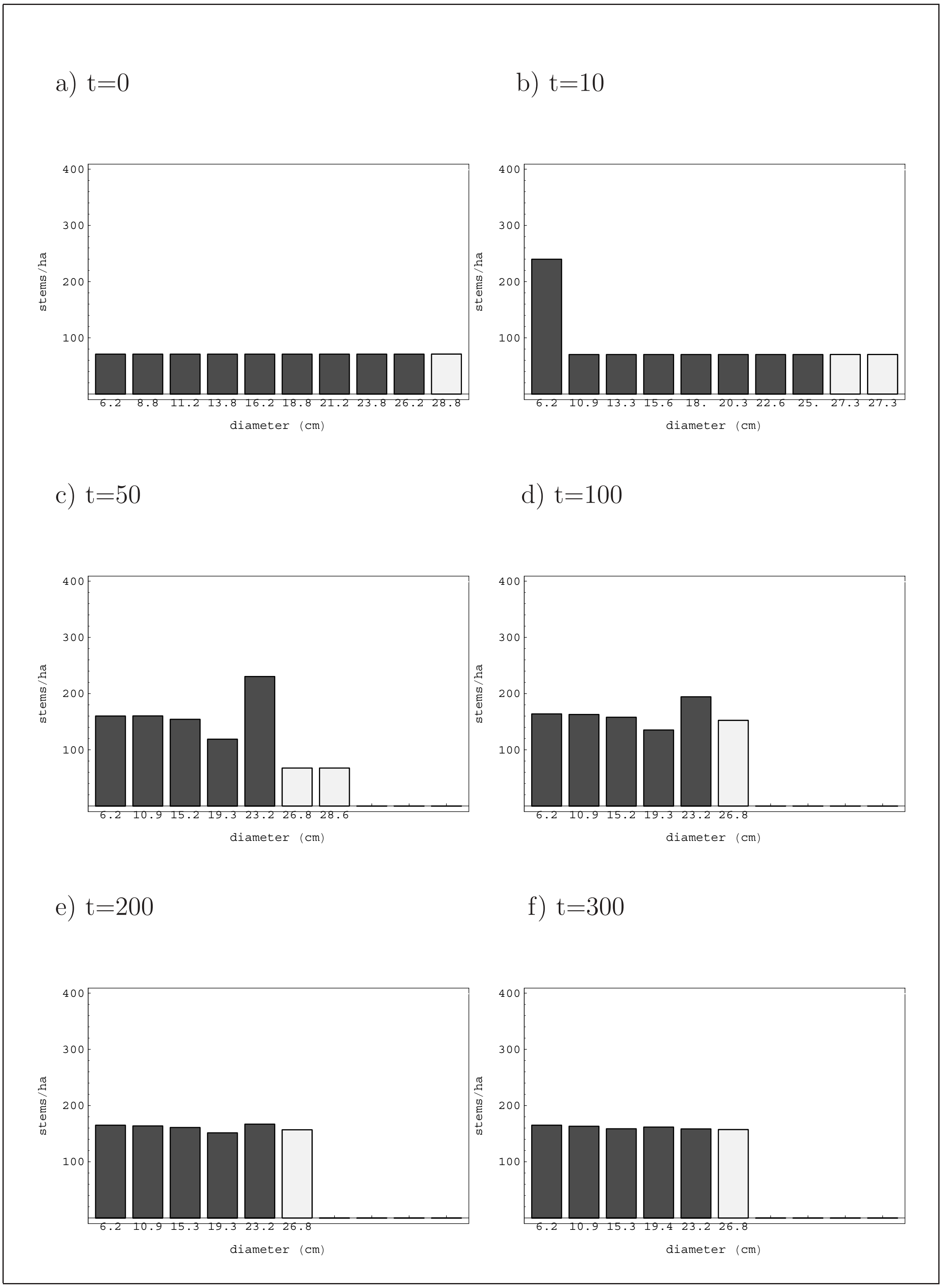


Figure 4.7: Evolution of the Optimal Diameter Distribution where $\delta = 0.03$

