Nonpoint source pollution, space, time, and asymmetric information - a deposit refund approach*

Renan Goetz† and Dolors Berga†

January 2004

Abstract

The incorporation of space allows the establishment of a more precise relationship between a contaminating input, a contaminating byproduct and emissions that reach the final receptor. However, the presence of asymmetric information impedes the implementation of the first-best policy. As a solution to this problem a site specific deposit refund system for the contaminating input and the contaminating byproduct are proposed. Moreover, the utilization of a successive optimization technique first over space and second over time enables definition of the optimal intertemporal site specific deposit refund system.

JEL Classification Number: Q15, Q25, Q28.
Keywords: nonpoint source pollution, spatial economics, deposit refund system, asymmetric information.

*We gratefully acknowledge the support of the Spanish Ministry of Technology research grant AGL20001-2333-C02-01 CICYT. We like to thank Elena Del Rey and two anonymous referees for their helpful comments and suggestions.

†Departament d'Economia, Campus de Montilivi, Universitat de Girona, 17071, Girona, Spain. (e-mail: renan.goetz@udg.es; dolors.berga@udg.es)

1. Introduction

Nonpoint source pollution problems are characterized by the fact that it is either impossible to observe the emissions or their observation is prohibitively expensive. As a result, alternative instruments to a Pigouvian tax on the emissions such as an ambient tax were proposed to correct this kind of externalities (Segerson 1988; Cabe and Harriges 1992). However, the application of this tax in practice is highly questioned (Horan and Shortle 2001) as there is, among other reasons, no direct relationship between individual behavior and the amount of the ambient tax. Other approaches utilize the amount of applied input as a proxy of the unobservable emissions within the framework of a principal agent model (Shortle and Dunn 1986; Dosi and Moretto 1993; Dosi and Moretto 1994). Yet, the amount of the purchased input is a poor approximation of the real emissions (Shortle and Abler 1998). In order to relate input use and emissions more precisely, it would be necessary to have complete information about the amount of input applied at a particular location, the way the input is applied, and to which activity it is applied. In order to elicit this information we propose the application of a deposit refund system. In the previous literature, it consists of a tax on the pollutant and the payment of a subsidy (refund) for the correct elimination of the pollutant. In contrast to the previous literature (Sigman 1998; Kolstad 2000), where a pollutant was analyzed, we apply this approach to a polluting input and a contaminating byproduct. Additionally we incorporate space to take account of the spatial heterogeneity of the land. The consideration of space, for instance in the form of a site vulnerability index, allows for the creation of a more precise relationship between the input use and the resulting emissions that reach the receptor, i.e. the place where environmental damage occurs.

Our proposal for the management of nonpoint source pollution is based on the figure of an authorized firm that provides the service of a correct[1] application of contaminating inputs with respect to its form and quantity. The authorized firm issues a certificate to the firm that commissioned its services. The certificate issued by the authorized firm allows observation of the correct application of the contaminating inputs. The incorrect application, however, cannot be observed.

Given this context we derive first the socially optimal distribution of production activities over space and their socially optimal intensity, i.e. the solution of the regional planner's decision problem. Due to the unobservability of the emis-

[1]Hereafter, we use the expressions "correct application" versus "incorrect application" to indicate that inputs are applied with more versus less guarantees to avoid emissions, i.e., the correct amount of inputs is applied and the inputs are applied such that emissions are minimal.

sions that reach the final receptor or due to the presence of asymmetric information with respect to the amount and the way of the application of the contaminating input, a policy that replicates the first-best outcome is not available. Therefore, a second-best policy based on a voluntary deposit refund system is proposed. The parameterization of space, based on site vulnerability, allows for targeting the policy site-specific, i.e., the deposit refund system is site-specifically differentiated.

Unfortunately, the introduction of spatially differentiated taxes gives rise to the possibility of arbitrage and thus a black market may emerge. To avoid black markets we propose ex ante uniform taxes and ex post spatially differentiated tax reductions, for instance in conjunction with the income tax.

Once we have obtained the optimal distribution of production activities over space and their socially optimal intensity we take account of the fact that many pollutants accumulate over time. Therefore, we introduce time into our spatial model. In order to obtain the socially optimal solution over space and time we employ a further developed optimization procedure in two stages, initially introduced by Goetz and Zilberman, 2000. This procedure enables an analysis of how the socially optimal spatial distribution of production activities and their corresponding optimal intensity develop over time. Given this knowledge we can design an intertemporally and spatially optimal deposit refund system.

This paper is organized as follows. In section 2 we introduce the concept of space based on certain characteristics of the land. Based on this concept, in section 3 we present our spatial economic model and in section 4 we study the case of asymmetric information where we propose a *deposit refund system.* In section 5 the intertemporal aspects of the problem are introduced. The paper closes out in section 6 with some conclusions.

2. The concept of space

We suppose that in a given region Ω, some production activities causing pollution take place. Region Ω reflects the origin of direct emissions of pollution and/or the space where emissions convert into a pollutant that accumulates at a receptor located in the region (for example, in the case of surface or underground water pollution, Ω would represent underlying watershed). In order to have a more tractable model and to concentrate on the proposal of a deposit refund system, we consider the case in which there is only one pollutant.[2] Since the pollutant accumulates at the receptor it will be necessary to consider time explicitly.

In our work we first concentrate on the spatial aspect in order to analyze the repercussion of space in the determination of ambient policies.

We start from the idea that region Ω can be represented by a line that starts at point 0, the urban center, and ends at point $\overline{\alpha}$, the limit of the region. Each location is identified by α, $\alpha \in [0, \overline{\alpha}]$. In order that variable α be sufficient to parameterize region Ω, we employ several variables of interest such as geophysics, topography and hydrology. In this way α can be interpreted as a georeferenciated index and not only as a form of coordinates. Thus, our index is a function from $\mathbb{R}^n$ to $\mathbb{R}$, where n is the amount of variables of interest, and $\mathbb{R}$ is the real line. For example, α can represent a land classification system that collects the relevant characteristics of each location with respect to the pollution process at the receptor. Thus, space is not introduced in the form of a standard parameterization but in the form of a parameterization of the site vulnerability of each location within the considered region. Site vulnerability captures the extent to which the application of a contaminating input leads to an increase in the concentration of a contaminant at the receptor.

All the economic transactions in the region except production activities take place at the urban center where extension collapses at a single point since it is small relative to the total region. Production activities generate emissions of a single pollutant that accumulates at a receptor that is located in the urban center (at $\alpha = 0$). The land outside the urban center is exclusively dedicated to production activities. The emphasis in our model is on consumers that live in the urban center, and are therefore affected by the pollutant at the receptor, versus the ones that live outside the urban center. This is based on the hypothesis that there are many more consumers living inside the urban center than outside.

Moreover, the different productive activities are part of a competitive system where changes in regional production or changes in regional demand for inputs do not affect the production or input prices of the competitive system. In other words, they are exogenously given within our model.

To reduce the complexity of the analysis, we concentrate on the emissions that reach the receptor (as in Hochman, Pines and Zilberman, 1977). That is, we consider final emissions, accepting that part of the emissions in the origin are lost (absorbed, decomposed or solidified) before they reach the receptor.

[2]The results with two or more pollutants would strongly depend on the existing interactions between the different pollutants. Thus, we would obtain very specific results for each particular case.

Given this situation, we determine the optimal location of different production activities in the region. For this end, we suppose that a regional planner maximizes net actual benefits from the different agricultural activities, taking into account that an environmental standard with respect to the emissions of the pollutant that reach the receptor, should not be exceeded.

However, the optimal allocation of the production activities over space is only the first stage of our optimization process over space and time. The regional planner's maximal net benefit is then reflected by a value function that depends on the optimal value of the decision variables and the exogenously defined parameters, for instance, the environmental standard with respect to the emissions of the pollutant that reach the receptor. Precisely this parameter becomes the decision variable in the second stage. This procedure, described in more detail by Goetz and Zilberman (2000) and Goetz and Zilberman (2002), allows the spatial and intertemporal optimization processes to split into two consecutive stages, enabling the obtainment of an analytical solution more easily.

3. The spatial economic model

According to the separation of the spatial and intertemporal optimization procedure we start out with the first stage where we optimize over space. For the sake of concreteness the production process is given by production activities with an infinite number of agents, say farmers. Each farmer cultivates at a given location α, where α is the parameterization of the region Ω, the total region for cultivation. Let L denote the number of hectares of arable land in Ω. Since our parameterization of space permits that not only a single point, but also a subarea of Ω corresponds to a location α, the size of a given location α in relation to the size of the region Ω is captured by the density function $g(\alpha)$ with $\int_0^{\overline{\alpha}} g(\alpha)d\alpha = 1$. Thus, for each α, $g(\alpha)L$ denotes the number of hectares of arable land at location α. The support of g is the interval $[0, \overline{\alpha}]$. In the discrete case, $g(\alpha)$ denotes the proportion of Ω associated with each α.

