Decision support for grid-connected renewable energy generators planning

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Abstract

Recent technological advances and the incremental demand for electrical energy are leading a growth in the prevalence of distributed generation. There are some off-the-shelf tools to support grid planners in locating and sizing a given number of Distributed Generators (DGs), but they approach the problem using a single set of the variables (either location, size or number of DGs). This paper reviews the problem and provides a new pathway for supporting grid planning with an integrated view; hence, a new planning problem is formulated to jointly determine how many new DGs are needed, of which type, their location and size, while attempting to maximise the profit of the generators, minimise the system losses and improve the voltage profile. Accompanying the new grid planning problem, solution approaches based on meta-heuristic methods are provided. A detailed performance analysis of the proposed approaches is carried out on 14- and 57-bus systems to illustrate what could be the outcomes of the new problem. In so doing, particle swarm optimisation-based approaches are able to find the best optimised solutions.

Keywords: Distributed generator, location and sizing, smart grid, particle swarm optimisation, genetic algorithm, simulated annealing

1. Introduction

Technological advances have made it possible to install small distributed generators. The installation of new DGs aims to improve the general performance of power grids in addition to obtaining a profit. In other words, installing new DGs, besides being economically profitable, can improve the quality of the voltage of a given grid and reduce power loss due to energy transport and distribution.

However, the potential benefits of installing new DGs depend on making good decisions regarding how many DGs should be installed, which kinds of DGs are the most appropriate (PV generators, wind turbines, fuel cells, etc.), where they should be connected and which generation capacity is the most appropriate. These questions cannot be solved in isolation since they depend on one another, the given grid (topology, load, etc.) and access to the availability of the sources harvested by generators. For example, the benefits of installing a PV generator depend on the

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Table 1: List of acronyms

DG	Distributed Generator
DGLS	Distributed Generation Location & Sizing
GA	Genetic Algorithm
GA+SAacc	Combination of GA and SAacc
GA+SAran	Combination of GA and SAran
LRS	Linear Random Search
PSO	Particle Swarm Optimisation
PSO+SAacc	Combinatin of PSO and SAacc
PSO+SAran	Combination of PSO and SAran
PV	Photovoltaic
SA	Simulated Annealing
SAacc	SA with the accumulating neighbourhood function
SAran	SA with the random replacement neighbourhood function
SFLA-DE	Shuffled Frog Leaping Algorithm - Differential Evolution

solar radiation, the correlation between the load and solar radiation, the bus where the generator is placed, the size of the generator, etc.

Therefore, there is a need for tools and methods that help grid planners to work out the number, type, location and size of new DGs in order to optimise the performance of the power grid (voltage profile and power loss) and the profit. However, due to the complexity of the problem, many efforts have been made to provide approaches to partially solve the problem (i.e. seeking the best locations and sizes of a set of DGs). In this way, this paper is the first attempt to globally tackle the problem taking into account the availability of the resources harvested by the candidate new DGs.

Summing up, the contribution of the paper is twofold: (i) a new formulation of the DGLS problem, which uses a set of types of generators, a grid, a time-dependent load and certain meteorological conditions to jointly determine the location, size, number and most appropriate type of DGs; and (ii) the recommendation of a meta-heuristic method to solve the problem based on the comparison analysis of several different meta-heuristic methods in two IEEE bus configurations.

This paper is organised as follows: first, Section 2 presents some work related to the DGLS problem; Section 3 formalises the problem and presents the formulation used; next, meta-heuristic approaches are described in Section 4, showing how they could be used for solving the posed problem; Section 5 explains the simulations performed in order to analyse the proposed approaches for providing a useful recommendation; finally, the paper ends with Section 6, which establishes the conclusions of the paper and makes suggestions for future work. Table 1 summarises the acronyms used in this paper.

2. Related Work

The literature presents several interesting models to the DGLS problem. These models aim to answer various questions related to the DGLS problem (see Figure 1), i.e., location of new DGs,

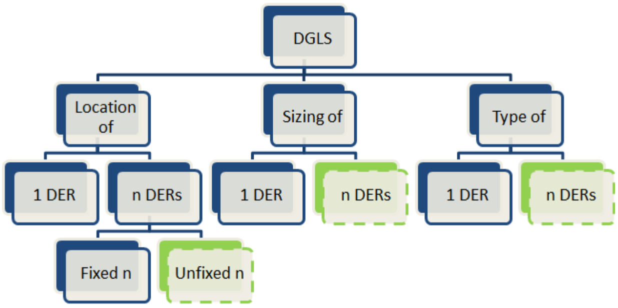

Figure 1: Classification of the questions tackled by the DGLS problem. The questions tackled in this paper are displayed in green.

the most suitable type of DG, etc. In particular, [1] presents a model that seeks to answer the same questions as this paper; that is, the best number, type, location and size of DGs to connect in a given grid with a given load. However, it restricts the type of DGs to small-size combined heat power generators and boilers. Conversely to [1], this paper proposes the placement of DGs, such as PV generators or wind turbines, that harvest renewable resources. Since the considered type of generators and the fuel or energy source are completely different, the formulation of the problem presented in [1] differs greatly to that presented in this paper. For example, the generation capacity of DGs harvesting renewable resources (henceforth renewable DGs) depends on the weather conditions, which are stochastic, while the fuel used by combined heat power generators is a storable energy source. As in this paper, [1] seeks to optimise the costs (or the profit) of installing new DGs and, as the authors do not consider the placement of renewable DGs, the formulation seeks to minimise the CO_2 emissions. This paper, however, does not incorporate the CO_2 emissions in the target function, but includes the power losses throughout the grid and the voltage index in each bus (voltage profile).

Authors in [2] present a model to seek the the most appropriate location, size and type of a given number of generators without restricting the type of DG to combined heat power generators. Instead, renewable DGs (i.e. PV generators) are considered in [2]. As a consequence, the formulation of the problem tackles the problem of the availability of the energy sources (i.e. solar radiation or wind speed) when there is the demand for energy. Accordingly, time-dependent load profiles are considered, as in this paper, and used to evaluate the suitability of the different types of generators. The model proposed in [2] seeks the minimisation of power losses using a mixed integer non-linear programming solver. Conversely to [2], this paper seeks to optimise more objectives (i.e. costs and voltage profile) and also seeks to find the best number of generators. Furthermore, this paper analyses the use of metaheuristic methods to solve the problem. Following the line of [2], authors in [3] also present a model to find the best location, size and type (considering different renewable DG) to minimise power losses and the costs of installing new DGs. Therefore, the work also tackle the problem of the availability of the energy sources. However, they propose the use of GA to solve the problem. Similarly, the authors in [4] present a combination of ant colony optimisation and artificial bee colony to determine the location, size and type of a given number of different DGs in order to optimise power losses, CO_2 emissions, voltage stability and energy costs. In particular, [4] considers gas turbines, fuel cells and wind turbines, but the authors do not perform and hourly analysis of the load and the generation. Instead, they consider constant loads and generation outputs and model the uncertainty of wind turbines production using a given probability density function and they deal with it using a point estimate method. Conversely to the presented paper, [4] limits the number of each type of DG.

The authors in [5] also seek the optimal location, type and size of a given number of DGs. However, they distinguish generators according to whether they can supply either active power or reactive power or both, instead of distinguishing them by the energy source. The authors consider them a constant load and output of the generators instead of hourly profiles and stochastic energy sources. Moreover, the authors propose to solve the problem by iteratively testing various promising locations and types while seeking the size that minimises power loss using PSO. Similarly to [5], the authors in [6] also seek to determine the best location, size and type of a given number of DGs distinguishing the type of DG according to their active and reactive power production. The authors also consider a constant load and output of the generators. Conversely to [5], the authors seek to optimise the voltage profile, power loss and required investment and propose to use the multi-objective GA called NSGA-II. In [7] it is also sought to find the best location, size and type of DGs in order to optimise power loss. The authors also distinguish DG types according to their active and reactive power generation and they consider a constant load and generation.