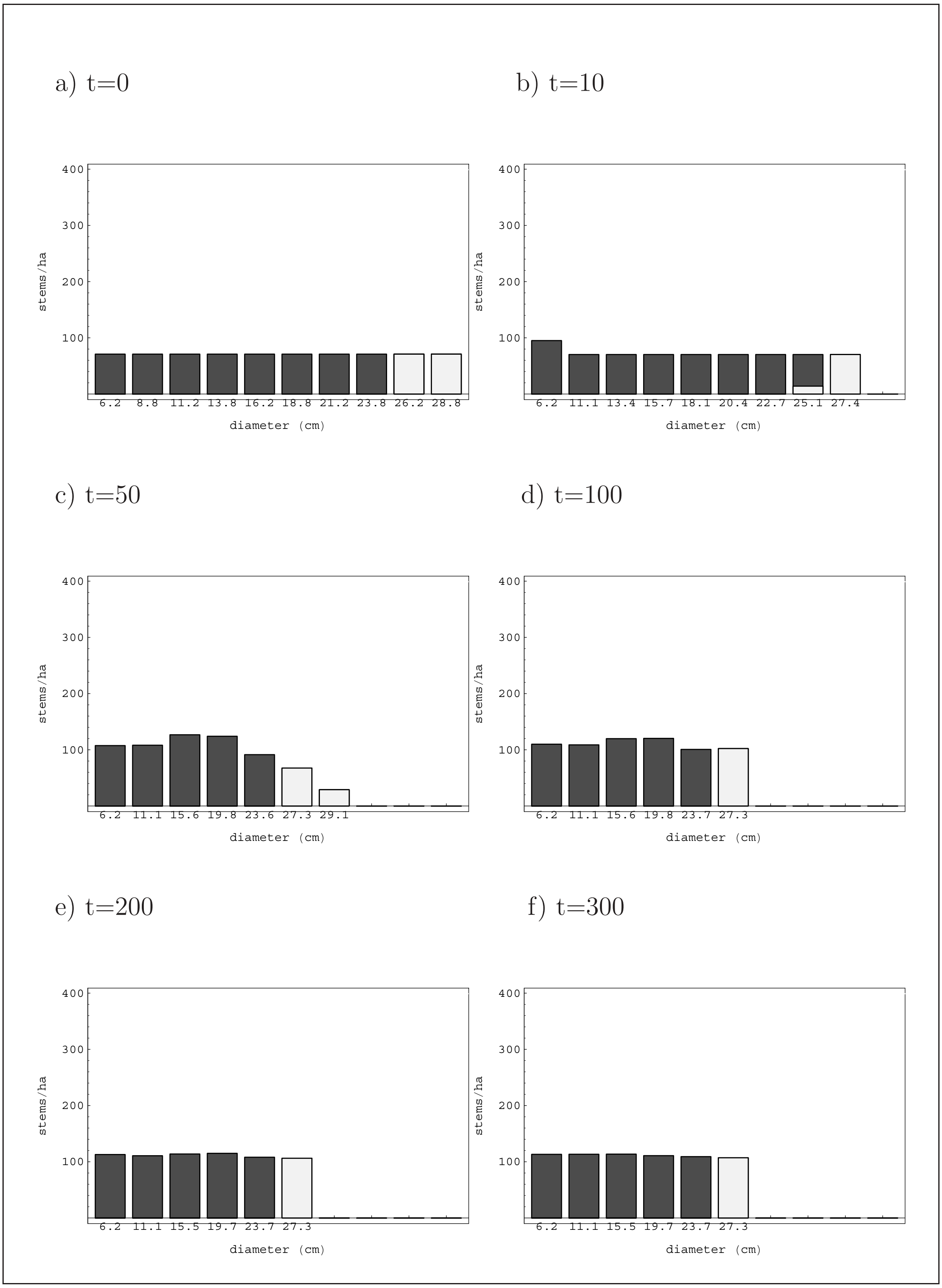


Figure 4.8: Evolution of the Optimal Diameter Distribution where $\gamma = \phi = 5$