Without loss of generality, we suppose that only a single farmer cultivates at a given location α and that there are two agricultural activities i, $i = 1, 2$ related to crop production, for instance the cultivation of wheat and corn.[3] The share of land

[3]Considering more than one farmer cultivating in α would complicate the analysis without obtaining any additional insight for the problem posed. If $m > 1$ farmers cultivate in α, the functions $g^{\overline{s}}(\alpha)$ defined for each farmer $\overline{s} = 1, ..., m$ satisfy $\sum_{s=1}^{m} g^{\overline{s}}(\alpha) = g(\alpha)$.

utilized for the cultivation of crop i at the location α is denoted by $\delta_i(\alpha) \in [0, 1]$ where $\sum_{i=1}^{2} \delta_i(\alpha) \leq 1$. The production of each crop per hectare is given by the function $\varphi\left(x_i(\alpha) + y_i(\alpha), \alpha; \beta_i\right)$, where $x_i(\alpha)$ and $y_i(\alpha)$ are the only inputs considered; they denote the amount per hectare of mineral fertilizer and the amount per hectare of organic fertilizer applied at location α to crop i, $i = 1, 2$, respectively. We suppose that φ presents constant returns to scale with respect to the size of the cultivated area. The production function is site-specific and therefore crop yields vary with location α. The parameter β_i presents a productivity index specific for each crop i.

The fixed costs per hectare associated with the cultivation of crop i are denoted by k_i, $i = 1, 2$, which stand for the annualized capital investment costs. We assume that the production function $\varphi(\cdot; \beta_i)$ is differentiable in x_i, y_i and α. Applying the chain rule we denote the derivative of φ with respect to either x_i or y_i by φ', which is usually positive but very high amounts of fertilizer may lead to negative marginal productivity. Moreover, φ is concave with respect to fertilizer, that is, $\varphi'' < 0$.

The farmer at location α has also the possibility to keep livestock. The benefits and costs of livestock at location α are denoted by $h(y\,(\alpha))$ and $z(y\,(\alpha))$, respectively, where $y\,(\alpha)$ indicates the amount of manure per hectare of arable land at α (which is equivalent to the stocking rate). Let us denote by $h'(y\,(\alpha))$ and $z'(y\,(\alpha))$ their derivative with respect to y. A one to one relationship exists between manure and the amount of animals. However, since we are interested in manure as a fertilizer as well as a source of water pollution we have opted to express the amount of animals in terms of manure. Farmers either use manure as organic fertilizer or if an excess exists, they dump it. Let $y^e(\alpha)$ denote the amount of manure in excess of organic fertilizer per hectare of arable land at α. The per hectare manure balance condition for the farm is given by:

$$y^e(\alpha) = y(\alpha) - \sum_{i=1}^{2} \delta_i(\alpha) y_i(\alpha) \text{ for any } \alpha \in [0, \overline{\alpha}]\,.$$

Moreover, the use of fertilizer is not only productive but also leads to emissions that are captured by two different emission functions. For mineral fertilizer applied to crop i at location α the final emission function per hectare is given by $\phi\left(x_i(\alpha), \alpha; \gamma_i^x\right)$ and reflects the final emissions due to the use of mineral fertilizer that reaches the receptor located in the urban center. Organic fertilizer applied to crop i at location α leads to the emission function $\phi\left(y_i(\alpha), \alpha; \gamma_i^y\right)$ and denotes the magnitude of the emissions per hectare due to the use of organic fertilizer with site quality characteristics α that reaches the receptor. We assume that $\phi\left(x_i(\alpha), \alpha; \gamma_i^x\right)$ and $\phi\left(y_i(\alpha), \alpha; \gamma_i^y\right)$ are differentiable in $x_i(\alpha)$, α and in $y_i(\alpha)$, α, respectively. We denote their partial derivatives with respect to x_i and y_i by $\phi_{x_i}(x_i(\alpha), \alpha; \gamma_i^x)$ and $\phi_{y_i}(y_i(\alpha), \alpha; \gamma_i^y)$, respectively, which we assume positive. Parameters γ_i^x and γ_i^y, $i = 1, 2$ denote a pollution index which captures the relationships between the amount of emissions and the amount per hectare of mineral or organic fertilizer applied in the production process, respectively. The dumping of excess manure leads to the emission function per hectare $\phi\left(y^e(\alpha), \alpha; \gamma^{y^e}\right)$, whose partial derivative with respect to y^e is denoted by $\phi_{y^e}\left(y^e(\alpha), \alpha; \gamma^{y^e}\right)$ and where for all $i = 1, 2$, $\phi\left(\overline{y}^e, \alpha; \gamma^{y^e}\right) > \phi\left(\overline{y}_i, \alpha; \gamma_i^y\right)$, for all $\overline{y}^e = \overline{y}_i$.

For all α, let $p_i(\alpha)$ and $\omega_i(\alpha)$ denote the output and mineral fertilizer prices, respectively, that are faced by the farmer located at α. We suppose that for all $\alpha > 0$, $p_i(\alpha)$ and $\omega_i(\alpha)$ differ from $p_i(0)$ and $\omega_i(0)$ only by the transportation costs that are taken care of by the farmer with $p_i'(\alpha) < 0$ and $\omega_i'(\alpha) \geq 0$.[4] Thus, we rewrite prices as $p_i(\alpha) \equiv p_i(\varsigma(\alpha))$ and $\omega_i(\alpha) \equiv \omega_i(\varsigma(\alpha))$ where $\varsigma(\alpha)$ is a function that takes only distance into account and no other variables of the land classification system. We also consider the application costs of each type of fertilizer x_i, y_i, for each crop $i = 1, 2$. They are defined by $c_i^x(x_i)$ and $c_i^y(y_i)$. We denote their derivatives with respect to x_i and y_i by $c_i^{x\prime}(x_i)$ and $c_i^{y\prime}(y_i)$, respectively.

Let $c^{y^e}(y^e)$ denote the costs of dumping of excess manure and by $c^{y^e\prime}(y^e)$ its derivative with respect to y^e. We suppose that $c^{y^e}(y^e)$ satisfies that for all $i = 1, 2$, $c^{y^e}(\overline{y}^e) < c^y(\overline{y}_i)$ for any $\overline{y}^e = \overline{y}_1 = \overline{y}_2$. For the sake of concreteness we assume that the single pollutant is given by nitrate NO_3^-.

Given this setup and taking into account an environmental standard that introduces an upper limit on the concentration of the pollutant at the receptor, the decision problem of the regional planner, say problem (R), consists in determining the optimal values for $x_i(\alpha)$ and $y_i(\alpha)$, in selecting $\delta_i(\alpha)$ the optimal activity itself together with its scale, and in choosing the optimal scale of husbandry $y(\alpha)$ in order to maximize the net benefits of the agricultural and livestock activities.

The solution of the decision problem of the regional planner allows for the design of environmental policies that induce individuals to behave optimally from a regional perspective by correcting the optimal intensity and the choice of the type and scale of the activity. See the appendix for details of the solution of problem (R) and possible environmental policies under the assumption of full information.

In contrast to the regulator, farmers do not take into account pollution at the receptor. To correct for this negative production externality, market intervention is required. Pigouvian taxes on emissions at the origin were considered insufficient to

[4] With respect to labor we assume that the labor requirements of the activities are predominantly met by the members of farm households.

correct the externality (see Henderson, 1977, Hochman and Ofek, 1979, and Tomasi and Weise, 1994). Hochman and Ofek (1979) show that an adequate tax should equal the aggregate of the spatially differentiated marginal damage at each location α. Like Hochman and Ofek (1979), Goetz and Zilberman (2002) introduced a final emissions function that relates the amount of a polluter at the urban center with the farmer's emissions at each location α.

In this way, the Pigouvian tax on the final emissions at the receptor is able to determine the optimal allocation of the use of land and fertilizers. However, since this tax is imposed on final emissions at the receptor it is constant through space, i.e. it is not spatially differentiated. Yet, these policies are not implementable because of the required information. As an alternative, input taxes (nitrogen tax) have been proposed. However, as shown in the appendix and by Goetz and Zilberman (2002) input taxes alone are not able to establish the social optimum. They need to be complemented by land-use taxes that, depending on the curvature of the emission function, are either positive (tax) or negative (subsidy), in order to achieve the socially optimal outcome.

While the land-use can be observed easily by the regulator, the amount of input applied to each crop cannot be observed easily. Thus, the presence of asymmetric information impedes that input taxes are able to establish the socially optimal outcome.

The literature has not yet developed a widely accepted solution to the problem of the optimal regulation of nonpoint source pollution.[5] All previous approaches share the fact that they do not take into account the way the contaminating inputs are applied. To a great extent it is not only the amount of input which is responsible for emissions but also the way the input is applied.