The best number, location and size of a single type of generator given a power grid and a load is sought in [8] using artificial neural networks. The presented formulation does not take into account meteorological conditions. However, the solutions found answer the interesting question of how many DGs should be placed in a given grid in order to optimise a given criteria, in particular power loss and voltage profile. Similarly, the authors in [9] also seek the best number, location and size of a single type of DG in order to optimise the voltage profile and power loss. But they propose the use of GA to find the best solutions. Conversely to them, this paper also considers the cost and revenue of installing new DGs, which is a key issue when the number of DGs is questioned. Following the same line, [10] also proposes a methodology for finding the best number, locations and sizes of a single type of DG; however, the authors in [10] only consider three possible sizes (5MW, 10MW or 15MW).

Despite these previous works, most of the literature only seeks to determine the location and size of a given number of DGs of the same type. These works then present similar formulations, which differ in the objective function, and their main contribution is the presented method to solve the posed problem. For example, the authors in [11] present a multi-objective PSO to improve the voltage profile, minimise power loss and maximise reliability; in [11], the authors propose a PSO to find the best location and size in order to maximise system loadability while minimising power loss; a similar PSO algorithm called teaching-learning-based optimisation is proposed in [12] to locate and size a given number of DGs to minimise power loss; in [13] an evolutionary algorithm is presented with the objective of minimising power loss, costs and carbon emissions; the authors in [14] present a PSO algorithm to improve voltage profile and stability of the network while minimising the costs of power loss; in [15] the authors propose a GA to locate and size DGs in order to minimise power loss while keeping an acceptable voltage profile; the authors in [16] use non-linear optimisation and GA to locate and size DGs to minimise power loss and investment cost and provide a comparison of the results obtained with the two approaches; and the authors in [17] present an approximate formulation in order to reduce the search space for determining the optimal location and size of a set of generators. Thus, they propose the use of this formulation and numerical methods to find the best solutions instead of metaheuristics methods.

Regarding the methods presented to solve the various DGLS problems, the main trends are the use of artificial intelligence techniques, i.e., metaheuristics or artificial neural networks, which are capable of finding very good, though not optimal, solutions to very complex problems. Within this group of techniques, swarm algorithms such as PSO [18] and evolutionary algorithms such as GA [19] stand out, since most approaches are based on these types of algorithms. This paper presents several metaheuristic approaches to tackle the posed problem in order to provide a comparison of their performance and identify the most appropriate method.

Furthermore, the authors in [20] propose a new analytical formulation that approximates the power flow model to make it simpler and less computing intensive. The authors then propose to find the optimal place and size for a new generator by first performing a sensitivity analysis to find the most appropriate candidate locations to place a new DG and, second, find the size of the placed DG that minimises power loss. Although the authors do not use a metaheuristic method with their formulation, it may be a formulation to take into account when using methods that perform many iterations.

Despite the literature focused on DG location and sizing, some works are focused in the location and sizing of storage units. An interesting example is [21] which proposes a three-stage method to determine the optimal location and capacity of a given number of storage units in order to optimise the congestion in a given system and the investment and operational costs of the storage units. The authors divide the complexity of solving the problem in three phases which are solved using mixed integer programming solvers. First, they determine optimal locations, second, they seek the optimal energy and power ratings of a given number of storage units placed in the locations found in the previous stage, and finally, they optimise the operation of the storage systems given their locations and features in order to analyse the advantages of the final solution.

A more general review of the literature related to DG technologies and DG location and sizing methodologies can be found in [22], where the authors summarise the advantages and disadvantages of the different DG technologies, highlight the advantages of the distributed generation and analyse the literature related to the location and sizing of DGs. The authors mainly provide a classification of the works according to the methodology they use to solve the problem.

3. Problem formulation

The problem this paper deals with aims to find the values of a set of variables (number, location, type and size of DGs in a grid) that achieve the best results for the criteria of the problem (profit, energy losses, voltage profile) while fulfilling the constraints of a given power system. Due to sustainable issues, available DG types include those that harvest renewable sources, what brings about the need of handling the non-controllable availability of these sources, which brings about the need for handling the non-controllable availability of these sources.

The following the section describes the variables, criteria, constraints and input data regarding the power system of the problem.

3.1. Nomenclature

t time index

i bus index

j bus index

k generator type index

$N_{DGtypes}$ number of types of generators

N_{bus} number of buses

$K^t_{i,j}$ power loss factor between bus i and j (MVA^{-1})

$S^t_{i,j}$ apparent power flow from bus i to bus j at time t

$S^{max}_{i,j}$ upper limit for apparent power flow from bus i to bus j

$R_{i,j}$ resistance of line from bus i to j

$Y_{i,j}$ admittance of line from bus i to j

$\theta_{i,j}$ phase angle of $Y_{i,j}$

V^t_i voltage magnitude in bus i at time t

V_i desired voltage magnitude at bus i

V^{min}_i lower limit for voltage of bus i

V^{max}_i upper limit for voltage of bus i

δ^t_i voltage phase in bus i at time t

$P^t_{i,k}$ active power output of generator type k at bus i at time t

$Q^t_{i,k}$ reactive power output of generator type k at bus i at time t

$L^t_{P,i}$ active power demand at bus i at time t

$L^t_{Q,i}$ reactive power demand at bus i at time t

$P^{max}_{i,k}$ upper limit for active power output of generator type k at bus i (production capacity)

$r^t_{i,k,forecast}$ expected resource availability for generator type k at bus i at time t

$C_{i,k}$ generation costs of generator type k at bus i (€/MWh)

$C_{i,k,amortisation}$ amortisation of installing generator type k at bus i (€/MW)

$C_{i,k,maintenance}$ yearly fixed maintenance cost of generator type k at bus i (€/MW)

$SUC_{i,k}$ start up cost of generator type k at bus i (€)

$SDC_{i,k}$ shut down cost of generator type k at bus i (€)

π^t energy price at time t (€/MWh)

ρ^t estimated cost of the energy produced at time t

$\beta^t_{i,k}$ binary decision variable which indicates whether generator at bus i and type k starts up at time t

$\gamma^t_{i,k}$ binary decision variable which indicates whether generator at bus i and type k shuts down at time t

$size^{max}_{i,k}$ maximum size of generator type k at bus i

3.2. Input data

The DGLS problem posed in this paper consists of determining the number, location, type and size of DGs given the following data:

- The available types of DGs with the associate amortisation $C_{i,k,amortisation}$, maintenance $C_{i,k,maintenance}$, production $C_{i,k}$, start up $SUC_{i,k}$ and shut down $SDC_{i,k}$ costs, which may depend on the bus that the DGs are connected to and the maximum allowed capacity $size_{i,k}^{max}$.
- The time-dependent active and reactive load profiles at each bus for each time t, $L_{P,i}^{t}$ and $L_{Q,i}^{t}$.
- The network features such as number of buses N_{bus}, conductivity parameters (i.e. maximum power flow $S_{i,j}^{max}$, resistance $R_{i,j}$, admittance $Y_{i,j}$), voltage limits (V_i^{min} and V_i^{max}), etc.
- The energy selling price π^t for each time t.
- The source availability forecast $r_{i,k,forecast}^{t}$. This is used to compute the generation capacity, at each time t, of DGs such as photovoltaic generators that use a stochastic energy source.

  It is important to highlight that renewable sources are, in general, not controllable and link generators to meteorological conditions. Therefore, weather information must be considered, as well as time-dependent energy price, in the planning of placing new generators.