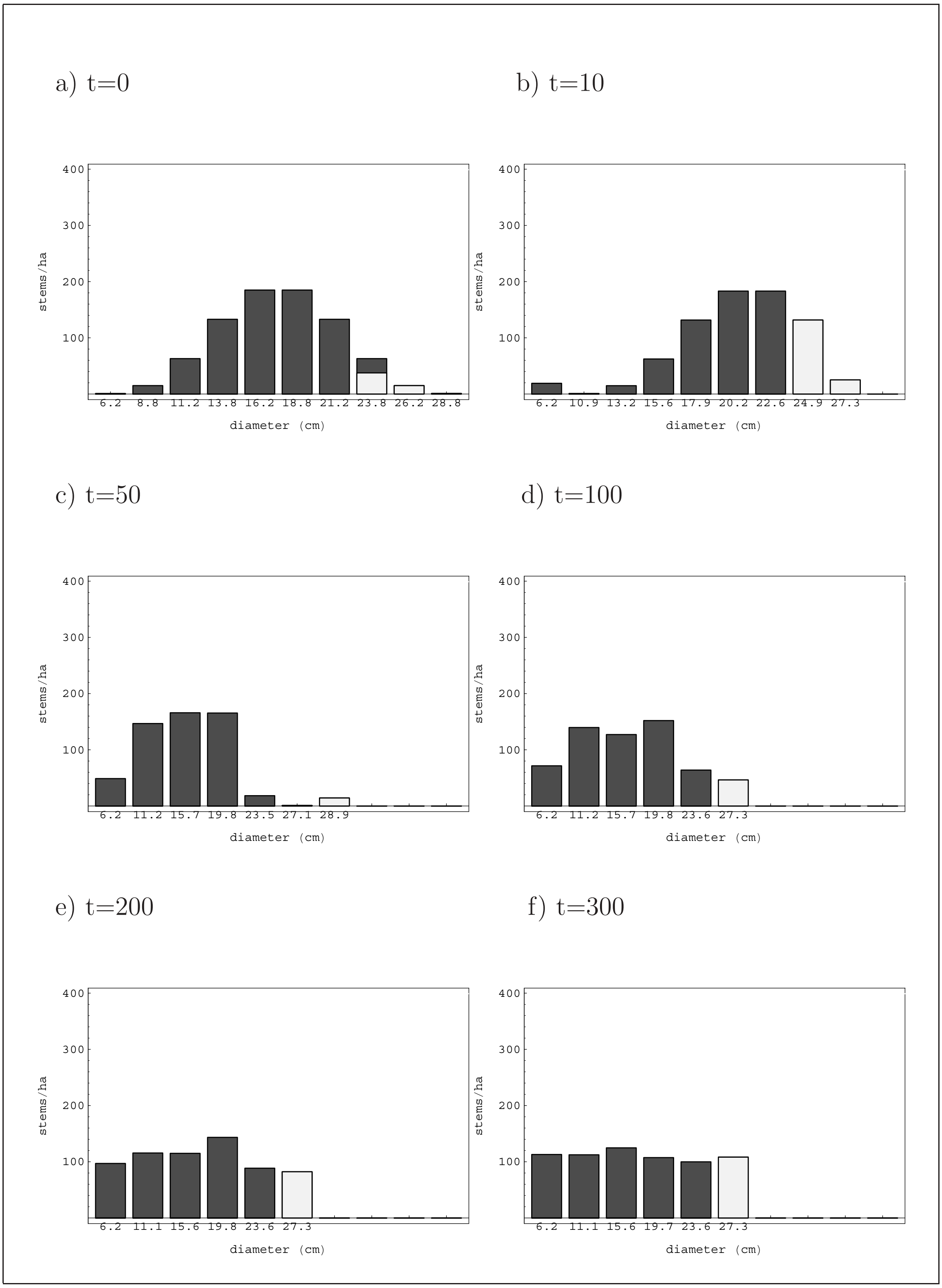


Figure 4.9: Evolution of the Optimal Diameter Distribution where $\gamma = 0.8$, $\phi = 2$