In order to give farmers incentives to reveal information about the correct application of the inputs with respect to the amount and the method of its application, we propose a deposit refund system that is presented formally in the following section. This approach is new to the literature of the optimal management of nonpoint source pollution. Sigman (1998) and Kolstad (2000) previously applied this approach in the literature to regulate a pollutant directly. In contrast, the analysis presented in this paper applies this approach to regulate a contaminating input and a contaminating byproduct with respect to the applied amount and the method of its application taking the spatial heterogeneity of the land into account. The correct application of the input has to be done by an authorized firm that certifies the amount of in-

[5]For a discussion of the advantages and disadvantages of the different existing approaches see Shortle and Abler (1998).

put applied, and the correct form of its application for a particular crop at a given location. For example, certain farmers or special firms may have been approved by the authorities to apply mineral and/or organic fertilizers. Moreover, the authorized firm issues a document that certifies the correct application of the input with respect to its form and quantity. In order to provide incentives to apply fertilizers correctly, the regulator gives a subsidy to reward the correct application. Farmers are only entitled to enjoy the subsidy if they present the certificate of the authorized firm to the regulatory body. In this way, the regulator can observe the correct application of the farm's fertilizers. The subsidy corresponds to the net savings of the social costs of correct versus incorrect application. The regulator can derive the remaining part of fertilizers that has not been applied correctly by observing the total amount of fertilizers purchased and the total amount of manure generated at location α. To reflect the social costs of the incorrect application of the fertilizer, a tax needs to be introduced. As the net savings of the social cost for the correct application are reimbursed via the subsidy, this tax is imposed on all fertilizers independently whether they have been applied correctly or incorrectly.[6]

4. Site-specific deposit refund system

In this section we depart from the spatial economic model previously discussed. The amount per hectare of mineral fertilizer applied correctly by an authorized firm to crop i at location α is denoted by $x_i^a(\alpha)$. Similarly, the amount per hectare of mineral fertilizer applied to crop i by the farm itself at location α is denoted by $x_i^f(\alpha)$. Thus, $x_i(\alpha) = x_i^a(\alpha) + x_i^f(\alpha)$ is the total amount per hectare of mineral fertilizer applied at location α. The price per unit of mineral fertilizer at location α is $\omega_i(\alpha)$. The price of the fertilizer is independent of whether the input is applied correctly by an authorized firm or not.

For location α, the authorized firm charges $c_i^{ax}\left(x_i^a(\alpha)\right)$ for the correct application of $x_i^a(\alpha)$.[7] Likewise, the application of $x_i^f(\alpha)$ by the farm leads to application costs

[6] Alternatively to a voluntary deposit refund system, one could think of an obligatory program where farmers are obliged to contract the services of an authorized firm. However, this solution would not be efficient since the externality caused even by the correct application is not internalized in the farmers' decision process.

[7] In order to concentrate on the proposal of a deposit refund system we assume that the net benefits of the authorized firm are zero at each period and that the farm costs do not depend on the demand by the farmers.

of $c_i^{fx}\left(x_i^f(\alpha)\right)$, with $c_i^{ax}\left(\overline{x}_i^a\right) > c_i^{fx}\left(\overline{x}_i^f\right)$ for each crop i, $i=1,2$ for every $\overline{x}_i^a = \overline{x}_i^f$. Similarly, $c_i^{ay}\left(y_i^a(\alpha)\right)$ and $c_i^{fy}\left(y_i^f(\alpha)\right)$ indicate the application costs of organic fertilizer for crop i at location α by an authorized firm or by the farm itself, respectively, with $c_i^{ay}\left(\overline{y}_i^a\right) > c_i^{fy}\left(\overline{y}_i^f\right)$ for each crop i, $i=1,2$ for every $\overline{y}_i^a = \overline{y}_i^f$. All cost functions are costs per hectare. For any $k=a,f$, we denote their derivatives with respect to x_i^k and y_i^k by $c_i^{kx\prime}(x_i^k)$ and $c_i^{ky\prime}(y_i^k)$, respectively. Thus, the per hectare production function now reads as $\varphi\left(x_i^a(\alpha)+x_i^f(\alpha)+y_i^a(\alpha)+y_i^f(\alpha),\alpha;\beta_i\right)$. As before, by the chain rule we denote the derivative of φ with respect to either x_i^a, x_i^f, y_i^a, or y_i^f by φ', which is usually positive but very high amounts of fertilizer may lead to negative marginal productivity. Moreover, φ is concave with respect to fertilizer, that is, $\varphi''<0$. The emission function can now be differentiated according to the type and form of the application of the fertilizer. For mineral fertilizer applied to crop i at location α by an authorized firm the final emission function is given by $\phi\left(x_i^a(\alpha),\alpha;\gamma_i^{ax}\right)$, and by $\phi\left(x_i^f(\alpha),\alpha;\gamma_i^{fx}\right)$ if the mineral fertilizer is applied by the farm itself. Organic fertilizer applied by an authorized firm to crop i at location α leads to the emission function $\phi\left(y_i^a(\alpha),\alpha;\gamma_i^{ay}\right)$, and to $\phi\left(y_i^f(\alpha),\alpha;\gamma_i^{fy}\right)$ if the farm applies the organic fertilizer itself. For any $k=a,f$, we denote their partial derivatives with respect to x_i^k, and y_i^k by $\phi_{x_i^k}\left(x_i^k(\alpha),\alpha;\gamma_i^{kx}\right)$ and $\phi_{y_i^k}\left(y_i^k(\alpha),\alpha;\gamma_i^{ky}\right)$, respectively. These emission functions satisfy $\phi_{x_i^f}(\cdot)$, $\phi_{x_i^a}(\cdot)\geq 0$, and $\phi\left(\overline{x_i^a},\alpha;\gamma_i^{ax}\right) < \phi\left(\overline{x_i^f},\alpha;\gamma_i^{fx}\right)$ for any $\overline{x_i^a}=\overline{x_i^f}$, and $\phi_{y_i^f}(\cdot)$, $\phi_{y_i^a}(\cdot)\geq 0$, and $\phi\left(\overline{y_i^a},\alpha;\gamma_i^{ay}\right) < \phi\left(\overline{y_i^f},\alpha;\gamma_i^{fy}\right)$ for any $\overline{y_i^a}=\overline{y_i^f}$. In this new framework, the manure constraint per hectare is given by

$$y(\alpha)=y^e(\alpha)+\sum_{i=1}^{2}\delta_i(\alpha)\left(y_i^a(\alpha)+y_i^f(\alpha)\right) \text{ for any } \alpha\in[0,\overline{\alpha}].$$

The spreading of $y^e(\alpha)$, the amount of excess manure dumped per hectare of arable land, leads to the emission function $\phi\left(y^e,\alpha;\gamma^{y^e}\right)$ whose partial derivative with respect to y^e is denoted by $\phi_{y^e}\left(y^e(\alpha),\alpha;\gamma^{y^e}\right)$ and such that for all $i=1,2$, $\phi\left(\overline{y}^e,\alpha;\gamma^{y^e}\right) > \phi\left(\overline{y}_i^f,\alpha;\gamma_i^{fy}\right)$ for all $\overline{y}^e=\overline{y}_1^f=\overline{y}_2^f$. Let $c^{y^e}(y^e(\alpha))$ denote the costs of spreading the excess manure and by $c^{y^e\prime}(y^e(\alpha))$ its derivative with respect to y^e. We suppose that for all $i=1,2$, $c^{y^e}(\overline{y}^e) < c_i^{fy}(\overline{y}_i^f)$ for any $\overline{y}^e=\overline{y}_i^f$.

Given this setup and taking into account that the regulator has imposed an environmental standard, the decision problem of the regional planner from now on referred to as (RA) is given by

$$\max_{\left\{x_i^k(\alpha),y_i^k(\alpha),y(\alpha),y^e(\alpha),\delta_i(\alpha)\right\}_{\substack{i=1,2\\k=a,f}}} \int_0^{\overline{\alpha}} \left\{\sum_{i=1}^{2} \delta_i(\alpha) \left[p_i(\alpha)\varphi\left(x_i^a(\alpha)+x_i^f(\alpha)+y_i^a(\alpha)+y_i^f(\alpha),\alpha;\beta_i\right) - \right.\right.$$

$$-k_i-\omega_i(\alpha)(x_i^a(\alpha)+x_i^f(\alpha))-c_i^{ax}\left(x_i^a(\alpha)\right)-c_i^{fx}\left(x_i^f(\alpha)\right)-c_i^{ay}\left(y_i^a(\alpha)\right)-$$

$$\left.\left.c_i^{fy}\left(y_i^f(\alpha)\right)\right]\right\} g(\alpha)Ld\alpha + \int_0^{\overline{\alpha}} \left\{h(y(\alpha))-z(y(\alpha))-c^{y^e}\left(y^e(\alpha)\right)\right\} g(\alpha)Ld\alpha \qquad \text{(RA)}$$

subject to

$$z_0 \geq \int_0^{\overline{\alpha}} \left\{\sum_{i=1}^{2} \delta_i(\alpha)\left[\phi\left(x_i^a(\alpha),\alpha;\gamma_i^{ax}\right)+\phi\left(x_i^f(\alpha),\alpha;\gamma_i^{fx}\right)+\phi\left(y_i^a(\alpha),\alpha;\gamma_i^{ay}\right)+\right.\right.$$

$$\left.\left.+\phi\left(y_i^f(\alpha),\alpha;\gamma_i^{fy}\right)\right]+\phi\left(y^e(\alpha),\alpha;\gamma^{y^e}\right)\right\} g(\alpha)Ld\alpha, \quad \text{(PC-RA)}$$

$$y^e(\alpha) \equiv y(\alpha)-\sum_{i=1}^{2}\delta_i(\alpha)\left(y_i^a(\alpha)+y_i^f(\alpha)\right) \text{ for any } \alpha\in[0,\overline{\alpha}], \qquad \text{(MC-RA)}$$

$$x_i^a(\alpha)\cdot g(\alpha)L \geq 0,\ x_i^f(\alpha)\cdot g(\alpha)L\geq 0,\ y_i^a(\alpha)\cdot g(\alpha)L\geq 0,\ y_i^f(\alpha)\cdot g(\alpha)L\geq 0,$$
$$\delta_i(\alpha)\cdot g(\alpha)L\geq 0, \text{ for } i=1,2;\ y(\alpha)\cdot g(\alpha)L\geq 0,\ y^e(\alpha)\cdot g(\alpha)L\geq 0, \text{ and}$$
$$\left(1-\sum_{i=1}^{2}\delta_i(\alpha)\right)\cdot g(\alpha)\geq 0 \text{ for any } \alpha\in[0,\overline{\alpha}]. \qquad \text{(LULC-RA)}$$