3.3. Decision variables

The decision variables of the posed problem are as follows: the number, location, type and size of the DGs. Considering that there is one DG per bus i and type k, the decision variables can be represented as the production capacity at each bus and for each type of DG, $P_{i,k}^{max}$. Note that considering a single DG per bus and type is equivalent to the aggregation of a collection of DGs of the same type connected to the same bus.

3.4. Constraints

Generation and load schedule have to fulfil a set of constraints:

- Bus voltages V_i^t must be within their limits

$$V_i^{min} \leq V_i^t \leq V_i^{max}, \quad \forall i \in [1, N_{bus}] \tag{1}$$

- Apparent power flow $\left|S_{i,j}^{t}\right| \leq \left|S_{i,j}^{max}\right|$ between buses i and j cannot exceed line thermal limit for all t

$$\left|S_{i,j}^{t}\right| \leq \left|S_{i,j}^{max}\right|, \quad \forall i,j \in [1, N_{bus}] \tag{2}$$

- Active and reactive power generation, $P_{i,k}^t$ and $Q_{i,k}^t$, must be balanced with active and reactive power demand, $L_{P,i}^t$ and $L_{Q,i}^t$, respectively

$$\sum_{k=1}^{N_{DGtypes}} P_{i,k}^t - L_{P,i}^t = \sum_j V_i^t V_j^t Y_{i,j} \cos\left(\delta_i^t - \delta_j^t - \theta_{i,j}\right), \quad \forall i \in [1, N_{bus}] \tag{3}$$

$$\sum_{k=1}^{N_{DGtypes}} Q_{i,k}^t - L_{Q,i}^t = \sum_j V_i^t V_j^t Y_{i,j} \sin\left(\delta_i^t - \delta_j^t - \theta_{i,j}\right), \quad \forall i \in [1, N_{bus}] \tag{4}$$

- Generation output cannot exceed the maximum power generation of the DG

$$P^t_{i,k} \leq P^{max}_{i,k}, \quad \forall i, k \tag{5}$$

- Power generation cannot exceed the expected generation due to the source availability forecast

$$P^t_{i,k} \leq P^{max}_{i,k} r^t_{forecast,i,k}, \quad \forall i, k \tag{6}$$

Note that the location and size of the DGs affect the maximum power output of distributed generation, Equation (5), and where (which bus) these inject power.

3.5. *Objective function*

The methodology presented in this paper aims to maximise the profits of distributed generation units f_1, minimise system energy loss f_2 and improve the voltage profile f_3.

First, the DGs' profit is the accumulated revenue of each DG for selling energy minus the cost of producing this energy, maintaining the DGs and the amortisation of the DGs.

$$\begin{aligned} f_1 =& \frac{1}{T} \sum_t \left(revenue^t - cost^t \right) - \\ & \sum_{i=1}^{N_{bus}} \sum_{k=1}^{N_{DGtypes}} P^{max}_{i,k} \cdot (C_{i,k,maintenance} + C_{i,k,amortisation}) \end{aligned} \tag{7}$$

where

$$revenue^t = \sum_{i=1}^{N_{bus}} \sum_{k=1}^{N_{DGtypes}} P^t_{k,i} \cdot \pi^t \tag{8}$$

$$cost^t = \sum_{i=1}^{N_{bus}} \sum_{k=1}^{N_{DGtypes}} \left(C_{i,k} \cdot P^t_{i,k} + \beta^t_{i,k} \cdot SUC_{i,k} + \gamma^t_{i,k} \cdot SDC_{i,k} \right) \tag{9}$$

Note that the $revenue^t$ and $cost^t$ do not directly depend on $P^{max}_{i,k}$, but $P^{max}_{i,k}$ $\forall i, k$ will act as limiters of the power production $P^t_{i,k}$ at each time t.

Second, system energy loss is the amount of energy lost in the system lines; it is formulated as follows:

$$f_2 = \sum_t \sum_{i=1}^{N_{bus}} \sum_{j=i+1}^{N_{bus}} K^t_{i,j} \left| S^t_{i,j} \right|^2 \rho^t \tag{10}$$

where the apparent power flow $\left| S^t_{i,j} \right|$ and the power loss factor $K^t_{i,j}$ are defined as:

$$\begin{aligned} \left| S^t_{i,j} \right|^2 =& \cos\left(\delta_i - \delta_j\right) \sum_k \left(P^t_{i,k} P^t_{j,k} + Q^t_{i,k} Q^t_{j,k} \right) + \\ & \sin\left(\delta_i - \delta_j\right) \sum_k \left(Q^t_{i,k} P^t_{j,k} + P^t_{i,k} Q^t_{j,k} \right) \end{aligned} \tag{11}$$

$$K^t_{i,j} = \frac{R_{i,j}}{\left|V^t_i V^t_j\right|} \tag{12}$$

It is important to point out that it is impossible to determine who has produced the energy lost in the system when there are multiple generators and, therefore, its cost. In this sense it is proposed to multiply the energy loss by an estimation of the cost of the energy injected in the system, ρ^t, to estimate the cost of such loss.

Third, to improve the voltage profile (the voltage at each bus at each time t) it is proposed to reduce the mean squared differences between the desired voltage V_i and the obtained voltage V^t_i.

$$f_3 = \frac{1}{T} \sum_{t=1}^{T} \sum_{i=1}^{N_{bus}} \left(V^t_i - V_i\right)^2 \tag{13}$$

However, any other voltage index could be used. See [9] for other indexes for improving voltage quality.

Similarly to $revenue^t$ and $cost^t$, f_2 and f_3 do not directly depend on $P^{max}_{i,k}$; however, the power production is constrained by $P^{max}_{i,k}$ and the voltage at each bus depends on the injection of power in the system.

Once the objective criteria are set, the optimisation problem consists of finding the size, type and location of the DGs that maximise the following objective function:

$$f = w_1 f_1 - w_2 f_2 - w_3 f_3 \tag{14}$$

where w_i is the weight of criterion f_i.

f (henceforth fitness) works as an indicator of the quality of a solution.

As previously stated, the decision variables are $P^{max}_{i,k} \forall i, k$ (where the values represent the size of the DG and the indices identify the type k and location i of the DG). However, the computation of the objective function f involves calculating the power output of each DG ($P^t_{i,k}$) at each time t (see Equations (7), (10) and (13)). The methodology for computing the production schedule $((P^t_{i,k} \forall t)$ is beyond the scope of this paper, despite it being used to calculate the fitness of a solution as explained in the experimental section.

4. Solution approaches

Once the problem is formulated, the next step consists of providing a method to find solutions; that is, finding the size, type, location (the bus that the DGs are connected to) and number of DGs that optimises the aggregation function expressed in Equation (14), given a grid, a load, time-dependent energy prices and weather conditions. This is a complex optimisation problem and the most suitable off-the-shelf methods are metaheuristics-based, which provide approximate solutions in a reasonable amount of time, while not involving many mathematical assumptions about the problem. However, the optimal solution remains unknown; therefore, it is mandatory to conduct a performance analysis of several methods to recommend the one that best fits the problem solving.

In particular, GA and PSO have been chosen because these are the most frequently used methods in the previous work on this topic. These algorithms are based on population of candidate

$$\begin{array}{cc} & N_{DGtypes} \\ N_{bus} & \begin{pmatrix} 0.5 & 0.0 \\ 0.2 & 0.0 \\ 0.0 & 0.0 \end{pmatrix} \end{array}$$

(a)

$$\begin{array}{c} N_{DGtypes} \times N_{bus} \\ \begin{pmatrix} 0.5 & 0.0 & 0.2 & 0.0 & 0.0 & 0.0 \end{pmatrix} \end{array}$$

(b)

Figure 2: (a) Example of a candidate solution of SA or GA with 2 DGs of type 1 of size 0.5MW and 0.2MW located at buses 1 and 2. (b) Example of the same candidate solution of (a) but as a 6-dimensional vector for PSO.

solutions, and finding the fitness (Equation (14)) of each solution is costly. In addition, Simulated Annealing (SA) has also been examined, since it considers a single solution in each iteration, allowing a large number of iterations per simulation. This section explains the particular implementation of these algorithms used to solve the posed problem.