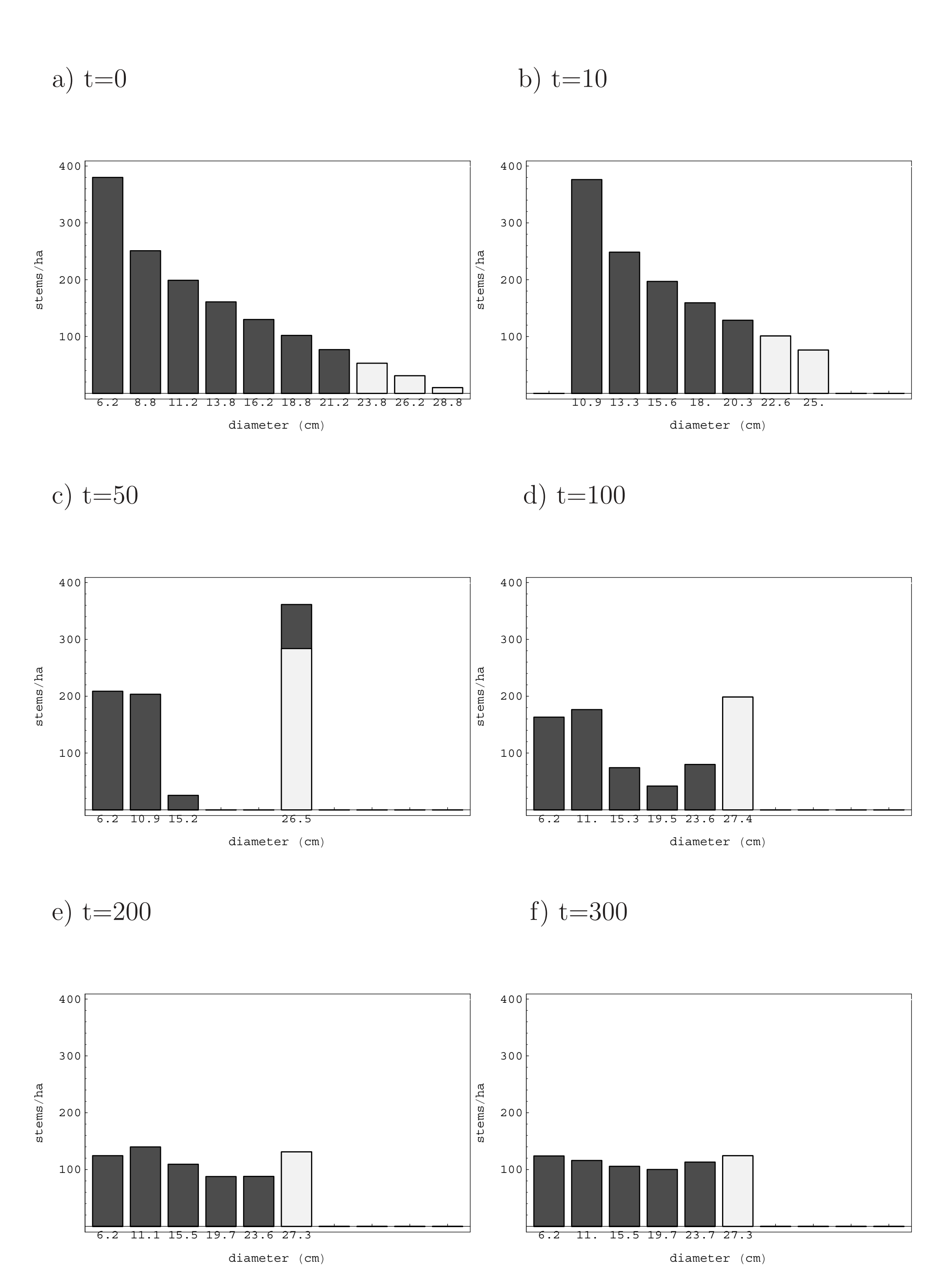
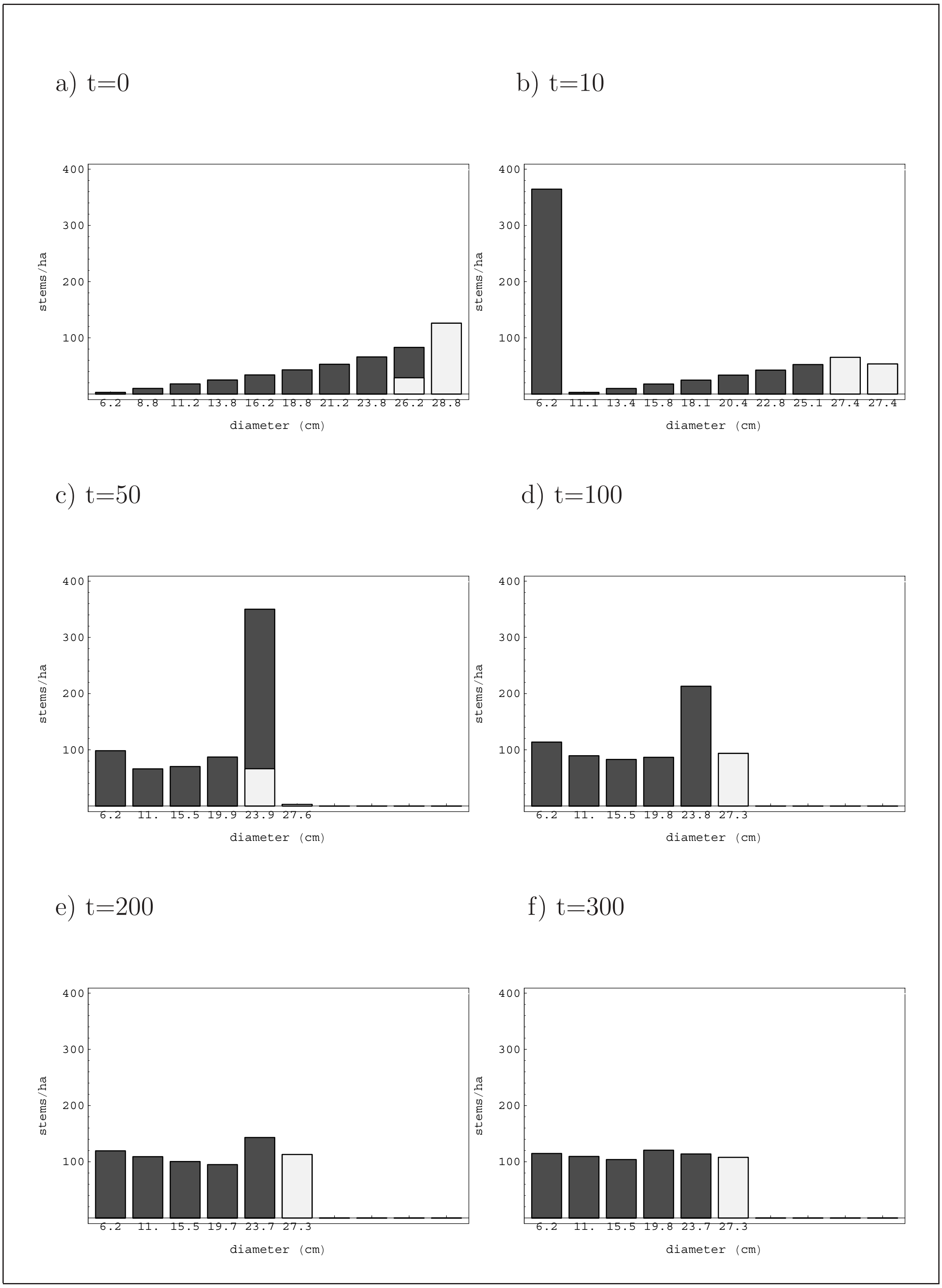