In order to solve this problem we substitute $y^e(\alpha)$ by $y(\alpha)-\sum_{i=1}^{2}\delta_i(\alpha)\left(y_i^a(\alpha)+y_i^f(\alpha)\right)$ as defined in the manure constraint (MC-RA).[8] We define the Lagrangian function $\mathcal{L}^1$ where we introduce the multipliers μ, λ_2, ξ_i^{kx}, ξ_i^{ky}, ξ_i^{δ}, ξ^y and χ for $i=1,2$ and for $k=a,f$. The multiplier μ is associated with the pollution constraint (PC-RA), while all other multipliers (ξ_i^{kx}, ξ_i^{ky}, ξ_i^{δ}, ξ^y, λ_2 and χ for $i=1,2$ and for $k=a,f$) are related with lower and upper limit constraints (LULC-RA).

The argument α of the variables/functions x_i^k, y_i^k, y, δ_i, p_i, ω_i, g and of the Lagrange multipliers will be omitted in order to simplify notation. The Lagrangian is therefore given by

[8] In what follows, for sake of simplicity, we will use the notation y^e if no ambiguity arises.

$$
\begin{aligned}
\mathcal{L}^1 \;=\; & \int_0^{\overline{\alpha}} \sum_{i=1}^{2} \delta_i \left[p_i \varphi(x_i^a + x_i^f + y_i^a + y_i^f, \alpha; \beta_i) - \right. \\
& \left. -k_i - \omega_i (x_i^a + x_i^f) - c_i^{ax}(x_i^a) - c_i^{fx}\left(x_i^f\right) - c_i^{ay}(y_i^a) - c_i^{fy}\left(y_i^f\right) \right] gLd\alpha + \\
& + \int_0^{\overline{\alpha}} \left[h(y) - z(y) - c^{y^e}(y^e) \right] gLd\alpha + \\
& +\mu \left[z_0 - \int_0^{\overline{\alpha}} \left\{ \sum_{i=1}^{2} \delta_i \left[\phi\left(x_i^a, \alpha; \gamma_i^{ax}\right) + \phi\left(x_i^f, \alpha; \gamma_i^{fx}\right) + \phi\left(y_i^a, \alpha; \gamma_i^{ay}\right) + \right. \right. \right. \\
& \left. \left. \left. +\phi\left(y_i^f, \alpha; \gamma_i^{fy}\right) \right] + \phi\left(y^e, \alpha; \gamma^{y^e}\right) \right\} gLd\alpha \right] + \\
& + \int_0^{\overline{\alpha}} \left\{ \sum_{i=1}^{2} \left[\xi_i^{ax} x_i^a + \xi_i^{fx} x_i^f + \xi_i^{ay} y_i^a + \xi_i^{fy} y_i^f + \xi_i^{\delta} \delta_i \right] + \right. \\
& \left. +\chi \left(1 - \sum_{i=1}^{2} \delta_i \right) + \xi^y y + \lambda_2 y^e \right\} gLd\alpha.
\end{aligned}
$$

We assume both that there exists a unique solution of this problem and that $g(\alpha) > 0$ for all α. Then, the solution has to comply with the following conditions at every location α, $\alpha \in [0, \overline{\alpha}]$:

$$
\begin{aligned}
\mathcal{L}^1_{x_i^k} = \delta_i \Big(p_i \varphi' \left(x_i^a + x_i^f + y_i^a + y_i^f, \alpha; \beta_i \right) - \omega_i - c_i^{kx\prime}(x_i^k) - \mu \phi_{x_i^k}\left(x_i^k, \alpha; \gamma_i^{kx}\right) \Big) + \\
+ \xi_i^{kx} = 0,\ i = 1, 2 \text{ and } k = a, f.
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}^1_{y_i^k} = \delta_i \Big(p_i \varphi' \left(x_i^a + x_i^f + y_i^a + y_i^f, \alpha; \beta_i \right) - c_i^{ky\prime}(y_i^k) + c^{y^e\prime}(y^e) - \mu \phi_{y_i^k}\left(y_i^k, \alpha; \gamma_i^{ky}\right) + \\
+\mu \phi_{y^e}\left(y^e, \alpha; \gamma^{y^e}\right) - \lambda_2 \Big) + \xi_i^{ky} = 0, \text{ for any } i = 1, 2 \text{ and } k = a, f.
\end{aligned}
$$

$$
\mathcal{L}^1_{y} = h'(y) - z'(y) - c^{y^e\prime}(y^e) - \mu \phi_{y^e}\left(y^e, \alpha; \gamma^{y^e}\right) + \xi^y + \lambda_2 = 0.
$$

$$\mathcal{L}^1_{\delta_i} = p_i\varphi\left(x_i^a + x_i^f + y_i^a + y_i^f, \alpha; \beta_i\right) - k_i - \omega_i\left(x_i^a + x_i^f\right) - c_i^{ax} - c_i^{fx} - c_i^{ay} - c_i^{fy} +$$
$$+ c^{y^e\prime}(y^e)\left(y_i^a + y_i^f\right) + \mu\phi_{y^e}\left(y^e, \alpha; \gamma^{y^e}\right)\left(y_i^a + y_i^f\right) -$$
$$- \mu\left(\phi\left(x_i^a, \alpha; \gamma_i^{ax}\right) + \phi\left(x_i^f, \alpha; \gamma_i^{fx}\right) + \phi\left(y_i^a, \alpha; \gamma_i^{ay}\right) + \phi\left(y_i^f, \alpha; \gamma_i^{fy}\right)\right) -$$
$$- \left(y_i^a + y_i^f\right)\lambda_2 + \xi_i^\delta - \chi = 0,\ i = 1, 2.$$

$$\mu \geq 0,\ \mu\left[z^0 - \int_0^{\overline{\alpha}}\left\{\sum_{i=1}^{2}\delta_i\left[\phi\left(x_i^a, \alpha; \gamma_i^{ax}\right) + \phi\left(x_i^f, \alpha; \gamma_i^{fx}\right) + \phi\left(y_i^a, \alpha; \gamma_i^{ay}\right)\ +\right.\right.\right.$$
$$\left.\left.\left. +\phi\left(y_i^f, \alpha; \gamma_i^{fy}\right)\right] + \phi\left(y^e, \alpha; \gamma^{y^e}\right)\right\} gLd\alpha\right] = 0.$$

In order to concentrate on the economic interpretations we assume that the Kuhn-Tucker conditions related to the restriction (LULC-RA) are satisfied.[9] Once the optimal value of all decision variables are obtained, these values can be used to obtain the optimal value of y^e, y^{e*} using the (MC-RA) condition.

We now analyze the implementation of a deposit refund system to give farmers incentives to contract an authorized firm that applies the inputs correctly. The following proposition defines a deposit refund system that establishes the optimal social outcome.

Proposition 1 *(Site-specific deposit refund system)*
Given the existence of an authorized firm and provided that (1) the total amount of mineral fertilizer and the total amount of organic fertilizer used by the farmer for each activity i can be observed at each location α, (2) and that technology choices, β_i, γ_i^{ax}, γ_i^{ay}, γ_i^{fx}, γ_i^{fy} and γ^{y^e} can also be observed at each location α and for each activity i, then an optimal policy can be obtained by the deposit refund system defined as follows:
(a1) a spatially differentiated and crop-specific tax over the total amount of mineral fertilizer applied to activity i at location α, $\tau_i^m(\alpha)$, equal to

$$\tau_i^m(\alpha) = \mu^*\phi_{x_i^f}\left(x_i^{f*}, \alpha; \gamma_i^{fx}\right),\ i = 1, 2,\ and$$

[9] We use an asterisk as a superindex of a decision variable or a multiplier to denote its optimal values.