4.1. Simulated annealing

SA [23] is a meta-heuristic algorithm that consists of iteratively improving a candidate solution by moving it around the problem space. To avoid local optima and flat zones, there is a chance it will make some *bad movements* and so not move towards the closest optimum.

SA starts with an initial random solution. Then, at each iteration it selects a *neighbour* solution s' and compares it with the current solution s. If s' is better, SA moves towards it and replaces s with s'. Otherwise, it also can move towards s' (*bad movement*) with a probability $e^{\frac{fitness(s')-fitness(s)}{T}}$ or stay on s with a probability of $1-e^{\frac{fitness(s')-fitness(s)}{T}}$, where T is the temperature of the environment and controls the probability of bad movements. Algorithm 1 shows the procedure of SA.

Algorithm 1 SA

Make an initial candidate solution S
while $T_f \leq T$ **do**
  Select a neighbour solution s'
  if $fitness(s) < fitness(s')$ **then**
    $s \leftarrow s'$
  else
    $x \leftarrow U(0,1)$
    if $x < e^{\frac{fitness(s')-fitness(s)}{T}}$ **then**
      $s \leftarrow s'$
    end if
  end if
  $T \leftarrow T \cdot \delta$
end while

For the particular case of the posed DGLS problem, a matrix gives a candidate solution, whose values indicate the size of the DG connected to the bus, which is indicated by the column index (Figure 2 shows an example). The type of generator is given by the row index. For example, if the value of column i and row k is zero, it indicates that no generator of type k is placed at bus i. Nevertheless, if the value is 0.5, it indicates that a DG of type k and with $P_{i,k}^{max}$ = 0.5MW is placed at bus i.

The fitness of each candidate solution is computed according to Equation (14).

A key point of every SA algorithm is selecting the neighbour solution for a given current one. This paper proposes two strategies for selecting neighbour solutions:

- Accumulation: this strategy consists of adding a random value given by a uniform distribution $U\left(-\frac{SIZE_{i,k}^{max}}{K}, \frac{SIZE_{i,k}^{max}}{K}\right)$ (where $SIZE_{i,k}^{max}$ is the maximum allowed value for $P_{i,k}^{max}$ and K is a real number greater than 1) to each value of the solution matrix for a given probability. The probability of modifying a particular value of the solution matrix is given by the relative generation capacity it represents in respect of the capacity of the whole solution. For example, a particular value a of the solution matrix has a probability of $\frac{a}{A}$ where A is the sum of all the values of the matrix. All values of the matrix must be within its corresponding $\left[0, SIZE_{i,k}^{max}\right]$. This approach is called SAacc.
- Random value: as in the previous strategy, each value of the solution matrix has a probability of being modified that depends on the relative generation capacity of the generator it represents. However, this mechanism modifies the values of the matrix by assigning them new random values within $\left[0, SIZE_{i,k}^{max}\right]$. The probability of 0 is 50% and the other values within $\left(0, SIZE_{i,k}^{max}\right]$ are uniformly distributed. This approach is called SAran.

Both strategies give the same chance to a generator of being modified and it depends on its relative importance. Small generators have more chance of being modified because it is supposed that their modification has fewer implications.

4.2. SA and linear random search

The problem posed in this paper consists of finding the optimal number, place, type and size of DGs in a given grid. It can be seen as a two-step problem consisting, first, of finding the places and types of the generators (which DGs should be placed) and, second, of setting the appropriate size of the DGs given from the first step. In this way, it is proposed to solve the first step through SA because it is a global search technique (bad movements enable it to avoid local optima and flat regions) and because it only needs to compute the fitness of a single solution at each iteration (conversely to GA or PSO, which are population-based algorithms). This point is very important because the fitness function is very time-consuming, requiring the computation of the power flow for each time step; thus it limits the number of times it can be computed. For the second step a linear random search (LRS) algorithm is proposed. It is a local search technique that consists of giving a solution, creating a new one that randomly modifies the size of the DGs (adding a random number to each one, i.e., $U\left(-\frac{SIZE_{i,k}^{max}}{K}, \frac{SIZE_{i,k}^{max}}{K}\right)$) and then moving to the new solution, if it is better than the current one. This algorithm does not need the fitness function to be linear and/or differentiable and, despite being a local search technique when the solution space is not convex, its convergence is very fast, enabling it to calculate good size values for the DGs given by the SA. Furthermore, despite LRS being a local search technique, its combination with SA can be considered a global search technique.

Algorithm 2 shows the combination of SA and LRS to solve the new DGLS problem. First it starts with an initial random solution. Then, iteration after iteration, it creates a neighbour solution s' (made up of 0s and 1s where 1 indicates the presence of a DG and 0 the absence). It then determines the size of the generators using LRS. Finally, it compares the quality of the new solution with the current one. As in SA, if the new solution is better, the algorithm replaces the current solution s with s'; otherwise, the algorithm only replaces s with s' with a probability given by $e^{\frac{fitness(s')-fitness(s)}{T}}$.

Note that this approach is very similar to SAacc, but it tends to perform a more exhaustive local search (in terms of size) for a given set of located generators.

Algorithm 2 SA+LRS

```
Make an initial and binary candidate solution s
while T_f ≤ T do
    Select a neighbour solution s′ (only made up of 0s and 1s)
    s′ ← LRS (s′)
    if fitness(s) < fitness(s′) then
        s ← s′
    else
        x ← U(0, 1)
        if x < e^{(fitness(s′)−fitness(s))/T} then
            s ← s′
        end if
    end if
    T ← T · δ
end while
```

4.3. Genetic algorithms

GA [19] starts from a population of chromosomes and evolves this population using operators inspired by natural evolution. Each chromosome represents a candidate solution. Chromosomes have a set of genes and each one represents a dimension of the problem space. Chromosomes are represented as $N_{DGtypes} \times N_{bus}$ matrices, where $N_{DGtypes}$ is the number of the available distributed generation types (Figure 2a shows an example). Thus, each gene represents a DG whose size is given by the value of the gene. The type and bus of the DG are represented by the row and column indices, respectively.

GA starts creating an initial population of chromosomes (a set of new random candidate solutions) and then it calculates the fitness of each one according to Equation (14). The size of the initial population is POP_{size}. After evaluating the members of the initial population, generation after generation, GA carries out reproduction and elitism to make the population evolve and improve in order to find better chromosomes. Algorithm 3 summarises the procedure of the GA explained and used in this paper.

Algorithm 3 GA

```
Make an initial set of chromosomes
Compute the fitness of each chromosome
for i ← 1 to N_generations do
    Select POP_size couples of parents
    Create a couple of children from each couple of parents using crossover and mutation
    Compute the fitness of each child chromosome
    Apply elitism to old and new individuals to obtain the new population
end for
```

Reproduction consists of three main steps:

- Selection of parents. At each generation the GA selects N_c couples of parents to breed N_c couples of children. The selection of each couple is made using the three tournament selection rule, which consists of randomly selecting three random chromosomes and choosing the best as the first parent of the couple. The process is repeated to select the second parent. This method has been chosen because it tends to keep more diversity than the roulette wheel selection [19].

- Crossover. After selecting the parents, each couple of parents breeds a couple of children exchanging their *genetic information* using the two point crossover [19]. Chromosomes are matrices instead of strings, but for crossover these are treated as strings of genes, whereby each gene represents a bus with all the generators connected to it.