Figure 4.10: Evolution of the Optimal Diameter Distribution where $\gamma = 2$, $\phi = 0.8$



A comparison of the Figures 4.7, 4.8, 4.9, and 4.10, shows that in the long-run, the optimal forest structure tends to a uniform distribution of the diameter of the trees. However, it takes more than 200 years to reach a steady-state distribution. One can observe that forests with a large proportion of big trees (Figure 4.10) require a large number of newly planted trees in the first 10 year period, in order to reach the optimal structure in the long-run. In Figure 4.8, however, one can observe that there is no plantation in the first 10 year period, since the share of small trees is higher than in the steady-state distribution.

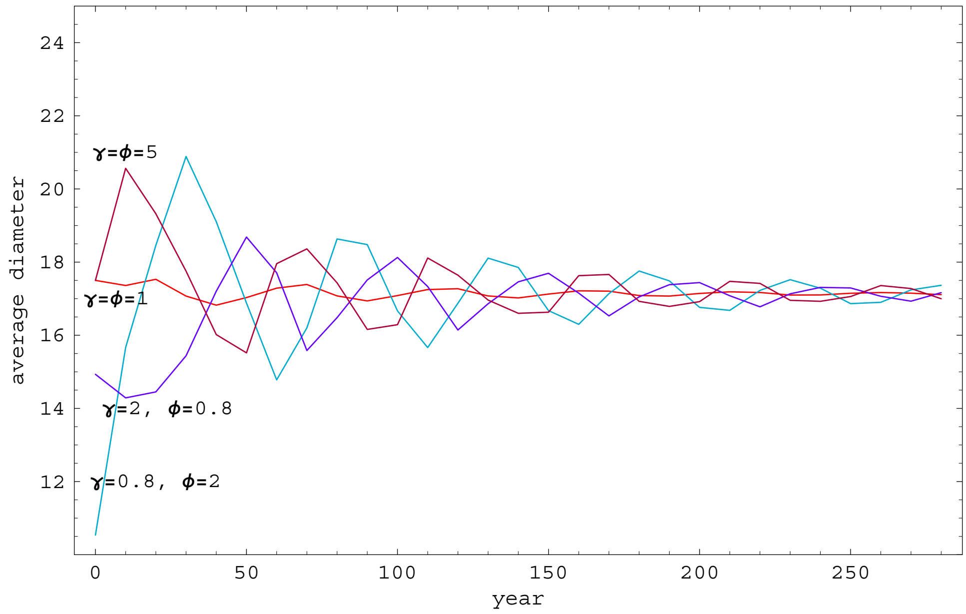
Figure 4.11 depicts the optimal evolution of the weighted average and variance of the diameter distribution over time, for the differen analyzed initial distributions. Figure 4.11 shows that the average diameter of the different distributions tends to converge after approximately 200 years, as the amplitude and phase of the cyclical behavior decreases. Additionally, Figure 4.11 shows that the variance of the distributions is governed by the same cyclical pattern. When the initial variance of the diameter distribution is similar to that of the steady-state, for instance, for the case where the parameters are $\gamma = \phi = 1$, the cyclical pattern of the mean and variance of the diameter is less pronounced, implying that the benefits will be more stable over time. In general, it can be observed that the long-run mean and variance tend to the values of 17 and 52 respectively, for all considered parameter values of γ and ϕ of the diameter distribution. Hence, the steady-state distribution of the diameter of the trees is independent of the initial distribution of the trees.