(a2) a spatially differentiated and crop-specific tax over the total amount of organic fertilizer applied to activity i at location α, $\tau_i^o(\alpha)$, equal to

$$\tau_i^o(\alpha) = \mu^* \phi_{y_i^f}\left(y_i^{f*}, \alpha; \gamma_i^{fy}\right), \; i = 1, 2, \text{ and}$$

(a3) a spatially differentiated tax on the excess manure, $\tau(\alpha)$, given by

$$\tau(\alpha) = \mu^* \phi_{y^e}\left(y^{e*}, \alpha; \gamma^{y^e}\right), \text{ and}$$

(b1) a spatially differentiated and crop-specific subsidy over the amount of mineral fertilizer correctly applied to activity i at location α, $s_i^m(\alpha)$, equal to

$$s_i^m(\alpha) = \tau_i^m(\alpha) - \mu^* \phi_{x_i^a}\left(x_i^{a*}(\alpha), \alpha; \gamma_i^{ax}\right), i = 1, 2, \text{ together with}$$

(b2) a spatially differentiated and crop-specific subsidy over the amount of organic fertilizer correctly applied to activity i at location α, $s_i^o(\alpha)$, equal to

$$s_i^o(\alpha) = \tau_i^o(\alpha) - \mu^* \phi_{y_i^a}\left(y_i^{a*}(\alpha), \alpha; \gamma_i^{ay}\right), i = 1, 2, \text{ and}$$

(c1) a spatially differentiated and crop-specific land-use tax or subsidy $\sigma_i(\alpha)$ given by

$$\begin{aligned}\sigma_i(\alpha) = \mu^* \Big(\phi\left(y_i^{a*}, \alpha; \gamma_i^{ay}\right) + \phi\left(y_i^{f*}, \alpha; \gamma_i^{fy}\right) + \phi\left(x_i^{a*}, \alpha; \gamma_i^{ax}\right) + \phi\left(x_i^{f*}, \alpha; \gamma_i^{fx}\right)\Big) - \\ - \tau_i^m(\alpha) \cdot \left(x_i^{a*}(\alpha) + x_i^{f*}(\alpha)\right) - \tau_i^o(\alpha) \cdot \left(y_i^{a*}(\alpha) + y_i^{f*}(\alpha)\right) + \\ + s_i^m(\alpha) \cdot x_i^{a*}(\alpha) + s_i^o(\alpha) \cdot y_i^{a*}(\alpha) \gtreqless 0, \; i = 1, 2.\end{aligned}$$

For simplicity of notation, the argument α of the taxes and subsidies τ_i^m, τ_i^o, τ, s_i^m, s_i^o, σ_i, $i = 1, 2$ are suppressed unless it is required for an unambiguous notation.

Proof. These instruments are obtained straightforward by comparing the first order conditions (*f.o.c.*) of the regional planner problem (RA) with the *f.o.c.* of the farmers' decision problems that is analyzed below where a deposit refund system and land-use taxes are implemented.
Each farmer cultivating at location α will choose the optimal strategy solving his/her private decision problem given by:[10]

$$\max_{\left\{x_i^k, y_i^k, y, \delta_i\right\}_{i=1,2,k=a,f}} gL \left\{ \sum_{i=1}^{2} \delta_i \left[p_i \varphi(x_i^a + x_i^f + y_i^a + y_i^f, \alpha; \beta_i) - \right.\right.$$

[10]For sake of simplicity, we suppress the Lagrange multipliers ξ_i^{kx}, ξ_i^{ky}, ξ_i^{δ}, ξ^y, λ_2, and χ for $i = 1, 2$ and $k = a, f$, as decision variables.

$$-k_i - \omega_i(x_i^a + x_i^f) - c_i^{ax}\left(x_i^a\right) - c_i^{fx}\left(x_i^f\right) - c_i^{ay}\left(y_i^a\right) - c_i^{fy}\left(y_i^f\right) -$$

$$-\tau_i^m\left(x_i^a + x_i^f\right) - \tau_i^o\left(y_i^a + y_i^f\right) + s_i^m x_i^a + s_i^o y_i^a - \sigma_i\Big] + \left[h(y) - z(y) - c^{y^e}(y^e) - \tau y^e\right] +$$

$$+\sum_{i=1}^{2}\left[\xi_i^{ax}x_i^a + \xi_i^{fx}x_i^f + \xi_i^{ay}y_i^a + \xi_i^{fy}y_i^f + \xi_i^{\delta}\delta_i\right] + \xi^y y + \chi\left(1 - \sum_{i=1}^{2}\delta_i\right) + \lambda_2 y^e\Bigg\},$$

where y^e is replaced by $y - \sum_{i=1}^{2}\delta_i\left(y_i^a + y_i^f\right)$ given equation (MC-RA). Being $\mathcal{L}$ the Lagrangian of this maximization problem,

$$\mathcal{L}_{x_i^a} = \delta_i\left(p_i\varphi'\left(x_i^a + x_i^f + y_i^a + y_i^f, \alpha; \beta_i\right) - \omega_i - c_i^{ax\prime}(x_i^a) - \tau_i^m + s_i^m\right) + \xi_i^{ax} = 0,\ i = 1, 2.$$

$$\mathcal{L}_{x_i^f} = \delta_i\left(p_i\varphi'\left(x_i^a + x_i^f + y_i^a + y_i^f, \alpha; \beta_i\right) - \omega_i - c_i^{fx\prime}(x_i^f) - \tau_i^m\right) + \xi_i^{fx} = 0,\ i = 1, 2.$$

$$\mathcal{L}_{y_i^a} = \delta_i\left(p_i\varphi'\left(x_i^a + x_i^f + y_i^a + y_i^f, \alpha; \beta_i\right) - c_i^{ay\prime}(y_i^a) + c^{y^e\prime}(y^e) - \tau_i^o + s_i^o + \tau - \lambda_2\right) + \\ + \xi_i^{ay} = 0,\ i = 1, 2.$$

$$\mathcal{L}_{y_i^f} = \delta_i\left(p_i\varphi'\left(x_i^a + x_i^f + y_i^a + y_i^f, \alpha; \beta_i\right) - c_i^{fy\prime}(y_i^f) + c^{y^e\prime}(y^e) - \tau_i^o + \tau - \lambda_2\right) + \\ + \xi_i^{fy} = 0,\ i = 1, 2.$$

$$\mathcal{L}_y = h'(y) - z'(y) - c^{y^e\prime}(y^e) - \tau + \xi^y + \lambda_2 = 0.$$

$$\mathcal{L}_{\delta_i} = p_i\varphi\left(x_i^a + x_i^f + y_i^a + y_i^f, \alpha; \beta_i\right) - k_i - \omega_i\left(x_i^a + x_i^f\right) - c_i^{ax} - c_i^{fx} - c_i^{ay} - c_i^{fy} + \\ + c^{y^e\prime}(y^e)\left(y_i^a + y_i^f\right) - \tau_i^m\left(x_i^a + x_i^f\right) - \tau_i^o\left(y_i^a + y_i^f\right) + s_i^m x_i^a + s_i^o y_i^a - \sigma_i + \xi_i^{\delta} - \chi + \\ - \lambda_2\left(y_i^a + y_i^f\right) + \tau\left(y_i^a + y_i^f\right) = 0,\ i = 1, 2.$$

■

It is important to note that the sign of the land-use tax, σ_i, can be either positive or negative. That is, it can be either a tax or a subsidy.

The implementation of the deposit refund system as defined in Proposition 1 requires that the regulator is able to observe for every farmer the amount of each type of fertilizer applied to each crop. However, the regulator cannot distinguish to which crop the fertilizer has been applied. Therefore, the regulator cannot tailor the environmental policy to each specific crop in terms of the tax. The tax of an implementable deposit refund system can be site-specific but not crop-specific. As a remedy we propose taxes $\tau^m(\alpha) = \mu^* \sum_{i=1}^{2} \delta_i \phi_{x_i^f} \left(x_i^{f*}, \alpha; \gamma_i^{fx} \right)$ and $\tau^o(\alpha) = \mu^* \sum_{i=1}^{2} \delta_i \phi_{y_i^f} \left(y_i^{f*}, \alpha; \gamma_i^{fy} \right)$ over the total amount of mineral fertilizer applied at location α and over the total amount of organic fertilizer applied at location α, respectively. That is, over $\sum_{i=1}^{2} \delta_i \left(x_i^a + x_i^f \right)$ and $\sum_{i=1}^{2} \delta_i \left(y_i^a + y_i^f \right)$, respectively. In other words, for each type of fertilizer, the fertilizer tax corresponds to the weighted average of the marginal costs of the final emissions of the two activities. Likewise, one can not observe the amount of manure that is applied by the farmer and the amount of excess manure. Therefore, it is proposed to define a single manure tax, $\overline{\tau}(\alpha)$, that does not distinguish between $y_i^f(\alpha)$ and $y^e(\alpha)$. It is given by $\overline{\tau}(\alpha) = \max\{\tau(\alpha), \tau^o(\alpha)\}$ and it is applied over the amount of manure not applied by the authorized firm at location α, that is, over $\sum_{i=1}^{2} \delta_i y_i^f + y^e$. At the end of the cultivating period the regulator can observe the total output. Thus, given the information about x_i^a, x_i^f, and y_i^a he can estimate y_i^f. With this information, the regulator may reimburse part of the collected tax, i.e., $[\overline{\tau}(\alpha) - \tau^o(\alpha)]\, y_i^f$ if $\overline{\tau}(\alpha) = \tau(\alpha)$ and $[\overline{\tau}(\alpha) - \tau(\alpha)]\, y^e$ if $\overline{\tau}(\alpha) = \tau^o(\alpha)$.