- Mutation. After each new child chromosome is created, it mutates, changing some of the genes of the solution. In particular two mutation operators have been used; the first consists of changing the value of a gene (changing the size of a DG) with a particular probability $mut_{size} = 0.1$. The second consists of interchanging the type of two generators of the same bus with a particular probability $mut_{type} = 0.01$. The mutation values have been selected after testing various combinations.

Elitism is used to keep the size of the population constant generation after generation, removing the worst members of the population. In this regard, it consists of removing all but the POP_{size} best members of the population.

4.4. Particle swarm optimisation

PSO is a swarm computation technique developed by Kennedy and Eberhart in 1995 [18]. Similar to GA, PSO is a population-based optimisation tool. It is inspired by the social behaviour of bird flocking or fish schooling. The members of the population explore the solution space by wandering around it. The moves are apparently random but, at the same time, they tend to wander towards their own best position and the best-known position of the swarm.

Regarding the problem posed in this paper, each individual (particle) in the PSO is composed of three D-dimensional vectors, where $D = N_{bus} \times N_{DGtypes}$ is the dimensionality of the search space (Figure 2b shows an example). These are the current position of the particle $\vec{x}_i$, its past best position $\vec{p}_i$ and the velocity $\vec{v}_i$. The position $\vec{x}_i$ (or the past best position $\vec{p}_i$) of each particle represents a possible solution to the problem of optimisation and the value of each slot j of the position vector corresponds to the size of the DG of type $k = j \pmod{N}_{DGtypes}$ connected to bus $i = \lceil \frac{j}{N_{DGtypes}} \rceil$. Thus, the matrices used in SA and GA are unfolded.

Algorithm 4 summarises the procedure of the algorithm. PSO starts with a group of particles, each having a random initial position $\vec{x}_i$ (step 1), and then it calculates the fitness of the positions of all the particles (step 2), in order to work out the best position of the swarm $\vec{p}_g$. Next, it starts a loop that, at each iteration, updates the previous best position of each particle $\vec{p}_i$ and the best position of the swarm (steps 5-10). It then computes the velocity of each particle $\vec{v}_i$ using $\vec{p}_i$ and $\vec{p}_g$ as attracting points:

$$\vec{v}_i \leftarrow \omega\vec{v}_i + \phi_1 \left(\vec{p}_i - \vec{x}_i\right) + \phi_2 \left(\vec{p}_g - \vec{x}_i\right) \tag{15}$$

where $\omega\vec{v}_i$ can be considered as the inertia of the particle and ω is called *inertia weight*. The term $\phi_1 \left(\vec{p}_i - \vec{x}_i\right) + \phi_2 \left(\vec{p}_g - \vec{x}_i\right)$ can be seen as an external force $\vec{F}_i$ that changes the velocity of the particle. In this regard, the change in a particle's velocity (particle acceleration) can be written as $\Delta\vec{v}_i = \vec{F}_i - (1 - \omega)$ and, therefore, the constant $1 - \omega$ acts as a friction coefficient and ω can be interpreted as the fluidity of the search space. Clerc and Kennedy analysed in [24] the convergence of PSO depending on the parameters ω, ϕ_1 and ϕ_2 and they concluded that $\frac{(\phi_1+\phi_2)}{\omega} > 4$ to ensure convergence. When this constriction method is used, the values are usually set to $\omega = 0.7298$, $\phi_1 = \phi_2 = 1.49618$. These values have also been used in this paper.

The fitness function used to evaluate the particles' positions is, as in SA and GA, Equation (14).

Algorithm 4 PSO

Initialise population with random positions
Compute the fitness of each particle
for $i \leftarrow 1$ **to** $N_{iterations}$ **do**
  for each particle **do**
    if $fitness(\vec{x}_i) > fitness(\vec{p}_i)$ **then**
      $\vec{p}_i \leftarrow \vec{x}_i$
      if $fitness(\vec{x}_i) > fitness(\vec{p}_g)$ **then**
        $\vec{p}_g \leftarrow \vec{x}_i$
      end if
    end if
  end for
  for each particle **do**
    $\vec{v}_i \leftarrow \vec{v}_i \cdot \omega + \Phi_1 (\vec{p}_i - \vec{x}_i) + \Phi_2 (\vec{p}_g - \vec{x}_i)$
    $\vec{x}_i \leftarrow \vec{x}_i + \vec{v}_i$
  end for
  Compute the fitness of each child chromosome
end for
Select the best position as solution

4.5. Combinations of algorithms

PSO and GA are population-based optimisation algorithms. These methods use populations of candidate solutions to perform a global search for the optimum. On the other hand, SA's search is based on improving a single solution instead of a collection. However, it can also be considered a global optimisation tool because it has mechanisms to avoid local optima and flat regions, though PSO and GA perform a much more global search.

Since PSO and GA are population-based algorithms and SA only improves a single solution, it makes sense to use a combination of these algorithms in order to perform, first, a global search using PSO or GA and then to polish such a search using SA, starting from the best solution found by PSO or GA.

This paper also analyses the performance of the combination of GA with SAran and SAacc (GA+SAran and GA+SAacc) and PSO with SAran and SAacc (PSO+SAran and PSO+SAacc) for solving the DGLS problem. The next section presents the analysis of such performances.

5. Application

The previous sections have proposed a formulation of the DGLS problem and some techniques to jointly tackle the various faces of it. This section presents the instantiation of the formulated problem in two concrete cases in order to test the techniques and show the solutions they can find.

The analysis is conducted by a comparison with an alternative state-of-the-art approach presented by [13], which has been proven to achieve very good results in determining the location and size of a set of generators. The work of [13] has been chosen for its similar problem formulation (constraints and target functions), which enables an easy adaptation of the method to our problem formalisation. The results of [13] (adapted to solve the problem posed in this paper) are presented under the label SFLA-DE, as this is the technique used to solve the problem in this previous work.

5.1. Cases

The two different power systems studied are the IEEE 14-bus and 57-bus systems [25]. The systems have been adapted to consider the features of the problem studied in this paper and

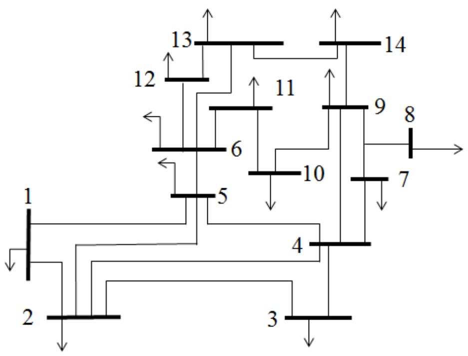

Figure 3: 14-bus diagram from University of Washington. Available at [25].

to simulate the system behaviour during a period of time that incorporates daily and seasonal variations of the load and energy sources. To that end, line data, bus voltage and power limits have been considered, but not the generators present in the system; instead, it has been considered to be a single default generator connected to the slack bus, able to provide sufficient energy to cover the internal demand. Furthermore, time-dependent loads with residential, commercial and industrial profiles have replaced the loads of each bus. The types of generators considered are on-shore wind turbines and photovoltaic generators with their corresponding investment, operation and maintenance costs. Appendix A includes precise information about the loads, DGs and meteorological data. The goal is to find the configuration of DGs (size, type, number) and their localisation, in order to remove the use of the default generator (or minimise its use).

To simulate the power system cases throughout time, it is necessary to define a production schedule for each DG, considering its location, type and size ($P^t_{i,k} \forall t$). The presented approach calculates $P^{max}_{i,k}$ but the scheduling is beyond the scope of this paper. Instead, a naive system has been used to simulate the power system operator. Given $P^{max}_{i,k}$ for all i and k, it calculates a power generation schedule where the load of the grid is proportionally shared with all the generators, considering their availability and physical constraints. If the DGs cannot cover the demand, it injects energy from the *main grid* through the slack bus (bus 1, default generator)[1].