4.5.5 Accounting for Biological Diversity

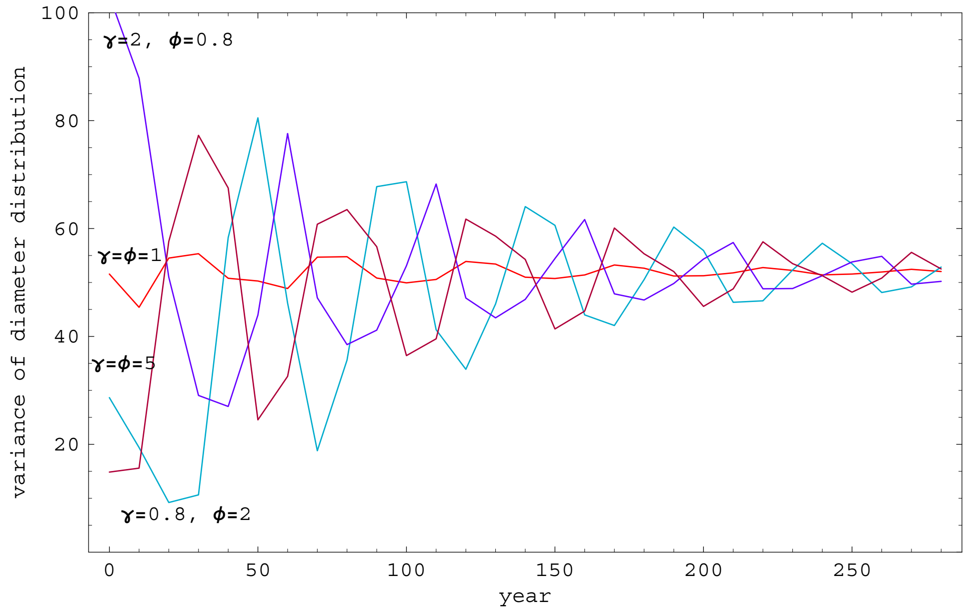
The previous section established the optimal selective-logging regime that maximizes the discounted sum of net private profits. However, the supplementary services that forests provide to the society have not been taken into account. One of the most important service that forest supply is natural habitat for numerous animal and plant

Figure 4.11: Evolution of the Weighted Average Diameter and Variance of the Diameter Distribution Over Time

a) Average diameter of the distribution after thinning



b) Variance of the distribution after thinning



species, and therefore the distribution of the diameter of the trees that maximizes the private profits does not necessarily maximizes the social net benefits of the forest, i.e., the net benefits where a monetary value of the habitat is considered in the decision problem. For instance, deers need small trees to feed on, and require large trees where they can hide. Taking these aspects into account supposedly leads to a higher weighted average and variance of the diameter of the trees. The measurement of these additional services is difficult. As a proxy one can measure diversity, as it is closely related with these services. Moreover putting a monetary value on diversity allows to consider the benefits of these services in the objective function of the decision problem, i.e. one determines the optimal selective-logging regime from a social point of view.

One of the most utilized indices for describing diversity is Shannon's H index (Marrugan, 1988). It is defined, for a general context, as:

$$H(t) \equiv -\sum_{i=1}^{s(t)} \frac{X_i(t)}{X(t)} \ln\left(\frac{X_i(t)}{X(t)}\right) \tag{4.17}$$

The maximum value that Shannon's index can reach is achieved when the number of trees in each cohort is identical. It is given by $\ln(s(t))$, where $s(t)$ denotes the number of cohorts at time t. However, the number of cohorts is not constant over time, since a new cohort, consisting of the newly planted trees, is added every 10 year period. As a consequence, Shannon's index increases over time. For instance, Shannon's index in the first 10 year period can reach a maximum value of 2.3, while the index in the final 10 year period (considering a time horizon of 200 years) can reach a maximum value of 3.4. Thus, in order to correct for this fact, a modification of the Shannon's index is proposed. The modified Shannon's index, denoted by $\tilde{H}(t)$, is obtained by dividing $H(t)$ by the natural logarithm of the potential maximum number of cohorts in each period, i.e., $\tilde{H}(t) = H(t)/\ln(m + t/\triangle t)$. In this way, the modified Shannon's index lyes within the range [0, 1].

One of the major problems of considering non-timber services to determine the optimal management of the forest is to find a correct monetary value for these non market services. For instance, the revenue from hunting licences, or from the sale of by-products such as mushrooms could be used to estimate the value of non-timber services. However, since this specific question is not of primary interest for the thesis, the exact amount of this monetary value is considered as a parameter which may vary accordingly to the question at hand. To demonstrate the applicability of the proposed methodological approach a particular value, derived form the timber value, was chosen. According to the data utilized in the empirical study, the timber value of the stand at time 0 is approximately €3000 per hectare. Assuming that non-timber services of the stand have a value of 20% of the timber value, a value of non-timber services of €600/ha is introduced in the model.

The optimal selective-logging regime where diversity is taken into account is calculated for the initial distribution of the diameter of the trees given in Figure 4.1, case b), that corresponds to a low variance diameter distribution, that is, 80% of trees pertain to the 4 central cohorts. Figure 4.12 depicts the evolution of the optimal diameter distribution when the diversity is taken into account. Thereafter a comparison of Figures 4.12 and 4.8 demonstrate the differences in the evolution of the diameter distribution of the trees when diversity is accounted for and when it is not. It shows that if diversity is accounted for the distribution of the diameter of the trees is more even than if diversity is disregarded. That is, in the case of considering diversity, the number of trees in each cohort from the beginning of the planning horizon is nearly the same, while in the case of no considering diversity, almost 200 years are needed to achieve a distribution of the diameter which is stable over time. Furthermore, Figure 4.12 illustrates that the introduction of diversity in the objective function, increases the number of cohorts present in the forest in the long-run. Although the number of trees in the steady state is slightly inferior, the average diameter of the forest increases from 14.9 to 16.1 cm, and the variance of the distribution increases from 37.8 to 45.4.