Consequently, the land-use tax has to be adapted accordingly. Moreover, since site-specific taxes may differ from one location to another, arbitrage may occur, i.e., there may exist black markets for mineral fertilizer where farmers with a high input tax are potential buyers and farmers with a low input tax are potential sellers. To avoid the problem of arbitrage we propose to fix a tax $\tau^m = \max_\alpha \tau^m(\alpha)$. Farmers with input taxes below $\max_\alpha \tau^m(\alpha)$ are able to claim back the difference between what they paid and what they should have paid in their tax declaration. Since the tax declaration is in retrospective the regulator already knows the cultivated crop and the obtained yield. Given this information the regulator can calculate what the farmer should have paid on the mineral fertilizer tax. Therefore, the regulator can verify within certain limits whether the claims of the farmer are justified or not.

Proposition 1 establishes the conditions to determine the optimal taxes and subsidies. However, before implementing this deposit refund system the regulator has to verify that the implementation of this system is socially and privately desirable. That is, the social and private net benefits are bigger with the deposit refund system compared to a system where no authorized firm exists. In other words, two types of inequality constraints have to be fulfilled (social implementation constraints and private participation constraints).

5. The Optimal Land Allocation over Space and Time

After having analyzed the optimal allocation of inputs and land over space and proposed an environmental policy to achieve a second best social outcome we now turn to the intertemporal optimization of the optimal spatial allocation.

The value function V from the first stage is now employed in the second stage of the regional planner's decision problem. The objective consists of the sum of the value function V that depends on z and a function $m(s)$ that captures the monetary damages caused by the pollution at the receptor and s denotes the concentration of the pollutant at the receptor. The left-hand side value of the constraint (PC-RA) of the first stage problem, z, becomes the decision variable in the second stage. It still denotes the emissions of the entire region that reach the receptor; however, it now depends on t like s. The terminal value function is given by $F(s(T))$. Given this context the regional planner's decision problem reads as:

$$\max_{z(t)} \int_0^T \Big(V(z(t)) - m(s(t)\Big) e^{-rt}\, dt - e^{-rT}\, F(s(T)),$$

subject to

$$\dot{s}(t) = z(t) - \xi s(t), \qquad s(0) = s_0, \qquad z(t) \in \mathcal{Z}, \tag{RT}$$

where a dot over a variable denotes the operator $\frac{d}{dt}$. The set $\mathcal{Z}$ presents the interval $[0, \bar{z}]$, where the upper limit of the set corresponds to the possible highest emissions that could reach the receptor. Argument t of all the dynamic variables is dropped to simplify notations whenever possible, without introducing an ambiguous notation. Hence, the current value Hamiltonian in the second stage $\mathcal{H}$ reads as $\mathcal{H} \equiv V(z) - m(s) - \psi(z - \xi s)$. Note, that a negative sign in front of the costate variable ψ has been introduced to facilitate its interpretation. The necessary conditions[11] for an

[11] See theorems 1 and 3 in Seierstad and Sydsæter (1987).

interior solution $0 < z < \bar{z}$ of problem (RT) are given by

$$\mathcal{H}_z = V_z - \psi = 0 \qquad \Rightarrow \qquad \mu(t) = \psi(t) \tag{2}$$

$$\dot{\psi} = \psi r + \mathcal{H}_s = \psi(r+\xi) - m'(s(t)) \tag{3}$$

$$\dot{s} = z - \xi s \ , \qquad s(0) = s_0, \tag{4}$$

where we made use of the dynamic envelope theorem to obtain the result of equation (2). This equation states that the marginal value of the final emissions of the entire region should equal its shadow cost ψ which, in turn, is equal to the shadow cost of the spatial allocation problem μ. Equation (3) explains the change in the shadow cost of a delayed reduction of a marginal unit of the pollution stock from period t to period $t+1$. It states that the change is equal to the extra interest and "decay" forgone paid on the shadow cost minus the cost of extra pollution associated with the delay. The transversality condition requires that

$$\psi(T) = F'(s(T)). \tag{5}$$

It shows that the shadow cost at the terminal point of time has to equal the marginal terminal value of the amount of pollutant at the receptor. Hence, the particular solution of differential equation (3) yields

$$\psi(t) = F'(s(T))e^{(r+\xi)(t-T)} + e^{(r+\xi)t}\int_t^T m'(s(\tau))e^{-(r+\xi)\tau}\,d\tau, \tag{6}$$

which states that shadow costs at time t correspond to the sum of the discounted marginal terminal value for the remaining time $(T-t)$ and the "present value" of the integral of the discounted marginal damage from time t to the end of the planning horizon. For both terms of the sum the discount rate consists of the social discount rate and the natural decay rate.

Knowing that the optimal values of $\mu(t)$ and $\psi(t)$ are identical we are able to write the dynamic version of the proposed deposit refund system, simply by replacing μ by $\mu(t)$ in proposition 1.

6. Summary and conclusions

For nonpoint source pollution neither the quantity nor the polluter is known. Furthermore the problem is exacerbated by the heterogeneity of the biophysical conditions that determine the transport and transformation process of the pollutant from its origin to the arrival at the receptor. As a solution to this problem this paper proposes to incorporate the dimension of space in order to target an environmental policy specific to the biophysical conditions at each location. In this way it is possible to relate more closely the amount of the contaminating inputs and of the contaminating byproduct with the emissions that reach the receptor.

However, the emissions that reach the receptor do not only depend on the amount of the applied inputs and byproduct but also on the way these are applied. Given the fact that the regulator can neither observe the quantity nor the way the inputs and the byproduct are applied there exists the problem of moral hazard. As a solution that is new to the literature of the management of nonpoint source pollution management, we propose a spatially differentiated deposit refund system. The results show that farmers who commission the service of an authorized firm to apply the contaminating input or the contaminating byproduct correctly should receive a subsidy equivalent to the net savings of the social costs of correct versus incorrect application. The social costs of the incorrect application of the input and the contaminating byproduct however, need to be imposed on the polluter in the form of a tax reflecting these costs on the input and on the byproduct. As the net savings of the social costs for the correct application are reimbursed via the subsidy, the tax is imposed on the input and the byproduct in general independently whether they are applied correctly or not.

While the subsidy is site and crop-specific, the implementable tax is only site-specific. To overcome this shortcoming a weighted tax is proposed. The different magnitude of the tax from one location to another may give rise to arbitrage, i.e. black markets may emerge. In order to avoid the formation of black markets it is proposed to impose the highest site-specific tax and allow for reimbursement via the tax declaration if the optimal site-specific tax should be below the highest tax. Likewise, the regulator can not observe whether the manure is used as a fertilizer by the farmer or whether it is dumped. Therefore, it is also proposed to utilize a single tax on manure not applied by the authorized firm. The information about the crop yields allows the regulator to reimburse part of the collected taxes after harvest.

Apart from policy questions, the paper also demonstrates the utilization of a two stage optimization technique that optimizes in the first stage the allocation of inputs and land over space and in the second stage, determines how this optimal spatial allocation changes over time. In this way the optimal spatial intertemporal form of the deposit refund system can be derived.

References

Cabe, R. and J. Harriges. (1992). "The regulation of nonpoint-source pollution under imperfect and asymmetric information", *Journal of Environmental Economics and Management,* **22**: 34-146.

Dosi, C. and M. Moretto. (1993). "NPS Pollution, Information Asymmetry, and the Choice of Time Profile for Environmental Fees" in *Theory, Modeling and Experience in the Management of Nonpoint-Source Pollution*, edited by C.S. Russell and J.F. Shogren, Dordrecht: Kluwer Academic Publishers.

Dosi, C. and M. Moretto. (1994). "Nonpoint Source Externalities and Polluter's Site Quality Standards Under Incomplete Information" in *Nonpoint Source Pollution Regulation: Issues and Policy Analysis,* edited by T. Tomasi and C. Dosi, Dortrecht: Kluwer Academic Publishers.

Goetz, R. and D. Zilberman. (2000). "The dynamics of spatial pollution: The case of phosphorus runoff from agricultural land", *Journal of Economic Dynamics and Control,* **24**: 143-163.

Goetz, R. and D. Zilberman. (2002). "The economics of land zoning: the dynamic approach", *Mimeo* Universitat de Girona and University of California at Berkeley.

Henderson, J. (1977). "Externalities in a spatial context: the case of air pollution", *Journal of Public Economics,* **7**: 89-110.

Hochman, E., D. Pines and D. Zilberman. (1977). "The effects of pollution taxation on the pattern of resource allocation: the downstream case", *Quarterly Journal of Economics,* **91** (4): 625-638.

Hochman, O. and H. Ofek. (1979). "A theory of the behavour of municipal governments: the case of internalizing externalities", *Journal of Urban Economics,* **6**: 416-431.