For simplicity and without loss of generality the energy price has been used as the estimation of the cost of the energy produced at each time ($\pi^t = \rho^t$). It is an overestimation of the cost of the energy. Moreover, the size of new DGs has been limited to $SIZE^{max}_{i,k} = 2.0$MW for all i, k, corresponding to a solar farm of approximately 45000 m^2. In the literature, the maximum size of DGs is usually limited to a value between 1MW and 3MW. The final value depends on the case and for the studied cases 2MW is the most appropriate integer value. The types of DGs are constrained to wind and photovoltaic power, since they are the most suitable renewable energy resources to harvest by DGs due to the climate of the region considered. The parameter K of algorithms SAacc and SA+LRS has been set to $K = 4$.

[1]In case of not considering the injection of power from the slack bus, it should be considered the interruptibility of some loads

5.2. *Results*

This section describes the solutions (DGs to be installed) found, for each case, by each technique with the corresponding values of the three considered objectives (power loss, voltage profile and profit).

5.2.1. *14-bus system*

Table 2 shows the location, type and size of the DGs of a solution found by each method for the 14-bus system and the value of the three components of the fitness function (f_1 profit, f_2 power losses, f_3 voltage profiles). The methods that identified the best solutions are in bold face. It can be seen that the preferred DG type is PV generator (27 DGs of 37 and 12 of the 12 DGs of the best solutions), which was expected as wind speeds greater than 4m/s are very scarce (see Figure A.7). The best solutions propose to place two DGs, one with a capacity of around 1MW, and a second with a capacity of around 0.5MW. All of these solutions propose to place the bigger DG at bus 5, which is one of the most centric buses. On the other hand, the second DG is placed on buses 1 or 2. Despite this disagreement between the best solutions, buses 1 and 2 are next to each other. In this regard, good solutions are very similar (same type, same sizes, same number of DGs and practically the same locations). Although the best solutions are very similar, the other solutions (those found by SA-based approaches and SFLA-DE) are very different, which indicates the incapacity of these methods to appropriately solve the problem.

5.2.2. *57-bus system*

Regarding the 57-bus system, Table 3 provides information about the number, types, size and localisation of DGs, as well as the fitness values of a solution found by PSO+SAran, the best found solution. According to them, the total capacity to install is about 8MW distributed throughout six buses. It is worth pointing out the predominance of installing 200kW to 2MW DGs, instead of many small DGs. This trend appears in the solutions of both systems. In addition, the preferred type of DG is also PV.

5.3. *Performance analysis of the proposed algorithms*

This subsection presents the results obtained by the presented approaches for the 14-bus and 57-bus system with a discussion of their performances. The results are based on the values obtained in the objective function Equation (14) and compared with SFLA-DE performance. It is noteworthy that optimal solutions are not known by meta-heuristics. Hence the comparison between algorithms is done through the fitness (the higher the better) of the solutions found along a set of runs. The fitness of the solutions found is represented through box-plots. However, it is difficult to achieve a conclusion due to the variance of the solutions. Thus, pair-response tests (which examine the distributions of the difference between the performances on the same problem values) were performed to accurately compare the results of each algorithm.

5.3.1. *14-bus system*

Figure 4 shows the box plot of the solutions' fitness achieved by the algorithms presented in this paper and the SFLA-DE method presented by [13] regarding the IEEE 14-bus system. Table 4 shows the results obtained after conducting paired-response tests where 1s indicate that the corresponding row algorithm outperforms the column algorithm. According to the results, the reader can conclude that the worst option for solving the DGLS problem is the use of SA with a neighbour function that assigns random values (SAran). The performance can be improved by

Table 2: Best distributed generation installation found by each method for 14-bus system. Methods with the best solutions are in bold face.

Method	f_1	f_2	f_3	Bus no.	DG type	DG size (MW)
GA	18414.13	413.37	1.98	1	PV	0.440
				5	PV	1.117
GA+SA ran.	18411.13	417.74	1.78	1	PV	0.502
				5	PV	1.038
SA ran.	16694.97	670.72	1.67	1	PV	0.399
				1	Wind	0.213
				4	PV	0.311
				4	Wind	0.465
				5	PV	0.331
				5	Wind	0.802
				6	PV	0.222
				6	Wind	0.332
				7	PV	0.232
				10	PV	0.370
				12	Wind	1.083
				13	Wind	0.178
GA+SA acc.	18429.19	409.62	2.05	2	PV	0.496
				5	PV	1.000
SA acc.	17947.94	467.22	2.31	1	PV	0.685
				5	PV	0.927
				6	PV	0.487
SA+LRS	17217.83	493.58	1.31	1	PV	1.041
				5	PV	0.540
				7	PV	0.163
				3	Wind	0.227
				4	Wind	0.463
				5	Wind	0.617
PSO+SA ran.	18449.97	409.98	1.87	2	PV	0.436
				5	PV	1.065
PSO+SA acc.	18440.42	415.90	1.85	2	PV	0.466
				5	PV	1.002
PSO	18441.19	413.98	1.94	2	PV	0.450
				5	PV	0.960
SFLA-DE	17787.13	409.44	2.28	1	PV	0.198
				5	PV	0.882
				6	Wind	0.110
				7	PV	0.417

Table 3: Distributed generation installation found by PSO+SAran for the 57-bus system.

f_1	f_2	f_3	DG type	Bus no.	DG size (MW)
68758.71	5974.05	1.05	PV	2	1.61
				3	0.49
				15	2.0
				17	2.0
			Wind	43	0.06
				53	2.0

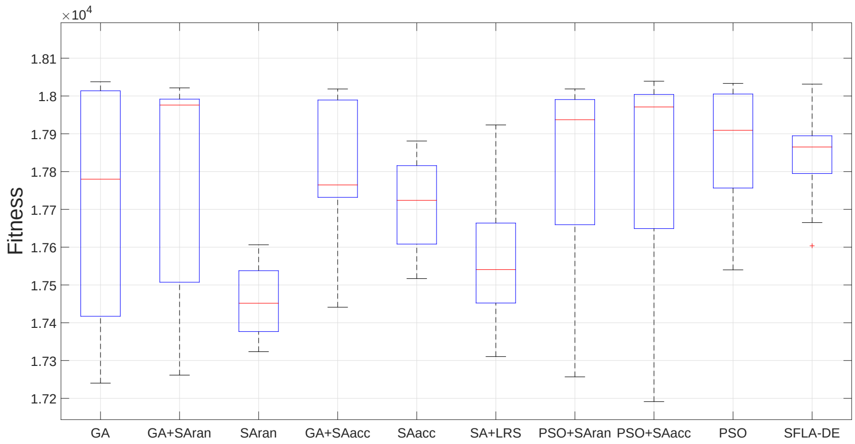

Figure 4: Fitness of the solutions found by different meta-heuristic algorithms in the 14-bus. Box plot over 50 solutions of each case.

combining SA with LRS; however, SA with the accumulative neighbour function (SAacc) outperforms SAran and SA+LRS. Thus, the combination of SA and LRS does not achieve a great improvement and the performance of SAran is usually worse than SAacc, even when combined with other algorithms (i.e. GA+SAacc is better than GA+SAran). Another conclusion drawn from Figure 4 is that the use of SAacc combined with another algorithm (PSO or GA) reduces the variance of the solutions. In the particular case of GA+SAacc, it can be seen that it outperforms GA. Furthermore, approaches that combine PSO with SA outperform other algorithms, according to Table 4, compared with PSO alone. However, paired-response tests are not able to determine which algorithm is the best; there is no single algorithm that outperforms all others. There is a trend indicating that combining GA or PSO with SA improves the performance, but also indicating that using SA after either PSO or GA improves the performance more than using only SA. Thus, it seems a good point to start the search using a population-based algorithm, such as GA or PSO, and then refine it using SA. This statement is also supported by the results achieved by the state-of-the-art hybrid method called SFLA-DE. It obtains better results than SA-based approaches. However, there is a dead heat between the population-based methods regarding the solving of the 14-bus DGLS problem (a *small* DGLS problem).