Figure 4.12: Evolution of the Optimal Diameter Distribution where Diversity is Taken into Account.

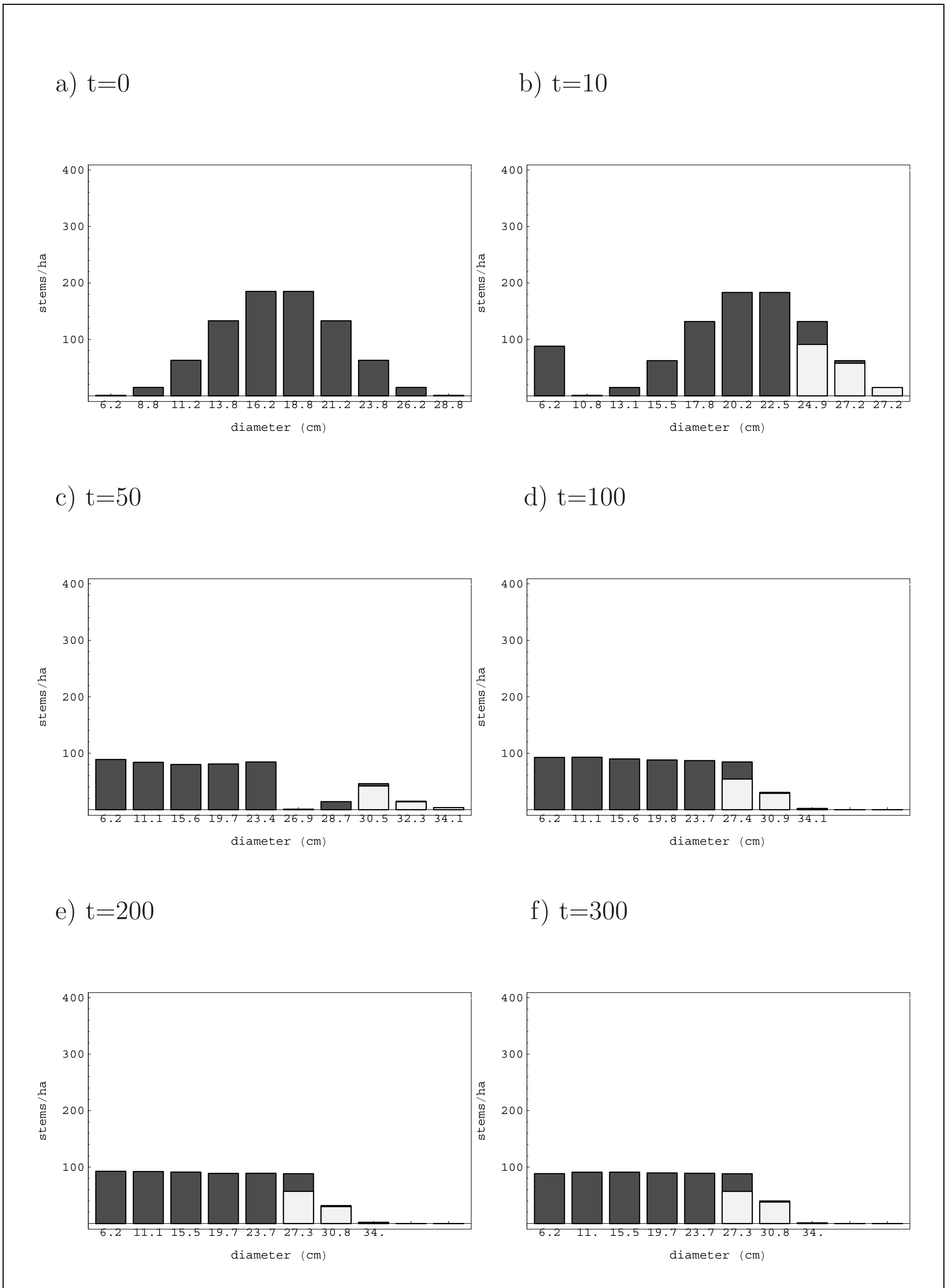


Table 4.3: Comparison of the Optimal Selective-Logging Regime where Diversity Value Is/Is Not Considered