Horan, R.D. and J.S. Shortle. (2001). "Environmental Instruments For Agriculture" in *Environmental policies for agricultural pollution control*, edited by J.S. Shortle and D. Abler.

Kolstad, C.D. (2000). *Environmental Economics*, Oxford University Press, New York.

Segerson, K. (1988). "Uncertainty and incentives for nonpoint pollution control", *Journal of Environmental Economics and Management,* **15**: 88-98.

Seierstad, A. and K. Sydsæter. (1987). "Optimal Control Theory With Economic Applications", Advanced textbooks in economics; 24; Amsterdam: North-Holland.

Shortle, J.S and D.G. Abler. (1998). "Nonpoint Pollution" in *The International Yearbook of Environmental and Resource Economics 1997/1998,* edited by H. Folmer and T. Tietenberg, Edward Elgar, Cheltenham UK.

Shortle, J.S. and J.W. Dunn. (1986). "The relative efficiency of agricultural source water pollution control policies", *American Journal of Agricultural Economics,* **68**: 668-677.

Sigman, H. (1998). "Midnight dumping: public policies and illegal disposal of used oil", *RAND Journal of Economics,* **29** (1): 157-178.

Tomasi, T. and A. Weise. (1994). "Water Pollution in a Spatial Model" in *Nonpoint Source Pollution Regulation: Issues and Analysis*, Chapter 7, edited by C. Dosi and T. Tomasi, Kluwer Academic Publishers, Dondrecht.

Appendix

The regional decision problem (R) can be stated as

$$\max_{\{x_i(\alpha),y_i(\alpha),\delta_i(\alpha),y^e(\alpha),y(\alpha)\}_{i=1,2}} \int_0^{\overline{\alpha}} \sum_{i=1}^{2} \delta_i(\alpha) \left(p_i(\alpha)\varphi(x_i(\alpha)+y_i(\alpha),\alpha;\beta_i)-\right.$$
$$\left.-k_i-\omega_i(\alpha)x_i(\alpha)-c_i^x(x_i)-c_i^y(y_i)\right) g(\alpha)Ld\alpha+$$
$$+\int_0^{\overline{\alpha}} \left[h\left(y(\alpha)\right)-z\left(y(\alpha)\right)-c^{y^e}\left(y^e(\alpha)\right)\right] g(\alpha)Ld\alpha \tag{R}$$

subject to

$$z_0 \geq \int_0^{\bar{\alpha}} \left\{ \sum_{i=1}^{2} \delta_i(\alpha) \left[\phi\left(x_i(\alpha),\alpha;\gamma_i^x\right)+\phi\left(y_i(\alpha),\alpha;\gamma_i^y\right)\right]+\phi\left(y^e(\alpha),\alpha;\gamma^{y^e}\right) \right\} g(\alpha)Ld\alpha, \tag{PC}$$

$$y^e(\alpha)=y(\alpha)-\sum_{i=1}^{2}\delta_i(\alpha)y_i(\alpha) \text{ for any } \alpha\in[0,\overline{\alpha}], \tag{MC}$$

$$x_i(\alpha)\cdot g(\alpha)L \geq 0,\ y_i(\alpha)\cdot g(\alpha)L \geq 0,\ \delta_i(\alpha)\cdot g(\alpha)L \geq 0,\ i=1,2;\ y(\alpha)\cdot g(\alpha)L \geq 0,$$

$$y^e(\alpha)\cdot g(\alpha)L \geq 0,\ \left(1-\sum_{i=1}^{2}\delta_i(\alpha)\right)\cdot g(\alpha)L \geq 0 \text{ for any } \alpha \in [0,\overline{\alpha}]\,, \quad \text{(LULC)}$$

where the maximal admissible concentration of the pollutant at the receptor, as introduced in the pollution constraint (PC), is denoted by z_0.

In order to solve this problem we substitute $y^e(\alpha)$ by $y(\alpha) - \sum_{i=1}^{2}\delta_i(\alpha)y_i(\alpha)$ as defined in the manure constraint (MC).[12] We define the Lagrangian function $\widehat{\mathcal{L}}$, where we introduce the Lagrange multipliers μ, λ_2, ξ_i^x, ξ_i^y, ξ_i^δ, ξ^y and χ for $i=1,2$. The multiplier μ is associated with the pollution constraint (PC), and all other multipliers (ξ_i^x, ξ_i^y, ξ_i^δ, ξ^y, λ_2 and χ for $i=1,2$) are associated with lower and upper limit constraints (LULC).

The argument α of the variables/functions x_i, y_i, y, δ_i, p_i, ω_i, g and of all Lagrange multipliers, will be omitted in order to simplify notation in the text below. Given the restrictions imposed on the decision variables the Lagrangian can be stated as

$$\widehat{\mathcal{L}} = \int_0^{\overline{\alpha}} \left\{ \sum_{i=1}^{2} \delta_i \left[p_i\varphi(x_i+y_i,\alpha;\beta_i) - k_i - \omega_i x_i - c_i^x(x_i) - c_i^y(y_i) \right] + \right.$$
$$\left. + h(y) - z(y) - c^{y^e}\left(y - \sum_{i=1}^{2}\delta_i y_i \right) \right\} gLd\alpha +$$

$$+\mu\left[z_0 - \int_0^{\overline{\alpha}} \left\{ \sum_{i=1}^{2} \delta_i \left[\phi\left(x_i,\alpha;\gamma_i^x\right) + \phi\left(y_i,\alpha;\gamma_i^y\right) \right] + \phi\left(y - \sum_{i=1}^{2}\delta_i y_i, \alpha; \gamma^{y^e} \right) \right\} gLd\alpha \right] +$$
$$+\int_0^{\overline{\alpha}} \left\{ \sum_{i=1}^{2} \left[\xi_i^x x_i + \xi_i^y y_i + \xi_i^\delta \delta_i \right] + \xi^y y + \chi\left(1 - \sum_{i=1}^{2}\delta_i\right) + \lambda_2\left(y - \sum_{i=1}^{2}\delta_i y_i \right) \right\} gLd\alpha.$$

We assume both that there exists a unique solution of problem (R) and that $g(\alpha) > 0$ for all α. Then, the solution has to comply with the following conditions at every location α, $\alpha \in [0,\overline{\alpha}]$:

$$\widehat{\mathcal{L}}_{x_i} = \delta_i \left[p_i\varphi'\left(x_i+y_i,\alpha;\beta_i\right) - \omega_i - c_i^{x\prime}(x_i) - \mu\phi_{x_i}\left(x_i,\alpha;\gamma_i^x\right) \right] + \xi_i^x = 0,\ i=1,2.$$

[12] In what follows, for sake of simplicity, note simply y^e if no ambiguity may arise.

$$\widehat{\mathcal{L}}_{y_i} = \delta_i \left[p_i \varphi' \left(x_i + y_i, \alpha; \beta_i \right) - c_i^{y\prime}(y_i) + c^{y^e\prime}(y^e) - \mu\phi_{y_i} \left(y_i, \alpha; \gamma_i^y \right) + \mu\phi_{y^e} \left(y^e, \alpha; \gamma^{y^e} \right) - \right.$$

$$\left. - \lambda_2 \right] + \xi_i^y = 0, \; i = 1, 2.$$

$$\widehat{\mathcal{L}}_y = h'(y) - z'(y) - c^{y^e\prime}(y^e) - \mu\phi_{y^e} \left(y^e, \alpha; \gamma^{y^e} \right) + \xi^y + \lambda_2 = 0.$$

$$\widehat{\mathcal{L}}_{\delta_i} = p_i \varphi \left(x_i + y_i, \alpha; \beta_i \right) - k_i - \omega_i x_i - c_i^x(y_i) - c_i^y(x_i) + c^{y^e\prime}(y^e) y_i - \mu\phi \left(x_i, \alpha; \gamma_i^x \right) -$$

$$- \mu\phi \left(y_i, \alpha; \gamma_i^y \right) + \mu\phi_{y^e} \left(y^e, \alpha; \gamma^{y^e} \right) y_i + \xi_i^\delta - \chi - \lambda_2 y_i = 0, \; i = 1, 2.$$

$$\mu \geq 0, \; \mu \left[z^0 - \int_0^{\overline{\alpha}} \left\{ \sum_{i=1}^{2} \delta_i \left[\phi \left(x_i, \alpha; \gamma_i^x \right) + \phi \left(y_i, \alpha; \gamma_i^y \right) \right] + \phi \left(y^e, \alpha; \gamma^{y^e} \right) \right\} gLd\alpha \right] = 0.$$

Additionally, we assume that the Kuhn-Tucker conditions, as a result of restrictions (LULC), hold.

The Lagrange multiplier μ is interpreted as the shadow price of the prespecified level of pollutant at the receptor. Thus, μ is constant over α since it is not evaluated at a specific location but over the whole interval of the domain. Like the rest of the multipliers, x_i^*, y_i^*, y^*, and δ_i^*, the optimal values of variables x_i, y_i, y, and δ_i, $i = 1, 2$, depend on α.[13]

Spatial environmental policy

In the following proposition we specify policies that would assure the optimal allocation of inputs and land.