Another point from Table 4 is that paired-response tests are not able to distinguish any difference between the results from the three approaches using PSO. Even an ANOVA analysis between the results of these three approaches concludes that it cannot be assumed that the results come from different distributions with a p-value of 0.9246. Such a p-value tells us that with a probability of 92.46%, the results obtained by the three approaches come from the same distribution. Thus, if the three approaches share the use of PSO, it means that the PSO dominates over SA regarding the final solutions found.

5.3.2. 57-bus system

Regarding the fitness, Figure 5 shows the box plot of the solutions found by all the approaches. Apparently, it can be seen that the best results are obtained by approaches that use PSO, followed

Table 4: Paired-response tests from the 14-bus and 57-bus system (14-bus|57-bus). 1 indicates that the row algorithm obtains better solutions than the column algorithm. 0 indicates it cannot be assumed that the row algorithm is better than the column algorithm.

<table>
<tr><th></th><th>GA</th><th>GA+SAran.</th><th>SAran.</th><th>GA+SAacc.</th><th>SAacc.</th><th>SA+LRS</th><th>PSO+SAran.</th><th>PSO+SAacc.</th><th>PSO</th><th>SFLA-DE</th></tr>
<tr><td>GA</td><td>-|-</td><td>0|0</td><td>1|1</td><td>0|1</td><td>0|1</td><td>1|1</td><td>0|0</td><td>0|0</td><td>0|0</td><td>0|1</td></tr>
<tr><td>GA+SA ran.</td><td>0|0</td><td>-|-</td><td>1|1</td><td>0|0</td><td>0|1</td><td>1|1</td><td>0|0</td><td>0|0</td><td>0|0</td><td>0|1</td></tr>
<tr><td>SA ran.</td><td>0|0</td><td>0|0</td><td>-|-</td><td>0|0</td><td>0|0</td><td>0|0</td><td>0|0</td><td>0|0</td><td>0|0</td><td>0|0</td></tr>
<tr><td>GA+SA acc.</td><td>1|0</td><td>1|0</td><td>1|1</td><td>-|-</td><td>1|1</td><td>1|1</td><td>0|0</td><td>0|0</td><td>0|0</td><td>0|1</td></tr>
<tr><td>SA acc.</td><td>0|0</td><td>0|0</td><td>1|1</td><td>0|0</td><td>-|-</td><td>1|1</td><td>0|0</td><td>0|0</td><td>0|0</td><td>0|0</td></tr>
<tr><td>SA+LRS</td><td>0|0</td><td>0|0</td><td>1|1</td><td>0|0</td><td>0|0</td><td>-|-</td><td>0|0</td><td>0|0</td><td>0|0</td><td>0|0</td></tr>
<tr><td>PSO+SA ran.</td><td>1|0</td><td>1|1</td><td>1|1</td><td>0|1</td><td>0|1</td><td>1|1</td><td>-|-</td><td>0|0</td><td>0|0</td><td>0|1</td></tr>
<tr><td>PSO+SA acc.</td><td>1|1</td><td>1|1</td><td>1|1</td><td>0|1</td><td>1|1</td><td>1|1</td><td>0|0</td><td>-|-</td><td>0|0</td><td>0|1</td></tr>
<tr><td>PSO</td><td>1|0</td><td>0|1</td><td>1|1</td><td>0|1</td><td>0|1</td><td>1|1</td><td>0|0</td><td>0|0</td><td>-|-</td><td>0|1</td></tr>
<tr><td>SFLA-DE</td><td>0|0</td><td>0|0</td><td>1|1</td><td>0|0</td><td>1|1</td><td>1|1</td><td>0|0</td><td>0|0</td><td>0|0</td><td>-|-</td></tr>
</table>

by those using GA. Figures 4 and 5 show that SAran is the worst approach for 14-bus and 57-bus cases. This mean that the use of an SA approach that determines the size of the DGs by adding/subtracting random values to the current sizes of the generators obtains better results than simply selecting new random sizes. That is consistent with the fact that PSO-based approaches obtain slightly better solutions than GA-based approaches.

Furthermore, it can be seen that SA+LRS outperforms SAran but not SAacc meaning it is better to use a good SA algorithm than combine it with LRS.

Comparing the presented methods with SFLA-DE, Figure 5 shows that GA and PSO based approaches outperform SFLA-DE, which is also supported by the pair-response test results of Table 4. However, SFLA-DE outperforms all SA-based approaches when solving the 57-bus DGLS problems, compared with the 14-bus problems. The fact that GA- and PSO-based approaches outperform SFLA-DE for the 57-bus problems and not for the 14-bus problems (where there is a tie) supports the conclusion that PSO and GA approaches are more appropriate for solving very complex problems. This is related to the results of [13], where the authors compare the SFLA-DE method with PSO and GA approaches to determine the location and size of a set of generators. The authors conclude that SFLA-DE obtains better results; however, the problem presented in this paper is more complex (including the number of generators and type) and GA and PSO methods outperform SFLA-DE when resolving large instances of the DGLS problem (57-bus problems and not 14-bus problems).

Table 4 shows that PSO+SAacc obtains better results than all other approaches, except those of PSO+SAran and PSO, from which it cannot be assumed that one is better than the others. Specifically, an ANOVA analysis of the results obtained by these three approaches returns a p-value of 0.5771, meaning that it could not be assumed that the results are from different distributions. Therefore, the use of any such techniques is equivalent for building a decision support tool for aiding grid planners.

5.3.3. Summary

Figure 6 shows a graph where a line $p \longrightarrow q$ indicates that algorithm p outperforms algorithm q (i.e., $SAran \longrightarrow SA + LRS$ means that SA+LRS outperforms SAran). This means that, according to Table 4, algorithm p outperforms q in both cases, or at least in one case and, in the other it is not outperformed by q. A dotted line between two algorithms p and q states that either p does not outperform q in any case and q does not outperform q, or p outperforms q in one case and in the other, q outperforms p, meaning that it cannot be assumed that one of the algorithms is better than the other.

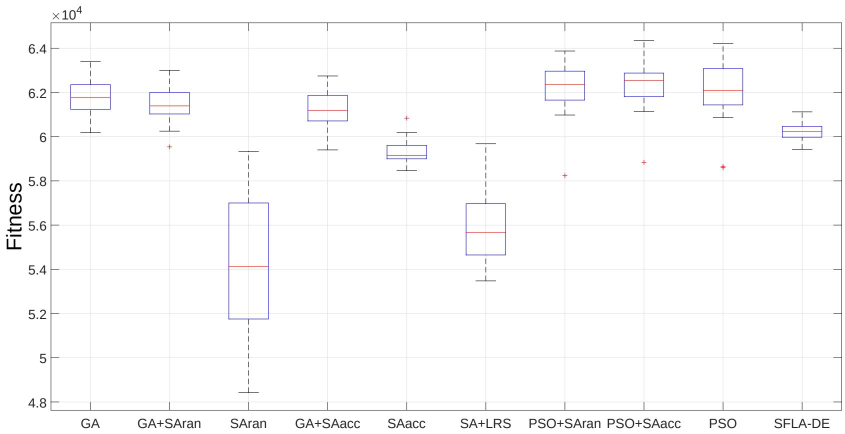

Figure 5: Fitness of the solutions found by different meta-heuristic algorithms in the 57-bus. Box plot over 50 solutions of each case.