Year	Number of trees[a]	Planted trees	Thinned trees	Net benefit of timber services (€/ha)	Discounted net benefits of timber services (€/ha)	Shannon's index	Discounted net benefits of non-timber services (€/ha)	Discounted social benefit (€/ha)
Optimal selective-logging regime when diversity is taken into account								
0	794	0	0	-641.18	-641.18	0.88	529.95	-111.23
10	874	88	165	1647.54	1220.53	0.83	367.23	1587.76
20	780	83	233	2647.08	1452.75	0.85	280.06	1732.81
30	622	82	180	2460.30	1000.28	0.86	209.94	1210.22
40	523	85	109	1763.25	531.08	0.82	148.39	679.47
50	500	89	61	1015.70	226.63	0.77	102.98	329.61
60	525	91	61	727.09	120.19	0.75	74.71	194.90
70	549	91	84	1149.29	140.74	0.74	54.16	194.90
80	553	92	82	1125.73	102.12	0.72	39.07	141.19
90	561	94	83	1119.67	75.25	0.70	28.26	103.51
100	568	93	85	1143.66	56.94	0.69	20.56	77.50
Sum over 300 years					4452		1911	6363
Optimal selective-logging regime when diversity is not taken into account								
0	794	0	54	7.74	7.74	0.84	502.80	510.54
10	752	19	157	1453.69	1076.92	0.74	327.19	1404.11
20	760	170	181	1921.22	1054.39	0.72	235.49	1289.88
30	742	169	192	2485.51	1010.53	0.70	170.68	1181.21
40	693	148	176	2601.10	783.44	0.61	110.74	894.18
50	560	49	15	-176.06	-39.28	0.63	83.69	44.41
60	606	67	18	-219.80	-36.33	0.64	63.52	27.19
70	739	157	162	1975.71	241.94	0.63	46.22	288.16
80	700	130	182	2244.05	203.58	0.63	34.32	237.90
90	675	141	141	1710.58	114.96	0.61	24.79	139.75
100	601	72	46	248.50	12.37	0.61	18.28	30.65
Sum over 300 years					4594		1810	6262

Table 4.3 summarizes the numerical results of both optimizations for the first 100 years of the planning horizon. It shows that the number of planted and thinned trees in the case where diversity is accounted for correspond each other more over time than in the case where diversity is not considered. The sum of discounted social benefits of forest management over 300 years, when non-timber services are taken into account, are €6363/ha, where the sum of discounted net benefits of timber services are €4452/ha, and the sum of the discounted net benefits of non-timber services are €1911/ha. If non-timber services are not taken into account, the forest owner obtains a sum of discounted private net benefits of timber services of €4594/ha. In this case, the forest management leads to a sum of discounted net benefits from diversity of €1810/ha, that can be considered as a by-product of the forest management in form of a public good. The sum of discounted social benefits is €6262/ha. Thus, ignoring diversity leads to a lower social benefits. However, discounted net benefits of timber services decrease if non-timber services are considered. Therefore, it is plausible to assume that forest owner will not follow the social management rules, since they lead to lower private net benefits. Thus, it is necessary to design policies that induce forest owners to take account of the multiple services that forest provides to the society.

4.6 Summary and Conclusions

This chapter presents a theoretical model that allows determining the optimal management of a diameter-distributed forest. The theoretical model can be formulated as a distributed optimal control problem where the control variables and the state variable depend on the two arguments, time and diameter of the tree. The resulting necessary conditions of this problem include a system of partial differential equations that usually cannot be solved analytically. For this reason, a numerical method (Escalator Boxcar Train) that is new to the economic literature and that can be programmed and handled easily is proposed. The numerical approach allows to transform the distributed optimal control problem into a classic optimization problem by transforming the independent argument, i.e., diameter, into a state variable of the problem that evolves over time, such that the resulting optimization problem can be solved utilizing standard mathematical programming techniques.

To find the optimal management of a diameter-distributed forest, an empirical analysis is conducted to determine the optimal selective logging, that is, the selective-logging regime that maximizes the discounted net benefits from timber production of a privately owned forest of *pinus sylvestris*. The study is characterized by a rigorous assessment of the complex growth process of trees. The empirical analysis shows that the clear-cutting regime, given by the Faustmann solution, leads to lower private benefits than the selective-logging regime. This is due to the fact that the selective logging permits the possibility of thinning some part of the forest at the 40^{th} year, while with clear-cutting the forest owner must wait until year 62 to cut the whole stand. Thus, the owner obtains the first benefits with a time lag of 22 years. As a result, the clear-cutting regime leads to a loss of approximately 38% of the benefits of the selective-logging regime.

It is also shown that the optimal long-run distribution of the diameter of the trees is unaffected by the initial distribution, provided that the initial basal area of the stand is the same. However, in most cases, more than 200 years are necessary to achieve a nearly stable distribution of the diameter of the trees.

Moreover, the study demonstrates how the optimal management of the forest changes when other than timber services are taken into account. It results in a more uniform distribution right from the beginning and throughout the planning horizon, and a higher average and variance of the diameter of the trees in the long-run. The sum of the discounted social net benefits obtained from the optimal management, if a monetary value of diversity is included in the objective function, are higher than the sum of the discounted benefits obtained when diversity is not considered.

However, the net benefits from timber services, that is, excluding the non-timber values, are lower in the first case and, therefore, it is easy to assume that forest owner will not follow the social management rules. Thus, it is necessary to design policies that induce forest owners to take account of the multiple services that forest offers to the society.

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