Proposition. *(Spatial policies)*
1. Provided that the final emissions can be observed, the first-best solution can be obtained by a Pigouvian tax τ_p on final emissions. The Pigouvian tax is $\tau_p = \mu^*$.
2. Provided that the inputs and technology choices, β_i, γ_i, and γ^{y^e}, can be observed at each location α and for each activity i, an optimal policy can be obtained by
(a) spatially differentiated and crop-specific input taxes $\tau_i^m(\alpha)$ and $\tau_i^o(\alpha)$ on the amount of used inputs x_i and y_i, respectively, equal to

$$\tau_i^m(\alpha) = \mu^* \phi_{x_i} \left(x_i^*, \alpha; \gamma_i^x \right), \; i = 1, 2, \; \alpha \in [0, \overline{\alpha}]$$

$$\tau_i^o(\alpha) = \mu^* \phi_{y_i} \left(y_i^*, \alpha; \gamma_i^y \right), \; i = 1, 2, \; \alpha \in [0, \overline{\alpha}] \; \textit{together with}$$

(b) a spatially differentiated tax $\tau(\alpha)$ on the dumped excess manure y^e given by

$$\tau(\alpha) = \mu^* \phi_{y^e} \left(y^{e*}, \alpha; \gamma^{y^e} \right), \; \alpha \in [0, \overline{\alpha}], \; \textit{and with}$$

[13]We use an asterisk as a superindex of a decision variable or a multiplier to denote its optimal values.

(c) a spatially differentiated and crop-specific land-use tax or subsidy $\sigma_i(\alpha)$ given by

$$\sigma_i(\alpha) = -\tau_i^m(\alpha)x_i^* - \tau_i^o(\alpha)y_i^* + \mu^*\left[\phi\left(x_i^*, \alpha; \gamma_i^x\right) + \phi\left(y_i^*, \alpha; \gamma_i^y\right)\right] \gtreqless 0,\ i = 1, 2,\ \alpha \in [0, \overline{\alpha}].$$

To make notation simpler, we will omit the argument α of taxes and/or subsidies τ_i^m, τ_i^o, τ, and σ_i, $i = 1, 2$ unless necessary to avoid confusions.

Proof. *Part 1*: It is straightforward when comparing the *f.o.c.* of problem (R) of the regional planner with the *f.o.c.* of the farmers' decision problems analyzed below when introducing a Pigouvian tax.
Each farmer cultivating at location α will choose his optimal strategy solving the following problem:[14]

$$\max_{\{x_i, y_i, y, \delta_i\}_{i=1,2}} gL\left[\sum_{i=1}^{2}\delta_i\left\{p_i\varphi(x_i + y_i, \alpha; \beta_i) - k_i - \omega_i x_i - c_i^x(x_i) - c_i^y(y_i)\right\} + \right.$$

$$+h(y) - z(y) - c^{y^e}\left(y - \sum_{i=1}^{2}\delta_i y_i\right) -$$

$$-\tau^P\left(\sum_{i=1}^{2}\delta_i\left[\phi\left(x_i, \alpha; \gamma_i^x\right) + \phi\left(y_i, \alpha; \gamma_i^y\right)\right] + \phi\left(y - \sum_{i=1}^{2}\delta_i y_i, \alpha; \gamma^{y^e}\right)\right) +$$

$$\left. + \sum_{i=1}^{2}\left[\xi_i^x x_i + \xi_i^y y_i + \xi_i^\delta \delta_i\right] + \xi^y y + \chi\left(1 - \sum_{i=1}^{2}\delta_i\right) + \lambda_2\left(y - \sum_{i=1}^{2}\delta_i y_i\right)\right].$$

Being $\widetilde{\mathcal{L}}$ the Lagrangian of this maximization problem, the first order conditions are the following:

$$\widetilde{\mathcal{L}}_{x_i} = \delta_i\left(p_i\varphi'\left(x_i + y_i, \alpha; \beta_i\right) - \omega_i - c_i^{x\prime}(x_i) - \tau^P\phi_{x_i}\left(x_i, \alpha; \gamma_i^x\right)\right) + \xi_i^x = 0,\ i = 1, 2.$$

$$\widetilde{\mathcal{L}}_{y_i} = \delta_i\left(p_i\varphi'\left(x_i + y_i, \alpha; \beta_i\right) - c_i^{y\prime}(y_i) + c^{y^e\prime}(y^e) - \tau^P\phi_{y_i}\left(y_i, \alpha; \gamma_i^y\right) + \right.$$

$$\left. + \tau^P\phi_{y^e}\left(y^e, \alpha; \gamma_i^{y^e}\right) - \lambda_2\right) + \xi_i^y = 0,\ i = 1, 2.$$

[14] For sake of simplicity, we suppress the Lagrange multipliers ξ_i^x, ξ_i^y, ξ_i^δ, ξ^y, λ_2, and χ, for $i = 1, 2$, as decision variables.

$$\widetilde{\mathcal{L}}_y = h'(y) - z'(y) - c^{y^e\prime}(y^e) - \tau^P \phi_{y^e}\left(y^e, \alpha; \gamma_i^{y^e}\right) + \xi^y + \lambda_2 = 0.$$

$$\widetilde{\mathcal{L}}_{\delta_i} = p_i \varphi\left(x_i + y_i, \alpha; \beta_i\right) - k_i - \omega_i x_i - c_i^x(x_i) - c_i^y(y_i) + c^{y^e\prime}(y^e) y_i -$$
$$- \tau^P \left(\phi\left(x_i, \alpha; \gamma_i^x\right) + \phi\left(y_i, \alpha; \gamma_i^y\right) - \phi_{y^e}\left(y^e, \alpha; \gamma_i^{y^e}\right) y_i\right) + \xi_i^\delta - \chi - \lambda_2 y_i = 0,\ i = 1, 2.$$

Part 2: It is straightforward when comparing the *f.o.c* of problem (R) of the regional planner with the *f.o.c* of the farmers' decision problems analyzed below when introducing input and land-use taxes or subsidies.
Each farmer cultivating at location α will choose the optimal strategy to solve the following problem:[15]

$$\max_{\{x_i, y_i, y, \delta_i\}_{i=1,2}} gL \left[\sum_{i=1}^{2} \{\delta_i \left[p_i \varphi(x_i + y_i, \alpha; \beta_i) - k_i - \omega_i x_i - c_i^x\left(x_i\right) - c_i^y\left(y_i\right) - \right.\right.$$

$$\left. - \tau_i^m x_i - \tau_i^o y_i - \sigma_i \right]\} + h(y) - z(y) - c^{y^e}\left(y - \sum_{i=1}^{2} \delta_i y_i\right) - \tau\left(y - \sum_{i=1}^{2} \delta_i y_i\right) +$$

$$\left. + \sum_{i=1}^{2} \left[\xi_i^x x_i + \xi_i^y y_i + \xi_i^\delta \delta_i\right] + \xi^y y + \chi\left(1 - \sum_{i=1}^{2} \delta_i\right) + \lambda_2\left(y - \sum_{i=1}^{2} \delta_i y_i\right)\right].$$

Being $\widetilde{\mathcal{L}}$ the Lagrangian of this maximization problem, the first order conditions are:

$$\widetilde{\mathcal{L}}_{x_i} = \delta_i \left(p_i \varphi'\left(x_i + y_i, \alpha; \beta_i\right) - \omega_i - c_i^{x\prime}\left(x_i\right) - \tau_i^m\right) + \xi_i^x = 0,\ i = 1, 2.$$
$$\widetilde{\mathcal{L}}_{y_i} = \delta_i \left(p_i \varphi'\left(x_i + y_i, \alpha; \beta_i\right) - c_i^{y\prime}\left(y_i\right) - \tau_i^o + c^{y^e\prime}(y^e) + \tau - \lambda_2\right) + \xi_i^y = 0,\ i = 1, 2.$$
$$\widetilde{\mathcal{L}}_y = h'(y) - z'(y) - c^{y^e\prime}(y^e) - \tau + \xi^y + \lambda_2 = 0.$$

$$\widetilde{\mathcal{L}}_{\delta_i} = p_i \varphi\left(x_i + y_i, \alpha; \beta_i\right) - k_i - \omega_i x_i - c_i^x - c_i^y + c^{y^e\prime}(y^e) y_i - \tau_i^m x_i - \tau_i^o y_i - \sigma_i + \tau y_i +$$
$$+ \xi_i^\delta - \chi - \lambda_2 y_i = 0, i = 1, 2.$$

■

[15] For sake of simplicity, we suppress the Lagrange multipliers ξ_i^x, ξ_i^y, ξ_i^δ, ξ^y, λ_2, and χ, for $i = 1, 2$, as decision variables.

The Pigouvian tax determines the optimal allocation of the use of land and fertilizers. Since the tax is imposed on final emissions at the receptor it is constant through space, not spatially differentiated. However, these policies are not implementable because of the information required. For this reason, alternative instruments have to be considered. In part 2 of this proposition we concentrate on the regulation of input use. However, inputs taxes are not enough to get the regional optimum. The selected taxes have to go along with land-use taxes or subsidies that determines the efficient allocation of land from the social viewpoint.