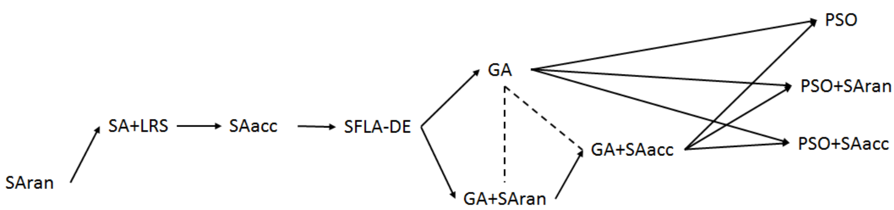

Figure 6: Graph indicating which algorithms outperform others. A line $p \longrightarrow q$ indicates that algorithm q outperforms p, and a dotted line between two algorithms states that it cannot be said that one algorithm outperforms the other.

According to Figure 6 it can be said that population-based algorithms, such as GA, PSO and SFLA-DE, obtain better solutions for the DGLS problem than SA, which is an algorithm based on improving a single candidate solution. It can also be seen that PSO-based approaches obtain better results than GA-based approaches and approaches using SAacc usually obtain better results than those using SAran. These results support the conclusion that those algorithms that perform the search modifying the candidate solution in a *continuous* way (PSO modifies the position of each particle according to a velocity that depends on its past velocity and two gravity centres and SAacc modifies the DGs' size of the candidate solution adding/subtracting small random values) obtain better results than those presenting more *discontinuities* in the search process. The finding that SA+LRS outperforms SAran also reinforces this conclusion. Moreover, the fact that PSO and GA approaches outperform SFLA-DE for the 57-bus problems, but not the 14-bus problems, leads to the conclusion that GA and PSO are more appropriate for very complex DGLS problems and that other methods, such as SFLA-DE, may be more appropriate for simpler problems.

5.4. Discussion

The results have shown that the problem formulated in the paper regarding the determination of the number of renewable DGs, type, size and location can be applied to very well-known power systems, such as 14-bus and 57-bus systems [25]. Furthermore, the results showed that, because of the cost of amortisation, it is better to place a few DGs of a significant size (from 200kW to 2MW), instead of placing many small DGs distributed throughout all buses. In addition, the preferred type is PV due to the scarcity of strong wind (more than 4m/s). However, other available DG types, weather conditions or load profiles will change this trend. Thus, there is a need for powerful optimisation methods, which enable us to solve the DGLS problem for each specific case.

Besides comparing the proposed approaches to solve the DGLS problem in this paper, it is worth pointing out that the DGLS problem is a very complex one, especially when there are so many variables to determine (location of the generators, type, size, etc.). It is also very time-consuming because of the need to evaluate the power flow of each candidate solution, which at the same time is not an easy problem. However, it has been demonstrated that it can be tackled using meta-heuristic algorithms. In particular, a decision support system based on a PSO+SAacc approach could provide recommendations to grid planners, as shown in Table 2.

6. Conclusions

Grid planning for new generators offers pathways to grid planners to meet evolving electric system needs. Accordingly, the way that planners decide how many distributed generators they need, where these are located and their size and type, is known as the DGLS problem. Due to sustainable social concerns, DGs harvesting renewable sources are of particular interest. However, their availability is linked to stochastic conditions (meteorological forecast uncertainties). Therefore, electrical engineers are facing a complex decision making process, and the research community is working towards providing support tools for this process.

The main contribution of this paper is a methodology for jointly tackling the problem of determining the number, location, type and size of DGs, whilst considering also the availability of the (renewable) sources that DGs harvest. Moreover, the time-dependent tariffs and optimisation goals, such as power loss, voltage profile quality and generator profits are also considered, as

Table A.5: Generators' costs considering an amortisation horizon of 10 years for PV DG and 20 for Wind turbines. Information from Open Energy Information (OpenEI).

	Investment (1000×€/MW)	Fixed maintenance cost (€/MW year)	Variable operating cost (€/MWh)
Wind	1570	11000	6.45
PV	2550	32000	0.00

in previous works. Therefore, this paper presents a formalisation of the DGLS problem with a broader scope than previous works in the literature.

As a consequence, the complexity of the problem to be handled increases. The second contribution of the paper consists of showing that the posed problem can be solved using current meta-heuristic techniques. In particular, experiments have been carried out using the IEEE 14- and 57-bus systems. The results show that the analysed algorithms provide optimal number of DGs, their locations, types and sizes in order to improve grid performance (voltage profile and power loss), whilst maximising the profitability of placing new DGs. The results show the kind of recommendations that engineers can obtain by formulating the problem as in this paper and using the solution approaches facilitated herein.

Among all the methods analysed, particle swarm optimisation-based approaches have been experimentally proven to be the best algorithms, especially with large instances of the problem.

The new formulation of the DGLS problem presented, however, linearly aggregates all optimisation criteria (power losses, voltage profiles and generator profits). In a future work, Pareto solutions could be considered by approaching the formulation of the problem following a multi-optimisation perspective. On the other hand, the complexity of the problem relies on the computation of the power flow. Thus, it remains open to achieve new ways to simplify the calculation of the to power flow, without losing the required rigour.

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Appendix A. System data information

This section completes the data information about the cases considered in Section 5. Table A.5 presents the investment, operation and maintenance costs of the DGs considered. Figure A.7 shows the wind and solar radiation profiles. The data have been extracted from the Catalan Meteorological Service[2]. The same wind speed probability distribution was considered for all buses and five different solar radiation profiles assigned to buses according to $i \mod 5$ where i is the bus number. Figure A.7 also shows the conversion curve from wind speed to the relative energy production and the three load profiles considered. This latter information is provided in with Table A.6, which shows the magnitude of each load profile considered in each bus.

[2]Catalan Meteorological Service: http://www.meteo.cat

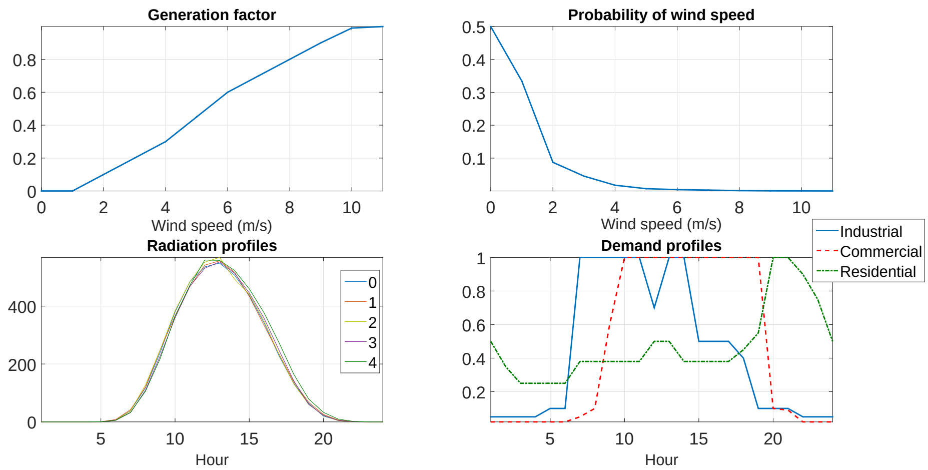

Figure A.7: Wind energy generation curve, probability of wind speed, daily radiation profiles and daily load profiles

Table A.6: Buses load profiles.

i mod 7	Industrial (MW)	Commercial (MW)	Residential (MW)
1	0.8	0.8	0.8
2	0.8	1.2	0.95
3	1.5	1.5	1.5
4	0.8	0.8	0.8
5	0.5	1.5	0.7
6	0.8	0.8	0.8
7	1.0	0.5	1.5

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